Improved ADMM-Based Algorithm for Multi-Group Multicast Beamforming in Large-Scale Antenna Systems

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***Abstract*—In this paper, we consider beamformer design for multi-group multicasting where a common message is transmitted to the users in each group. We propose a novel effective alternating direction method of multipliers (ADMM) formulation in order to reduce the computational complexity of the existing state-of-the-art algorithm for multi-group multicast beamforming with per-antenna power constraints. The proposed approach is advantageous for the scenarios where the dimension of the channel matrix is less than the number of antennas at the base station. This case is always valid when the number of users is less than that of antennas, which is a practical situation in massive-MIMO systems. Simulation results show that the proposed method performs the same with significantly less computational time compared to the benchmark algorithm.**

***Index Terms*—Multi-group multicast beamforming, ADMM, large arrays.**

I. Introduction

Over the last decade, physical layer multicasting using beamforming has become an important research area [1]-[5]. Multi-group multicast beamforming where distinct common information signals are sent to multiple multicast groups is first considered in [1] and later it is applied in different scenarios [2]-[5]. Since the beamforming optimization is a non-convex quadratically constrained quadratic programming (QCQP) problem, the global optimum solution may not be easily found. Recently, in [6], a general algorithmic framework based on alternating direction method of multipliers (ADMM) is proposed for QCQP problems. ADMM is known as a powerful first-order method [7] and considered in other several works [4], [8], [9]. Although the algorithm in [6] has superior performance, it requires large number of auxiliary variables. In [4], the authors proposed a special ADMM reformulation for multi-group multicast beamforming problem with per-antenna power constraints. This effective approach is shown to maintain the same performance as existing benchmarks with a reduced complexity. Hence, it is the current state-of-the-art solution to the addressed problem and selected as the benchmark algorithm.

In this paper, we propose a novel ADMM algorithm which has a lower computational time compared to the one in [4]. Our method is advantageous when the dimension of the subspace spanned by the channel vectors of the users is less than the number of antennas. In fact, this condition corresponds to a very practical scenario in 5G where massive number of antennas are used [10]. Besides, other case may result an infeasible optimization problem due to severe interference.

The proposed method is based on a new reformulation of the problem such that ADMM iterations are carried through lower dimensional vectors. For this, the original beamformer vectors are decomposed into the subspace of the channels and its nullspace. The nullspace is only used in per-antenna power updates. The steps of the new algorithm admit optimum closed-form solutions. Furthermore, some rearrangements are made in the steps and variables to further reduce its computational complexity. Secondly, we tackle the original non-convex problem directly instead of using both inner and outer iterations as in [4]. ADMM is a powerful method and can be applied safely for non-convex problems [6], [8], [9]. It also can be shown that the ADMM algorithm converges KKT point of the non-convex problem under certain conditions [6], [9]. Applying ADMM directly results a simplified algorithm with reduced computational complexity. Simulation results show that the same performance can be obtained by the proposed algorithm with a significantly less computational time.

II. System Model and Problem Formulation

We consider a multicasting system comprising a base station (BS) equipped with N transmit antennas and M multicast groups of single-antenna users. The BS transmits a common multicast message to the users in each group. Let $\mathcal{G}_m$ denote the m^{th} multicast group of users for all $m \in \mathcal{M} = \{1, ..., M\}$ and assume that there are K users in total. Each user is in only one multicast group, i.e., $\mathcal{G}_m \bigcap \mathcal{G}_{m'} = \emptyset$ for $m \neq m'$, $\forall m, m' \in \mathcal{M}$. Narrowband block-fading channel is considered. The signal transmitted from the antenna array of BS is $\mathbf{x} = \sum_{m=1}^{M} \mathbf{w}_m s_m$ where s_m is the information signal for the users in $\mathcal{G}_m$ and $\mathbf{w}_m$ is the corresponding $N \times 1$ complex beamformer weight vector for the m^{th} multicast group. It is assumed that the information signals $\{s_m\}_{m=1}^{M}$ are mutually uncorrelated each with zero mean and unit variance, $\sigma_{s_m}^2 = 1$. In this case, the total transmitted power is $P_{tot} = \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{w}_m$. The received signal at the k^{th} user is given as,

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad \forall k \in \mathcal{K} \tag{1}$$

where $\mathbf{h}_k$ is the $N \times 1$ complex channel vector between BS and the k^{th} user. $\mathcal{K} = \{1, ..., K\}$ is the index set of all

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the users. n_k is the additive zero mean Gaussian noise at the k^{th} user's antenna with variance σ_k^2. n_k is assumed to be uncorrelated with the information signals. The received signal-to-interference-plus-noise ratio (SINR) of the k^{th} user is expressed as,

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{w}_{m_k}|^2}{\sum_{m' \neq m_k} |\mathbf{h}_k^H \mathbf{w}_{m'}|^2 + \sigma_k^2}, \quad \forall k \in \mathcal{K} \tag{2}$$

where m_k denotes the index of multicast group to which the k^{th} user belongs. In this paper, we consider quality-of-service (QoS)-aware beamformer design where the aim is to minimize the total transmitted power subject to receive-SINR and per-antenna power constraints. The QoS-aware design problem can be formulated as follows,

$$\min_{\{\mathbf{w}_m\}_{m=1}^M} \sum_{m=1}^M \mathbf{w}_m^H \mathbf{w}_m \tag{3a}$$

$$s.t. \quad \frac{|\mathbf{h}_k^H \mathbf{w}_{m_k}|^2}{\sum_{m' \neq m_k} |\mathbf{h}_k^H \mathbf{w}_{m'}|^2 + \sigma_k^2} \geq \gamma_k, \quad \forall k \in \mathcal{K} \tag{3b}$$

$$\sum_{m=1}^M |w_{m,n}|^2 \leq P_n, \; \forall n \in \mathcal{N} \tag{3c}$$

where γ_k is the minimum required SINR for the k^{th} user and P_n is the maximum allowable power at the n^{th} transmit antenna of BS. $w_{m,n}$ is the n^{th} element of the vector $\mathbf{w}_m$ and $\mathcal{N} = \{1, ..., N\}$ is the index set for all the transmit antennas. The problem in (3) is not convex and hence should be handled appropriately for an effective and fast solution. Recently, an efficient ADMM-based algorithm is proposed for general QCQP problems by using consensus optimization and decomposing the original problem into QCQP subproblems with only one constraint [6]. Later in [4], an improved technique is proposed for multi-group multicasting problem in (3). As stated in [4], one of the main disadvantages of consensus-ADMM algorithm in [6] is that it requires a local copy of the optimization variables and a corresponding dual vector variable for each constraint. In [4], a new ADMM framework which requires less auxiliary variables is proposed by introducing $\{\{\Gamma_{k,m} = \mathbf{h}_k^H \mathbf{w}_m\}_{k=1}^K\}_{m=1}^M$ and expressing the SINR constraints in (3b) in terms of them. This new ADMM is applied for a sequence of convex subproblems obtained by convex-concave procedure (CCP). The method in [4] performs significantly better compared to [6] with less computational complexity. In this paper, we reduce the computational complexity more by an effective reformulation of the problem. Our new algorithm directly deals with the original problem instead of solving a sequence of subproblems which requires both inner and outer loop iterations as in [4]. It is shown that the efficiency is improved significantly in terms of computational saving.

III. Improved ADMM-Based Algorithm for (3)

Note that all the ADMM updates are carried through $N \times 1$ vectors for the algorithm in [6]. Similarly, $N \times 1$ vectors are used for the update of the main variables and per-antenna power constraints in [4]. When the number of antennas, N, is very large, these updates become extremely costly due to matrix inversions and multiplications. In this paper, we reduce the complexity of the ADMM iterations by decomposing beamformer vectors into the subspace spanned by the channel vectors and its nullspace. For this method to be efficient, it is required that the dimension of the subspace of the channel vectors to be less than N. Let $\mathbf{H}$ denote the $N \times K$ matrix which is formed by stacking all the channel vectors $\mathbf{h}_k$, $\forall k \in \mathcal{K}$, as its columns, i.e., $\mathbf{H} = [\mathbf{h}_1 \; \mathbf{h}_2 \; ... \; \mathbf{h}_K]$. If L denotes the dimension of the column space of $\mathbf{H}$, there are two possible cases for $L < N$. In case the number of antennas, N, is greater than the number of users, K, L is always less than N. This is a very practical scenario in modern wireless communications which involves massive antenna systems. For the second case, i.e., $N < K$, L may not be less than N. However, it is possible for the scenarios where some users are clustered in close groups. In such a case, the corresponding channel vectors are highly correlated and the rank of $\mathbf{H}$ gets smaller. Now, let us consider the singular value decomposition of $\mathbf{H}$ as follows,

$$\mathbf{H} = \begin{bmatrix} \mathbf{U}_A & \mathbf{U}_B \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_B \end{bmatrix} \begin{bmatrix} \mathbf{V}_A^H \\ \mathbf{V}_B^H \end{bmatrix} \tag{4}$$

where $\mathbf{\Sigma}_A$ and $\mathbf{\Sigma}_B$ are the diagonal matrices whose elements are the positive and zero singular values of $\mathbf{H}$, respectively. Let us express $\{\mathbf{w}_m\}_{m=1}^M$ as $\mathbf{w}_m = \mathbf{U}_A \mathbf{v}_{A,m} + \mathbf{v}_{B,m}$ where $\mathbf{v}_{A,m} \in \mathbb{C}^L$ and $\mathbf{v}_{B,m} \in \mathbb{C}^N$ for $m \in \mathcal{M}$ are the newly introduced auxiliary variables. $\mathbf{v}_{B,m}$ is in the nullspace of $\mathbf{U}_A$, i.e., $\mathbf{U}_A^H \mathbf{v}_{B,m} = \mathbf{0}$. The optimization problem in (3) can be reformulated as follows,

$$\min_{\{\mathbf{w}_m, \mathbf{v}_{A,m}, \mathbf{v}_{B,m}\}_{m=1}^M} \sum_{m=1}^M \mathbf{w}_m^H \mathbf{w}_m \tag{5a}$$

$$s.t. \quad \frac{|(\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m_k}|^2}{\sum_{m' \neq m_k} |(\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m'}|^2 + \sigma_k^2} \geq \gamma_k, \quad \forall k \in \mathcal{K} \tag{5b}$$

$$\mathbf{w}_m = \mathbf{U}_A \mathbf{v}_{A,m} + \mathbf{v}_{B,m}, \; \forall m \in \mathcal{M} \tag{5c}$$

$$\mathbf{U}_A^H \mathbf{v}_{B,m} = \mathbf{0}, \; \forall m \in \mathcal{M} \tag{5d}$$

$$\sum_{m=1}^M |w_{m,n}|^2 \leq P_n, \; \forall n \in \mathcal{N} \tag{5e}$$

where $(\mathbf{\Sigma}_A \mathbf{V}_A^H)_k$ denotes the k^{th} column of $\mathbf{\Sigma}_A \mathbf{V}_A^H$. In order to make the problem in (5) appropriate for ADMM algorithm, we will define additional auxiliary variables $\Gamma_{k,m} \triangleq (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m}$, $\forall k \in \mathcal{K}, \forall m \in \mathcal{M}$ using the same approach in [4]. In addition, we introduce $\tilde{\mathbf{v}}_{A,m} \triangleq \mathbf{U}_A \mathbf{v}_{A,m}$ and $\tilde{\mathbf{v}}_{B,m} \triangleq \mathbf{v}_{B,m}$. The significance of these definitions will be explained later. Using the new variables, the problem in (5) can be expressed as follows,

$$\min_{\substack{\{\mathbf{v}_{A,m}, \tilde{\mathbf{v}}_{A,m}, \mathbf{v}_{B,m}, \tilde{\mathbf{v}}_{B,m}, \\ \{\Gamma_{k,m}\}_{k=1}^K\}_{m=1}^M}} \sum_{m=1}^M (\tilde{\mathbf{v}}_{A,m}^H \tilde{\mathbf{v}}_{A,m} + \tilde{\mathbf{v}}_{B,m}^H \tilde{\mathbf{v}}_{B,m}) \tag{6a}$$

$$s.t. \quad \Gamma_{k,m} = (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m}, \; \forall k \in \mathcal{K}, \forall m \in \mathcal{M} \tag{6b}$$

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$$\frac{|\Gamma_{k,m_k}|^2}{\sum_{m' \neq m_k} |\Gamma_{k,m'}|^2 + \sigma_k^2} \geq \gamma_k, \quad \forall k \in \mathcal{K} \tag{6c}$$

$$\tilde{\mathbf{v}}_{A,m} = \mathbf{U}_A \mathbf{v}_{A,m}, \ \forall m \in \mathcal{M} \tag{6d}$$

$$\tilde{\mathbf{v}}_{B,m} = \mathbf{v}_{B,m}, \ \forall m \in \mathcal{M} \tag{6e}$$

$$\mathbf{U}_A^H \tilde{\mathbf{v}}_{B,m} = \mathbf{0}, \ \forall m \in \mathcal{M} \tag{6f}$$

$$\mathbf{U}_B^H \tilde{\mathbf{v}}_{A,m} = \mathbf{0}, \ \forall m \in \mathcal{M} \tag{6g}$$

$$\sum_{m=1}^{M} |\tilde{v}_{A,m,n} + \tilde{v}_{B,m,n}|^2 \leq P_n, \ \forall n \in \mathcal{N}. \tag{6h}$$

Note that the constraint in (6g) is redundant. However, the inclusion of it will simplify the updates in ADMM algorithm. Similar to [4], the variables in (6) can be split into two blocks, $\{\mathbf{v}_{A,m}, \mathbf{v}_{B,m}\}_{m=1}^M$ and $\{\{\Gamma_{k,m}\}_{k=1}^K, \tilde{\mathbf{v}}_{A,m}, \tilde{\mathbf{v}}_{B,m}\}_{m=1}^M$ such that the updates of ADMM algorithm are separable. Now, the steps of ADMM algorithm for the problem (6) in scaled-form [7] can be given as follows,

$$\begin{aligned} \{\Gamma_{k,m}\}_{m=1}^M \leftarrow \underset{\{\Gamma_{k,m}\}_{m=1}^M}{\arg\min} \sum_{m=1}^{M} |\Gamma_{k,m} - & \\ (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m} + \lambda_{k,m}|^2 & \\ s.t. \ \ |\Gamma_{k,m_k}|^2 \geq \gamma_k \sum_{m' \neq m_k} |\Gamma_{k,m'}|^2 + \gamma_k \sigma_k^2 & \\ \forall k \in \mathcal{K} & \end{aligned} \tag{7a}$$

$$\begin{aligned} \{\tilde{\mathbf{v}}_{A,m}, \tilde{\mathbf{v}}_{B,m}\}_{m=1}^M \leftarrow \underset{\{\tilde{\mathbf{v}}_{A,m}, \tilde{\mathbf{v}}_{B,m}\}_{m=1}^M}{\arg\min} \sum_{m=1}^{M} \Big(\tilde{\mathbf{v}}_{A,m}^H \tilde{\mathbf{v}}_{A,m} & \\ + \tilde{\mathbf{v}}_{B,m}^H \tilde{\mathbf{v}}_{B,m} + \rho ||\tilde{\mathbf{v}}_{A,m} - \mathbf{U}_A \mathbf{v}_{A,m} + \mathbf{z}_{A,m}||^2 & \\ + \rho ||\tilde{\mathbf{v}}_{B,m} - \mathbf{v}_{B,m} + \mathbf{z}_{B,m}||^2 \Big) & \\ s.t. \ \sum_{m=1}^{M} |\tilde{v}_{A,m,n} + \tilde{v}_{B,m,n}|^2 \leq P_n, \ \forall n \in \mathcal{N} & \\ \mathbf{U}_A^H \tilde{\mathbf{v}}_{B,m} = \mathbf{0}, \quad \mathbf{U}_B^H \tilde{\mathbf{v}}_{A,m} = \mathbf{0}, \ \forall m \in \mathcal{M} & \end{aligned} \tag{7b}$$

$$\begin{aligned} \mathbf{v}_{A,m} \leftarrow \underset{\mathbf{v}_{A,m}}{\arg\min} \sum_{k=1}^{K} |\Gamma_{k,m} - (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m} + \lambda_{k,m}|^2 & \\ + ||\tilde{\mathbf{v}}_{A,m} - \mathbf{U}_A \mathbf{v}_{A,m} + \mathbf{z}_{A,m}||^2, \qquad \forall m \in \mathcal{M} & \end{aligned} \tag{7c}$$

$$\mathbf{v}_{B,m} \leftarrow \underset{\mathbf{v}_{B,m}}{\arg\min} ||\tilde{\mathbf{v}}_{B,m} - \mathbf{v}_{B,m} + \mathbf{z}_{B,m}||^2, \ \forall m \in \mathcal{M} \tag{7d}$$

$$\begin{aligned} \lambda_{k,m} \leftarrow \lambda_{k,m} + \Gamma_{k,m} - (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m}, & \\ \forall k \in \mathcal{K}, \ \forall m \in \mathcal{M} & \end{aligned} \tag{7e}$$

$$\mathbf{z}_{A,m} \leftarrow \mathbf{z}_{A,m} + \tilde{\mathbf{v}}_{A,m} - \mathbf{U}_A \mathbf{v}_{A,m}, \qquad \forall m \in \mathcal{M} \tag{7f}$$

$$\mathbf{z}_{B,m} \leftarrow \mathbf{z}_{B,m} + \tilde{\mathbf{v}}_{B,m} - \mathbf{v}_{B,m}, \qquad \forall m \in \mathcal{M} \tag{7g}$$

where $\{\{\lambda_{k,m}\}_{k=1}^K\}_{m=1}^M$, $\{\mathbf{z}_{A,m}\}_{m=1}^M$ and $\{\mathbf{z}_{B,m}\}_{m=1}^M$ are the scaled dual variables corresponding to the equality constraints in (6b), (6d), and (6e), respectively. $\rho > 0$ is the penalty parameter used in augmented Lagrangian [6], [7]. In the following, we will present the closed form expressions for the updates in (7a-d), respectively.

In [4], the solution of the optimization problem (7a) is found as follows,

$$\Gamma_{k,m_k} \leftarrow \begin{cases} \zeta_{k,m_k} & \text{if } \phi_k(0) \geq 0 \\ \frac{\zeta_{k,m_k}}{1-\mu_k^\star} & \text{if } \phi_k(0) < 0 \end{cases} \tag{8a}$$

$$\Gamma_{k,m'} \leftarrow \begin{cases} \zeta_{k,m'} & \text{if } \phi_k(0) \geq 0 \\ \frac{\zeta_{k,m'}}{1+\gamma_k \mu_k^\star} & \text{if } \phi_k(0) < 0 \end{cases}, \ \forall m' \neq m_k, \tag{8b}$$

$$\forall k \in \mathcal{K}$$

where $\zeta_{k,m} \triangleq (\mathbf{\Sigma}_A \mathbf{V}_A^H)_k^H \mathbf{v}_{A,m} - \lambda_{k,m}$ is defined for ease of notation. In (8a-b), $\phi_k(\mu) = \frac{|\zeta_{k,m_k}|^2}{(1-\mu)^2} - \gamma_k \sum_{m' \neq m_k} \frac{|\zeta_{k,m'}|^2}{(1+\gamma_k \mu)^2} - \gamma_k \sigma_k^2$ and $\mu_k^\star$ is the unique solution of $\phi_k(\mu) = 0$ in $0 < \mu < 1$ in case $\phi_k(0) < 0$. Note that $\mu_k^\star$ can easily be found by solving a quartic equation.

Now, let us consider the optimization problem in (7b). In order to simplify (7b), let us assume that $\mathbf{z}_{A,m}$ and $\mathbf{z}_{B,m}$ are initialized such that they lie in the column space of $\mathbf{U}_A$ and $\mathbf{U}_B$, respectively without loss of generality. Following (7f-g), they continue to remain in the same subspaces if they are initialized in this way. Assume also that initial value of $\mathbf{v}_{B,m}$ is selected from the column space of $\mathbf{U}_B$ in accordance with the constraints (6e-f). In this case, (7b) can be expressed as follows,

$$\begin{aligned} \min_{\{\tilde{\mathbf{v}}_{A,m}, \tilde{\mathbf{v}}_{B,m}\}_{m=1}^M} \sum_{m=1}^{M} (\tilde{\mathbf{v}}_{A,m} + \tilde{\mathbf{v}}_{B,m})^H (\tilde{\mathbf{v}}_{A,m} + \tilde{\mathbf{v}}_{B,m}) & \\ + \rho ||\tilde{\mathbf{v}}_{A,m} + \tilde{\mathbf{v}}_{B,m} - (\mathbf{U}_A \mathbf{v}_{A,m} - \mathbf{z}_{A,m} + \mathbf{v}_{B,m} - \mathbf{z}_{B,m})||^2 & \end{aligned} \tag{9a}$$

$$s.t. \ \sum_{m=1}^{M} |\tilde{v}_{A,m,n} + \tilde{v}_{B,m,n}|^2 \leq P_n, \ \forall n \in \mathcal{N} \tag{9b}$$

$$\mathbf{U}_A^H \tilde{\mathbf{v}}_{B,m} = \mathbf{0}, \quad \mathbf{U}_B^H \tilde{\mathbf{v}}_{A,m} = \mathbf{0}, \ \forall m \in \mathcal{M} \tag{9c}$$

Now, let us define $\tilde{\mathbf{w}}_m \triangleq \tilde{\mathbf{v}}_{A,m} + \tilde{\mathbf{v}}_{B,m}$. If we further define $\tilde{\mathbf{z}}_m \triangleq \mathbf{U}_A \mathbf{v}_{A,m} - \mathbf{z}_{A,m} + \mathbf{v}_{B,m} - \mathbf{z}_{B,m}$ for ease of notation, the objective function in (9a) can be expressed as $(1+\rho)||\tilde{\mathbf{w}}_m - \frac{\rho}{1+\rho}\tilde{\mathbf{z}}_m||^2 + \frac{\rho}{1+\rho}||\tilde{\mathbf{z}}_m||^2$. The second term is constant and can be removed. Note that $\tilde{\mathbf{v}}_{A,m}$ and $\tilde{\mathbf{v}}_{B,m}$ lie in the column space of $\mathbf{U}_A$ and $\mathbf{U}_B$, respectively. Hence, they can be expressed in terms of new variables as $\tilde{\mathbf{v}}_{A,m} = \mathbf{U}_A \boldsymbol{\upsilon}_{A,m}$ and $\tilde{\mathbf{v}}_{B,m} = \mathbf{U}_B \boldsymbol{\upsilon}_{B,m}$. Using these variables, (9) can be reformulated as follows,

$$\min_{\{\tilde{\mathbf{w}}_m, \boldsymbol{\upsilon}_{A,m}, \boldsymbol{\upsilon}_{B,m}\}_{m=1}^M} \sum_{m=1}^{M} \left|\left|\tilde{\mathbf{w}}_m - \frac{\rho}{1+\rho}\tilde{\mathbf{z}}_m\right|\right|^2 \tag{10a}$$

$$s.t. \ \sum_{m=1}^{M} |\tilde{w}_{m,n}|^2 \leq P_n, \ \forall n \in \mathcal{N} \tag{10b}$$

$$\boldsymbol{\upsilon}_{A,m} = \mathbf{U}_A^H \tilde{\mathbf{w}}_m, \quad \boldsymbol{\upsilon}_{B,m} = \mathbf{U}_B^H \tilde{\mathbf{w}}_m, \ \forall m \in \mathcal{M} \tag{10c}$$

In the formulation (10), it is clearly seen that (10c) does not have any affect on both the objective function and the other constraints in (10b). Hence, the optimum solution is found by solving (10a-b). (10c) is used to obtain the optimum $\{\tilde{\mathbf{v}}_{A,m}, \tilde{\mathbf{v}}_{B,m}\}_{m=1}^M$.

Note that the problem (10a-b) can be decomposed into N subproblems. If we define $\hat{\mathbf{w}}^n \triangleq [\ \tilde{w}_{1,n}\ \tilde{w}_{2,n}\ ...\ \tilde{w}_{M,n}\ ]^T$ and $\hat{\mathbf{z}}^n \triangleq \frac{\rho}{1+\rho}[\ \tilde{z}_{1,n}\ \tilde{z}_{2,n}\ ...\ \tilde{z}_{M,n}\ ]^T$, $\forall n \in \mathcal{N}$, the n^{th} subproblem is given as follows,

$$\min_{\hat{\mathbf{w}}^n} \left|\left|\hat{\mathbf{w}}^n - \hat{\mathbf{z}}^n\right|\right|^2 \quad (11a)$$

$$s.t.\ \ ||\hat{\mathbf{w}}^n||^2 \leq P_n. \quad (11b)$$

Following [4], the optimum solution of (11) is given by $\hat{\mathbf{w}}^n = \min\left\{\frac{\sqrt{P_n}}{||\hat{\mathbf{z}}^n||_2}, 1\right\}\hat{\mathbf{z}}^n$. Using this and (10c), the optimum update in (7b) is given as,

$$\hat{\mathbf{w}}^n \leftarrow \min\left\{\frac{\sqrt{P_n}}{||\hat{\mathbf{z}}^n||_2}, 1\right\}\hat{\mathbf{z}}^n,\ \forall n \in \mathcal{N} \quad (12a)$$

$$\tilde{\mathbf{w}}_m \leftarrow [\ \hat{w}_m^1\ \hat{w}_m^2\ ...\ \hat{w}_m^N\ ]^T,\ \forall m \in \mathcal{M} \quad (12b)$$

$$\tilde{\mathbf{v}}_{A,m} \leftarrow \mathbf{U}_A\mathbf{U}_A^H\tilde{\mathbf{w}}_m,\ \forall m \in \mathcal{M} \quad (12c)$$

$$\tilde{\mathbf{v}}_{B,m} \leftarrow \mathbf{U}_B\mathbf{U}_B^H\tilde{\mathbf{w}}_m,\ \forall m \in \mathcal{M} \quad (12d)$$

Note that defining auxiliary variables $\tilde{\mathbf{v}}_{A,m}$ in (6d) and $\tilde{\mathbf{v}}_{B,m}$ in (6e), $\forall m \in \mathcal{M}$ resulted the Euclidean projection problem in (11) whose closed-form optimum solution exists.

The update in (7c) can easily be expressed as follows,

$$\mathbf{v}_{A,m} \leftarrow \left(\mathbf{I}_L + \mathbf{\Sigma}_A^2\right)^{-1}\Big(\mathbf{U}_A^H(\tilde{\mathbf{v}}_{A,m} + \mathbf{z}_{A,m}) + \sum_{k=1}^{K}(\mathbf{\Sigma}_A\mathbf{V}_A^H)_k(\Gamma_{k,m} + \lambda_{k,m})\Big),\ \ \forall m \in \mathcal{M} \quad (13)$$

Note that matrix inverse in (13) is computationally efficient since the matrix inside the inverse operation is diagonal unlike its counterpart in [4]. Similarly, the update in (7d) is given as follows,

$$\mathbf{v}_{B,m} \leftarrow \tilde{\mathbf{v}}_{B,m} + \mathbf{z}_{B,m},\ \ \forall m \in \mathcal{M} \quad (14)$$

At this point, all the steps of ADMM algorithm are expressed in closed-form. In the following part, we will arrange the algorithm variables in order to reduce its computational complexity.

First, let us consider the dual variable update in (7f). Here, $\mathbf{z}_{A,m}$ is a $N \times 1$ complex vector. In fact, it is possible to carry out the update through a low dimensional dual vector. Let us define $\mathbf{u}_m \triangleq \mathbf{U}_A^H\mathbf{z}_{A,m}$ $\forall m \in \mathcal{M}$. Remember that $\mathbf{z}_{A,m}$ lies in the column space of $\mathbf{U}_A$ if it is initialized properly. Hence, we can write $\mathbf{z}_{A,m} = \mathbf{U}_A\mathbf{u}_m$, $\forall m \in \mathcal{M}$. Using this and (12c), the update in (7f) becomes

$$\mathbf{u}_m \leftarrow \mathbf{u}_m + \mathbf{U}_A^H\tilde{\mathbf{w}}_m - \mathbf{v}_{A,m},\qquad \forall m \in \mathcal{M} \quad (15)$$

Using the newly introduced dual variable, the update in (13) can be expressed as follows,

$$\mathbf{v}_{A,m} \leftarrow \left(\mathbf{I}_L + \mathbf{\Sigma}_A^2\right)^{-1}\Big(\mathbf{U}_A^H\tilde{\mathbf{w}}_m + \mathbf{u}_m + \sum_{k=1}^{K}(\mathbf{\Sigma}_A\mathbf{V}_A^H)_k(\Gamma_{k,m} + \lambda_{k,m})\Big),\ \ \forall m \in \mathcal{M} \quad (16)$$

Now, we can easily see that there is no need to compute $\tilde{\mathbf{v}}_{A,m}$. Furthermore, as we show in the following part, there is also no need for the dual variable $\mathbf{z}_{B,m}$ in the iterations. Suppose $\mathbf{z}_{B,m}^0$ is the initial value of the dual variable $\mathbf{z}_{B,m}$. Then, we obtain $\mathbf{v}_{B,m}^1 = \tilde{\mathbf{v}}_{B,m}^1 + \mathbf{z}_{B,m}^0$ in the first iteration by (14). After that, $\mathbf{z}_{B,m}$ is updated by (7g) as $\mathbf{z}_{B,m}^1 = \mathbf{z}_{B,m}^0 + \tilde{\mathbf{v}}_{B,m}^1 - \mathbf{v}_{B,m}^1 = \mathbf{z}_{B,m}^0 + \tilde{\mathbf{v}}_{B,m}^1 - (\tilde{\mathbf{v}}_{B,m}^1 + \mathbf{z}_{B,m}^0) = 0$. In the first iteration, $\mathbf{z}_{B,m}$ becomes 0 and it continues in this way. Hence, we can omit this dual variable in the algorithm. Now, the simplified steps of the ADMM algorithm are given below. Note that neither $\mathbf{v}_{B,m}$ nor $\tilde{\mathbf{v}}_{B,m}$ are kept in memory. Instead, $\tilde{\mathbf{w}}_m^j - \mathbf{U}_A\mathbf{U}_A^H\tilde{\mathbf{w}}_m^j$ is used in place of $\mathbf{v}_{B,m}^j$ in (17c). The number of dual complex variables in the counterpart algorithm in [4] is $M(N+K)$ whereas it is $M(L+K)$ in the proposed one as can be seen in (17f-g). Furthermore, in case $N \geq K$, the number of complex multiplications is approximately $M(2NK + \min(N^2, 2NK))$ per ADMM iteration in [4]. Here, it is $M(2NL + 2KL + L)$ which is usually smaller due to $L \leq \min(N, K)$. When the number of antennas is relatively large compared to that of users, $L \ll N$, and the proposed algorithm becomes significantly efficient in computational complexity.

Algorithm 1: ADMM for the Problem (6)

Initialization: Initialize $\tilde{\mathbf{w}}_m^0 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, $\mathbf{v}_{A,m}^0 = \mathbf{U}_A^H\tilde{\mathbf{w}}_m^0$, $\lambda_{k,m}^0 \leftarrow 0$, $\forall k \in \mathcal{K}$, $\mathbf{u}_m^0 \leftarrow \mathbf{0}$, $\forall m \in \mathcal{M}$. Set the iteration number $j \leftarrow 0$ and the penalty parameter ρ.
Repeat

$$\Gamma_{k,m_k}^{j+1} \leftarrow \begin{cases} \zeta_{k,m_k}^j & \text{if } \phi_k^j(0) \geq 0 \\ \frac{\zeta_{k,m_k}^j}{1-\mu_k^\star} & \text{if } \phi_k^j(0) < 0 \end{cases} \quad (17a)$$

$$\Gamma_{k,m'}^{j+1} \leftarrow \begin{cases} \zeta_{k,m'}^j & \text{if } \phi_k^j(0) \geq 0 \\ \frac{\zeta_{k,m'}^j}{1+\gamma_k\mu_k^\star} & \text{if } \phi_k^j(0) < 0 \end{cases},\ \forall m' \neq m_k, \quad (17b)$$

$\forall k \in \mathcal{K}$

$$\tilde{\mathbf{z}}_m^{j+1} \leftarrow \mathbf{U}_A(\mathbf{v}_{A,m}^j - \mathbf{u}_m^j) + \tilde{\mathbf{w}}_m^j - \mathbf{U}_A\mathbf{U}_A^H\tilde{\mathbf{w}}_m^j \quad (17c)$$

$$(\hat{\mathbf{w}}^n)^{j+1} \leftarrow \min\left\{\frac{\sqrt{P_n}}{||(\hat{\mathbf{z}}^n)^{j+1}||_2}, 1\right\}(\hat{\mathbf{z}}^n)^{j+1},\ \forall n \in \mathcal{N} \quad (17d)$$

$$\mathbf{v}_{A,m}^{j+1} \leftarrow \left(\mathbf{I}_L + \mathbf{\Sigma}_A^2\right)^{-1}\Big(\mathbf{U}_A^H\tilde{\mathbf{w}}_m^{j+1} + \mathbf{u}_m^j + \sum_{k=1}^{K}(\mathbf{\Sigma}_A\mathbf{V}_A^H)_k(\Gamma_{k,m}^{j+1} + \lambda_{k,m}^j)\Big),\ \ \forall m \in \mathcal{M} \quad (17e)$$

$$\lambda_{k,m}^{j+1} \leftarrow \lambda_{k,m}^j + \Gamma_{k,m}^{j+1} - (\mathbf{\Sigma}_A\mathbf{V}_A^H)_k^H\mathbf{v}_{A,m}^{j+1},\quad \forall k \in \mathcal{K},\ \forall m \in \mathcal{M} \quad (17f)$$

$$\mathbf{u}_m^{j+1} \leftarrow \mathbf{u}_m^j + \mathbf{U}_A^H\tilde{\mathbf{w}}_m^{j+1} - \mathbf{v}_{A,m}^{j+1},\qquad \forall m \in \mathcal{M} \quad (17g)$$

Set $j \leftarrow j+1$.
Until convergence criterion is met.

IV. Simulation Results

The number of antennas is set as $N = 100$ and power limit for each antenna is $P_n = -10$ dBW throughout the simula-

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tions. The channel vectors for all the users are assumed to be independent and zero-mean unit variance complex Gaussian vectors. The noise variance is $\sigma_k^2 = 1, \forall k$. The target SNR for each user is $\gamma_k = 10$ dB. Note that an initialization procedure is employed by iterating ADMM steps for a feasibility problem without considering the objective function in (6a). A similar implementation is also done in [4]. The penalty parameter for both the proposed method and the one in [4] is selected as $\rho = 0.2$. In the figures, each point presents the results for the average of randomly generated 100 channels and PM stands for the proposed method. We compare PM with the benchmark algorithm in [4].

In Fig. 1, the number of users per multicast group is kept constant at $K/M = 10$ and the number of multicast groups, M, is varied. The left side of the y-axis represents transmit power in dBW while the right side is for computational time of the ADMM algorithms in seconds. As shown in Fig. 1, the transmit power is nearly the same for both methods. However, PM is significantly efficient in terms of computational time. The gap between two methods increases dramatically with M and PM reduces the complexity by 11 fold for $M = 6$.

In the second experiment, the number of multicast groups is set as $M = 5$ and the number of users per multicast group is changed from $K/M = 8$ to $K/M = 13$. Similarly, the y axes represents the transmit power and computational time, respectively in Fig. 2. Although the gap is small, PM results less transmit power compared [4]. Again, PM provides a significant amount of computational saving approaching 18 fold decrease when $K/M = 13$.

From both figures, it is observed that PM provides at least the same performance in terms of transmit power while it is computationally more efficient. In particular, the reduction in computational time is striking when the problem size increases.

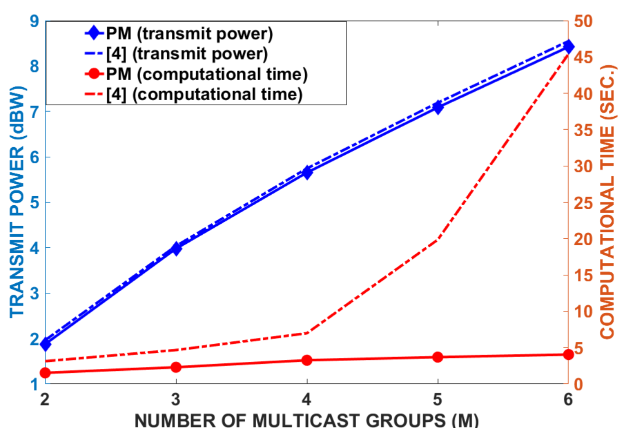

Fig. 1. Transmit power and computational time versus number of multicast groups, M for $K/M = 10$.

V. Conclusion

We propose a novel ADMM based algorithm for the multi-group multicast beamforming problem with per-antenna power constraints. This new ADMM form decomposes the vector variables into smaller size by exploiting the fact that only a lower dimension subspace of design space is required for SINR updates. Furthermore, it takes the advantage of subproblems with each of them having optimum closed-form solution. After presenting the steps of the ADMM algorithm, we make some arrangements for the updates and variables to further reduce the computational complexity. The proposed method provides significantly less computational time especially when the number of multicast groups and users increases.

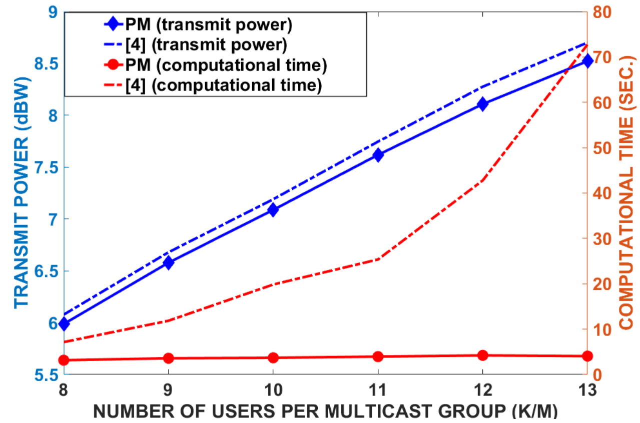

Fig. 2. Transmit power and computational time versus number of users per multicast group, K/M for $M = 5$.

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