Modeling Time of Arrival Probability Distribution and TDOA Bias in Acoustic Emission Testing

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***Abstract*—Acoustic emission testing is widely used by industry to detect and localize faults in structures, but estimated source positions often show significant bias in real tests as a consequence of Time Difference of Arrival (TDOA) bias. In this work, a model for TDOA bias is developed considering the time of arrival was estimated using the fixed threshold algorithm, as well as theoretical upper and lower bounds for it. In addition, we derive the time of arrival probability distribution function in terms of the noise distribution and acoustic emission waveform for the fixed threshold algorithm, showing that, contrary to usual practice, it in general cannot be well approximated by a Gaussian distribution.**

I. Introduction

Acoustic emission testing is a non-destructive testing method used to detect several kinds of faults in structures such as piping, bridges and aerospace structures. Its main advantages over other non-destructive testing methods are the high sensibility, the wide coverage region and the possibility of monitoring structures in real time [1]–[6]. Elastic waves that propagate through the medium are detected by acoustic emission sensors, and as the signal is acquired at each sensor its time of arrival is estimated. If the wave velocity at the medium is known and at least three sensors are used, localization algorithms can estimate the fault position in a surface based on the time difference of arrival (TDOA) estimates, which is defined as the difference between the estimated times of arrival at two different sensors. [7]–[13]. However, localization methods usually provide estimated positions that may have large variance and bias. For this reason, several authors have been working in new localization methods [14]–[19].

In real tests, the time of arrival estimates are subject to uncertainties deriving from signal distortion and noise. While some authors estimate time of arrival applying the fixed threshold method to simulated or real signals [14]–[16], [20], others assume the time of arrival is Gaussian-distributed instead of processing the received signals [11], [17]–[19]. To the best of our knowledge, no study about the time of arrival distribution expression has been published yet. Knowing the time of arrival pdf expression would allow the development of new statistical source position estimators that may have better performance than traditional source localization methods. For this reason, in this work we derive the time of arrival pdf considering noise, sampling and waveform distortion, and show it is not Gaussian distributed. We also deduce an approximated and simplified expression for the pdf that depends on less parameters, being easier to apply in practical situations than the exact distribution.

The fixed threshold method is a popular detection algorithm where the time of arrival is estimated as the first time the received signal absolute value crosses a fixed threshold [20]. However, sampling rate, attenuation and envelope modulation due to sensor frequency response can add bias to estimated Time Difference of Arrival (TDOA) and consequently to estimated position, since the position is estimated using TDOA measurements. It is interesting to reduce TDOA bias because it may lead to an estimated source position bias reduction, and thus a smaller localization error. In [19], a bias reducing algorithm for TDOA-based localization is developed, but TDOA itself was assumed unbiased and Gaussian distributed. Our second contribution is to derive an approximated model for the TDOA bias for the fixed threshold algorithm considering our more realistic time of arrival model, as well as lower and upper bounds for it.

Notation: $\mathbb{E}\{\cdot\}$ denotes the expected value, $\mathbb{P}(A)$ is the probability of A, the operator $*$ represents convolution, $\Gamma(\cdot)$ is the Gamma function and $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ are respectively the floor and ceil operators.

II. Time of arrival probability distribution

The objective of this section is to derive an expression for the probability distribution of the time of arrival considering it was estimated using a fixed threshold, as well as an approximated closed-form expression for it. The threshold value is considered to be set high enough so that the noise has approximately zero probability of crossing it.

A. Signal Model

We consider an acoustic source located at a deterministic position $\mathbf{X} = (x_s, y_s)$ emitting an elastic wave at $t = 0$ that propagates throughout the 2D isotropic material with velocity c. Let $u_s(t)$ and $u_i(t)$ be the displacement respectively at the source position and at the i-th sensor position $\mathbf{X}_i = (x_i, y_i)$. We assume a frequency-independent attenuation model so that

$$u_i(t) = a_i u_s(t - \tau_i), \tag{1}$$

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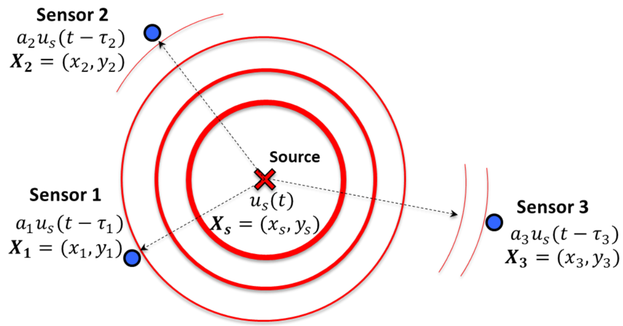

Figure 1. The elastic wave generated by the source at $t = 0$ reaches the sensors at different instants and with different amplitudes.

where

$$a_i = \frac{e^{-\alpha\|\mathbf{X}_s - \mathbf{X}_i\|}}{\|\mathbf{X}_s - \mathbf{X}_i\|^{\frac{1}{2}}} \tag{2}$$

is the attenuation function adopted in [21]. The constant α is the attenuation coefficient and $\tau_i = \frac{1}{c}\|\mathbf{X}_s - \mathbf{X}_i\|$ is the time the wave takes to propagate from the source to the i-th sensor, as illustrated in Figure 1. Assuming all sensors have the same frequency response $h(t)$, the electrical signal $r_i(t)$ sensed by the i-th sensor in a noiseless scenario is

$$r_i(t) = [h * u_i](t) = a_i[h * u_s](t - \tau_i) = a_i\psi_i(t), \tag{3}$$

where $\psi_i(t) = [h * u_s](t - \tau_i)$. Defining $A = \max_t |\psi_i(t)|$ and letting the normalized received signal be $\psi(t - \tau_i) = \frac{\psi_i(t)}{A}$ (A is the equivalent source amplitude) and assuming the signal at each sensor is sampled at the instants $t = nT + t_0$, where n is a non-negative integer, $T = \frac{1}{F}$ is the sampling interval and t_0 is the initial sample instant, the signal sampled by sensor i and corrupted by a white noise $w_i[n]$ is

$$r_i[n] = a_i A\psi(nT + t_0 - \tau_i) + w_i[n]. \tag{4}$$

Throughout this paper, we assume that the noise probability density function $f_W(w)$ is symmetric, i.e. $f_W(w) = f_W(-w)$, or equivalently, $F_W(w) = 1 - F_W(-w)$, where $F_W(w)$ is the noise cumulative distribution function (cdf). We also assume that t_0 is uniformly distributed in $[0, T]$. In addition, we consider the time of arrival was estimated using the fixed threshold method. This algorithm estimates the time of arrival at the i-th sensor as the smaller instant such that the $|r_i[n]|$ is greater than the threshold K.

B. Time of Arrival Probability Distribution

We first calculate the pdf of the time of arrival t_i given the initial sample time t_0. Using the fixed threshold method, the probability of detection at the sample n is the probability of $|r_i[n]| > K$ and $|r_i[j]| < K$ for all $j < n$.

Let $\Phi_i(t)$ be the probability of the absolute value of a sample acquired at instant t be less than the threshold. $\Phi_i(t)$ can be expressed in terms of the noise cdf $F_W(w)$:

$$\Phi_i(t) = \mathbb{P}[|a_i A\psi(t - \tau_i) + w_i[F(t - t_0)]| < K] = F_W(K - a_i A\psi(t - \tau_i)) - F_W(-K - a_i A\psi(t - \tau_i)). \tag{5}$$

As the threshold is assumed to be much larger than the noise standard deviation, we can approximate $F_W(K) = 1 - F_K(-K)$ to one. This means that $F_W(K - a_i A\psi(t - \tau_i))$ is approximately one if $a_i A\psi(t - \tau_i) < 0$, otherwise $F_W(-K - a_i A\psi(t - \tau_i))$ is approximately zero. The noise symmetry implies that $F_W(w) = 1 - F_W(-w)$, leading to a simplified approximation for $\Phi_i(t)$:

$$\Phi_i(t) \approx F_W(K - |a_i A\Psi(t - \tau_i)|). \tag{6}$$

Recalling that the estimated time of arrival is constrained to the discrete instants $t = kT + t_0$, the probability mass function (pmf) of t_i given t_0 can be calculated as

$$\mathbb{P}(t_i = t|t_0) = (1 - \Phi_i(t))\prod_{j=1}^{\infty}\Phi_i(t - jT). \tag{7}$$

Taking into account that $r_i(t) = 0$ for $t < \tau$, the product upper bound can be substituted by $\lfloor F(t - \tau)\rfloor$ since $\Phi_i(t - jT) = 1$ for $j > F(t - \tau)$. The time of arrival pdf $f_{t_i}(t)$ can be calculated using $\mathbb{P}(t_i = t|t_0)$ and the initial sample instant pdf $f_{t_0}(t)$:

$$f_{t_i}(t) = \sum_{k=-\infty}^{+\infty}\mathbb{P}(t_i = t|t_0 = t - kT)f_{t_0}(t - kT). \tag{8}$$

As $f_{t_0}(t - kT) = 0$ for $t - kT \notin [0, T]$, all the elements of the sum are equal to zero, except for k such that $0 \le t - kT \le T$, which is only possible for $k = \lfloor Ft \rfloor$. Substituting the pmf in (7) into (8) yields

$$f_{t_i}(t) = \frac{1}{T}[1 - \Phi_i(t)]\prod_{j=1}^{\lfloor F(t-\tau)\rfloor}\Phi_i(t - jT). \tag{9}$$

We compare $f_{t_i}(t)$ from (9) with the experimental distribution obtained in simulation in section IV.

C. Simplified Case

(9) is a complicated expression that describes the time of arrival pdf in terms of the noise cdf for any signal and for any symmetric noise distribution, and does not lead to closed-form solutions in general. A simple expression can be obtained assuming the noise level is low enough so that $\psi(t)$ can be approximated by a first-order Taylor polynomial centered at $\bar{t}_i$, where $\bar{t}_i$ is the noiseless time of arrival at the i-th sensor (i.e., disregarding the effect of noise, but not the errors due to sampling and waveform attenuation. See figure 2). Note that this hypothesis is not true if $\bar{t}_i - \tau_i$ occurs near a maximum of $|\psi(t)|$.

Let $\psi'(t) = \frac{\mathrm{d}\psi}{\mathrm{d}t}$ and consider the first-order Taylor approximation for the signal $\psi(t)$ centered at $\bar{t}_i - \tau_i$:

$$\psi(t) \approx \psi(\bar{t}_i - \tau_i) + \psi'(\bar{t}_i - \tau_i)(t - \bar{t}_i + \tau_i). \tag{10}$$

Defining the constants $b_i = \psi'(\bar{t}_i - \tau_i)$ and $\bar{\tau}_i = \bar{t}_i - \tau_i - \frac{\psi(\bar{t}_i - \tau_i)}{\psi'(\bar{t}_i - \tau_i)}$, this expression can be rewritten as

$$\psi(t) \approx b_i(t - \bar{\tau}_i). \tag{11}$$

We also need to approximate $F_W(w)$ in order to simplify the product in (9). $F_W(w)$ may be approximated as the

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first order Taylor polynomial centered at $w = 0$, if w is assumed to be bounded on an interval to guarantee that $0 \leq F_W(w) \leq 1$.

$$F_W(w) \approx \begin{cases} 0, & w < -\frac{1}{2f_W(0)}. \\ \frac{1}{2} + wf_W(0), & -\frac{1}{2f_W(0)} \leq w \leq \frac{1}{2f_W(0)}. \\ 1, & w > \frac{1}{2f_W(0)}. \end{cases} \tag{12}$$

Using this approximation for $F_W(w)$ is equivalent to approximate the noise distribution as a uniform distribution bounded between $-\Delta$ and $+\Delta$, with $\Delta = \frac{1}{2f_W(0)}$. Substituting these approximations in (9) and defining the constants $t_i^- = \bar{\tau}_i + \tau_i + \frac{K-\Delta}{|Ab_i|}$ and $t_i^+ = \bar{\tau}_i + \tau_i + \frac{K+\Delta}{|Ab_i|}$, we have

$$f_{t_i}(t) \approx \frac{B_i(t)}{T} \prod_{j=1}^{\lfloor F(t-t_i^-) \rfloor} \frac{|b_i|}{2\Delta}(t_i^+ - t + jT), \tag{13}$$

where $B_i(t)$ is defined by

$$B_i(t) = \begin{cases} \frac{b_i(t-t_i^-)}{2\Delta}, & t_i^- \leq t \leq t_i^+. \\ 1, & t_i^+ < t < t_i^+ + T. \\ 0, & t < t_i^- \text{or } t \geq t_i^+ + T. \end{cases} \tag{14}$$

Applying the arithmetic progression product formula [22] to (13), we obtain

$$f_{t_i}(t) \approx \frac{B_i(t)}{T} \left[\frac{b_i T}{2\Delta}\right]^{N(t)} \frac{\Gamma(1 + F(t_i^+ - t) + N(t))}{\Gamma(1 + F(t_i^+ - t))}, \tag{15}$$

where $N(t) = \lfloor F(t - t_i^-) \rfloor$. This expression can be further simplified assuming the sample rate is high enough so that $N(t) \approx F(t - t_i^-)$, resulting in

$$f_{t_i}(t) \approx \frac{B_i(t)}{T} \left[\frac{b_i T}{2\Delta}\right]^{F(t-t_i^-)} \frac{\Gamma(1 + F(t_i^+ - t_i^-))}{\Gamma(1 + F(t_i^+ - t))}. \tag{16}$$

Unlike the exact time of arrival pdf expression (9), this approximated pdf does not depend on the entire source waveform, but only on the noiseless time of arrival and the source waveform derivative at that point. This closed-form approximation may simplify the development of better location estimators.

III. Time Difference of Arrival Bias

In the previous section an exact pdf for the estimated time of arrival was derived. We are also interested in obtaining the bias in the TDOA, which is more important than the time of arrival bias when the localization algorithm is based on the TDOA, since two time of arrival estimates with the same bias would produce an unbiased TDOA.

We can obtain the TDOA pdf using the time of arrival pdf expression in (9) or even its approximation in (16). However, this approach does not lead to a closed-form expression, so in the remaining of this section we concentrate on finding a closed-form approximation for TDOA bias.

The signal $\psi(t)$ is modeled here as an envelope $m(t)$ that is increasing and thus invertible in the interval $[0, t_{max}]$

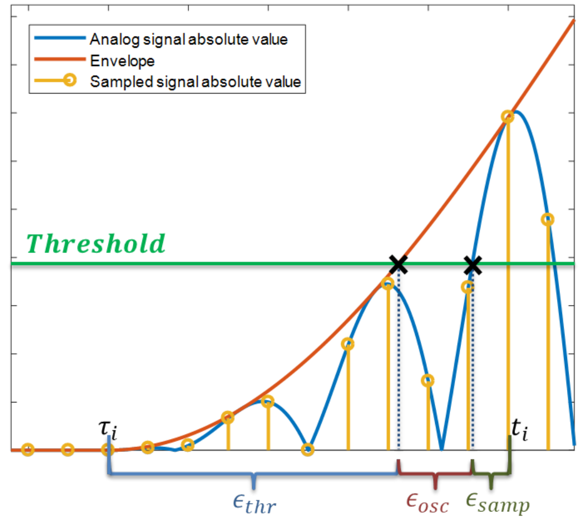

Figure 2. Illustration of (18) in a noiseless case

and whose maximum is at $t = t_{max}$, modulated by a cosine whose frequency f_0 represents the sensor resonance frequency. Since the time of arrival has a low probability to be after the peak of the waveform, we do not make any restriction to $\psi(t)$ for $t > t_{max}$.

$$\psi(t) = m(t)\cos(2\pi f_0 t), \qquad 0 \leq t \leq t_{max} \tag{17}$$

The time τ_i the wave takes to reach the sensor can be seen as the optimal value for t_i. The measured time of arrival deviation from τ_i can be approximately decomposed into a sum of errors cause by different factors:

$$t_i - \tau_i \approx \epsilon_{\text{thr}} + \epsilon_{\text{osc}} + \epsilon_{\text{samp}} + \epsilon_{\text{noise}}. \tag{18}$$

The factor $\epsilon_{\text{thr}} = m^{-1}(\frac{K}{Aa_i})$ is the time the wave envelope $m(t)$ takes to reach the threshold since the wave has arrived, ϵ_{osc} is the time the signal $\psi(t)$ takes to cross the threshold after $m(t)$ crosses it, ϵ_{samp} is the time the sampled signal takes to cross the threshold after the analog signal crosses it, and ϵ_{noise} is the time of arrival deviation cause by sampled noise. Figure 2 illustrates (18) in a noiseless case.

Consider the noise level is small enough so that $\psi(t)$ can be approximated by its tangent line at $t = \bar{t}_i - \tau_i$ as in equation (10). As $|Aa_i\psi(\bar{t}_i - \tau_i)| = K$, approximating the time of arrival deviation caused by sampled noise as the one caused by a zero-mean continuous-time noise $w_i(t)$ leads to

$$Aa_i\psi'(\bar{t}_i - \tau_i)(t_i - \bar{t}_i) + w_i(t) = 0 \tag{19}$$

Applying the expected value into both sides of the equation we obtain $\mathbb{E}\{t_i\} = \bar{t}_i$ and conclude that $\mathbb{E}\{\epsilon_{\text{noise}}\} \approx 0$.

Assuming the sampling rate is higher enough than f_0 so that the first time $|\psi(t)|$ crosses the threshold it has a high probability to stay above it for at least one sampling period, the deviation ϵ_{samp}, which is always positive, will have a high probability to be less than T.

Since $m(t)$ is an increasing function in $[0, t_{max}]$, the deviation ϵ_{osc}, which is also always positive, is limited by

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half of the oscillation period because in the worst case the signal will be detected in the next period of $|\psi(t)|$. Hence, under the assumed hypothesis, the measured time of arrival deviation from τ_i is bounded by

$$m^{-1}\left(\frac{K}{Aa_i}\right) \leq t_i - \tau_i \leq T + \frac{1}{2f_0} + m^{-1}\left(\frac{K}{Aa_i}\right). \quad (20)$$

The TDOA bias bounds can then be calculated using (20):

$$-T - \frac{1}{2f_0} + \Delta\epsilon_{\text{thr}} \leq \Delta t - \Delta t^{opt} \leq T + \frac{1}{2f_0} + \Delta\epsilon_{\text{thr}}, \quad (21)$$

where $\Delta t^{opt} = \tau_2 - \tau_1$ is the optimal TDOA, $\Delta t = t_2 - t_1$ is the estimated TDOA and $\Delta\epsilon_{\text{thr}}$ is the time of arrival deviation due to the amplitude difference between received waves at different sources, given by

$$\Delta\epsilon_{\text{thr}} = m^{-1}\left(\frac{K}{Aa_2}\right) - m^{-1}\left(\frac{K}{Aa_1}\right). \quad (22)$$

If the attenuation factors a_1 and a_2 were the same, $\Delta\epsilon_{\text{thr}}$ would be zero. This would only happen if the source were equally distant from the sensors. Note that a_i and hence $\Delta\epsilon_{\text{thr}}$ depend on the source position.

The average of the lower and upper TDOA bias bounds, equal to $\Delta\epsilon_{\text{thr}}$, can be used as an estimator for the TDOA bias. This estimator should show good performance if $\Delta\epsilon_{\text{thr}} \gg T + \frac{1}{2f_0}$.

IV. Simulations

A. Time of arrival probability distribution

In order to verify if (9) successfully represents the time of arrival distribution for a generic signal and to assess the approximation in (16), a noisy signal was generated and detected by one sensor in simulation. The signal $\psi(t)$ was modeled as in (17), and the chosen envelope $m(t) = \sin(\frac{\pi t}{L})$ was a hanning window, as done in [20].

We used the following parameters: $f_0 = 150$kHz (a popular resonance frequency for Acoustic Emission sensors), $L = 50\mu s$, $Aa_i = 0.04$, $F = 1$MHz, $K = 0.0178$ (equivalent to a 45dB threshold) and $\tau_i = 100\mu s$ (equivalent to a source whose distance from the sensor is $0.5m$ if $c = 5000\frac{m}{s}$). Two simulations were run with different noise levels, aiming to verify that the approximation in (16) only holds for low level noise. For both simulations the generated noise is Gaussian distributed with zero mean, and its standard deviation is 10^{-3} in one simulation and 5×10^{-3} in the another one.

Figures 3 and 4 show the comparison between the distribution of the time of arrivals obtained in simulation, the theoretical time of arrival pdf (9) and the approximated one (16). These figures also show the value of $\bar{t}_i$, the time of arrival that would be estimated if there was no noise.

The simulated time of arrival distribution is not Gaussian shaped and coincides with the theoretical one. In both cases the noiseless time of arrival does not coincide with the time of arrival expected value. We conclude that the presence of a zero-mean noise modifies the time of arrival expected value.

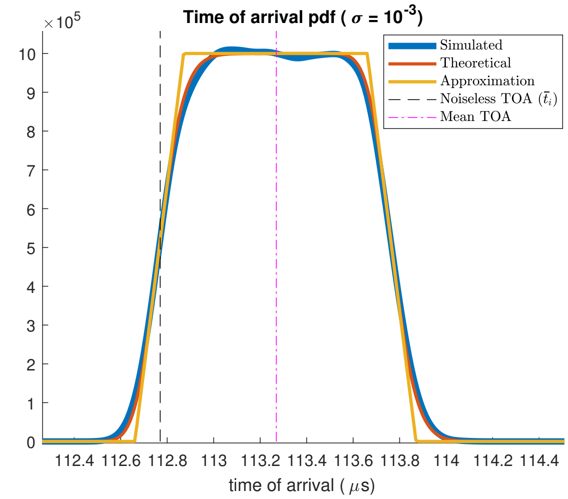

Figure 3. Time of arrival (TOA) pdf with low noise level

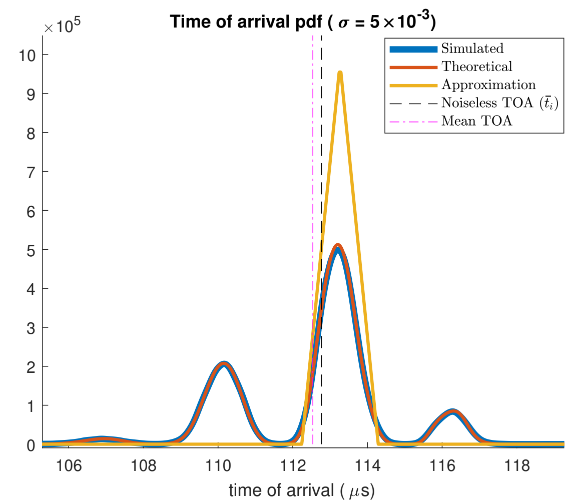

Figure 4. Time of arrival (TOA) pdf with high noise level

In the scenario where the noise variance is low (figure 3), the approximated pdf from (16) seems to be a good estimation of the theoretical one. However, when the noise variance is high (figure 4), the approximated pdf only fits the main lobe of the theoretical one. When the noise level is high, the threshold can be triggered in different oscillation periods, creating secondary lobes in the time of arrival pdf spaced by half of the oscillation period (in this case, $\frac{1}{2f_0} = 3.33\mu s$).

B. TDOA bias

Another simulation was performed aiming to verify if the TDOA bias theoretical upper and lower bounds described in (21) really limit the bias, as well as assess the performance of the proposed TDOA bias estimator performance. Two sensors were placed in the x-axis at $x = 0$m and $x = 1$m, and the source position was swept from $x = 0.01$m to $x = 0.99$m along the x-axis. The attenuation coefficient was chosen as $\alpha = 2$ m^{-1}, and the noise is Gaussian-

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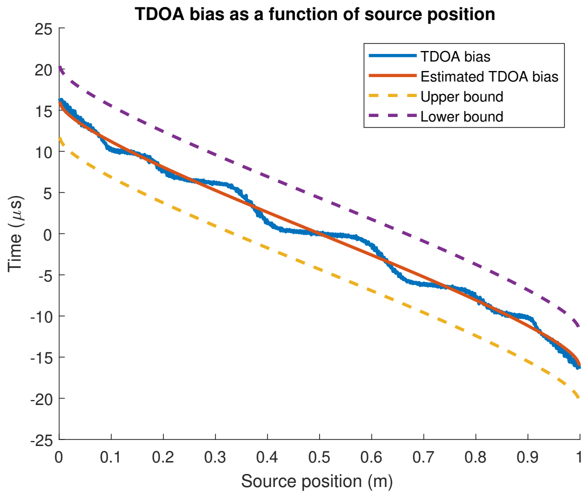

Figure 5. Simulated and estimated time difference of arrival and its theoretical bounds for different source positions

distributed with standard deviation $\sigma = 10^{-3}$. The rest of the parameters are the same as in the previous simulation.

Figure 5 shows this simulation results. The TDOA bias obtained in simulation respects the theoretical bounds deduced in (21). Moreover, the theoretical bias in this scenario is $T + \frac{1}{2f_0} + \Delta\epsilon_{\text{thr}} = 4.33\mu s + \Delta\epsilon_{\text{thr}}$, but the obtained bias is much larger than $4.33\mu s$ in general, showing that most of the bias is due to $\Delta\epsilon_{\text{thr}}$, i.e. caused by the different signal amplitudes received by each sensor on account of the difference of attenuation. That is why the TDOA bias estimator fits well the obtained TDOA bias curve. Another interesting observation is that the bias is minimum at $x = 0.5$m, when the wave propagation path length is the same for both sensors, resulting in no attenuation difference.

Conclusion

We deduced the time of arrival probability distribution expression for the fixed threshold method using a realistic model for an acoustic waveform and signal propagation, allowing the future development of new statistical TDOA and source position estimators that can have better performance than usual methods. A simplified and approximated expression for the time of arrival pdf under low noise level condition was also developed, making it possible to test localization algorithms without the need to simulate the signals using a more reliable model for the uncertainty on the time of arrival estimates than the Gaussian distribution. Theoretical bounds for TDOA bias were calculated under a low noise level condition, and a TDOA bias estimator was derived using the new bounds. The bounds can be used to limit position error, aiding algorithms that group multiple estimated fault positions originated from the fault. The TDOA bias estimator makes it possible to correct localization bias and thus reduce the distance between the estimated and the real source position. Eliminating the position bias might also allow the application of the Cramer-Rao Lower Bound and help the search for better unbiased TDOA and source position estimators.

References

[1] W. H. Prosser, *Nondestructive Evaluation — Theory, Techniques, and Applications*. New York: Marcel Dekker, Inc., 2001, ch. Acoustic Emission, pp. 369–446.
[2] M. Rabiei and M. Modarres, "Quantitative methods for structural health management using in situ acoustic emission monitoring," *International Journal of Fatigue*, vol. 49, pp. 81–89, 2013.
[3] R. Pullin, K. M. Holford, S. L. Evans, M. Baxter, and J. Hensman, "Advanced location and characterisation of damage in complex metallic structures using acoustic emission," in *Proceedings of the 13th International Conference on Experimental Mechanics ICEM*, vol. 13, 2007, pp. 925–926.
[4] M. J. Eaton, R. Pullin, and K. M. Holford, "Towards improved damage location using acoustic emission," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 226, no. 9, pp. 2141–2153, 2012.
[5] D. G. Aggelis, E. Z. Kordatos, and T. E. Matikas, "Acoustic emission for fatigue damage characterization in metal plates," *Mechanics Research Communications*, vol. 38, no. 2, pp. 106–110, 2011.
[6] C. Farrar and K. Worden, *Structural Health Monitoring: A Machine Learning Perspective*. Wiley, 2012.
[7] P. Stoica and J. Li, "Lecture Notes - Source Localization from Range-Difference Measurements," *IEEE Signal Processing Magazine*, vol. 23, no. 6, pp. 63–66, Nov. 2006, 00141.
[8] A. Beck, P. Stoica, and J. Li, "Exact and Approximate Solutions of Source Localization Problems," *IEEE Transactions on Signal Processing*, vol. 56, no. 5, pp. 1770–1778, May 2008.
[9] T. Kundu, "Acoustic source localization," *Ultrasonics*, vol. 54, no. 1, pp. 25–38, Jan. 2014, 00079.
[10] T. Kundu, H. Nakatani, and N. Takeda, "Acoustic source localization in anisotropic plates," *Ultrasonics*, vol. 52, no. 6, pp. 740–746, Aug. 2012, 00079.
[11] B. Friedlander, "A passive localization algorithm and its accuracy analysis," vol. 12, no. 1, pp. 234–245, Jan. 1987, 00354.
[12] M. D. Gillette and H. F. Silverman, "A linear closed-form algorithm for source localization from time-differences of arrival," vol. 15, pp. 1–4, 2008.
[13] H. W. Wei and S. F. Ye, "Comments on "A Linear Closed-Form Algorithm for Source Localization From Time-Differences of Arrival"," vol. 15, pp. 895–895, 2008.
[14] V. Salinas, Y. Vargas, J. Ruzzante, and L. Gaete, "Localization algorithm for acoustic emission," *Physics Procedia*, vol. 3, no. 1, pp. 863 – 871, 2010, international Congress on Ultrasonics, Santiago de Chile, January 2009.
[15] K. Grabowski, M. Gawronski, I. Baran, W. Spychalski, W. J. Staszewski, T. Uhl, T. Kundu, and P. Packo, "Time–distance domain transformation for acoustic emission source localization in thin metallic plates," *Ultrasonics*, vol. 68, pp. 142 – 149, 2016.
[16] A. Deng, L. Zhao, and Y. Zhao, "A new algorithm of ae localization using sub-gradient projection," in *2009 International Conference on Computational Intelligence and Software Engineering*, Dec 2009, pp. 1–5.
[17] A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localization problems," *IEEE Transactions on Signal Processing*, vol. 56, no. 5, pp. 1770–1778, May 2008.
[18] L. Rui and K. C. Ho, "Bias analysis of source localization using the maximum likelihood estimator," in *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, March 2012, pp. 2605–2608.
[19] K. C. Ho, "Bias reduction for an explicit solution of source localization using tdoa," *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2101–2114, May 2012.
[20] M. A. Torres Arredondo, "Acoustic emission testing and acousto-ultrasonics for structural health monitoring," 01 2013.
[21] M. Gresil and V. Giurgiutiu, "Prediction of attenuated guided waves propagation in carbon fiber composites using rayleigh damping model," *Journal of Intelligent Material Systems and Structures*, vol. 26, no. 16, pp. 2151–2169, 2015.
[22] M. A. Asiru, "Asymptotic formulas for sequence factorial of arithmetic progression," *International Journal of Mathematical Education in Science and Technology*, vol. 45, no. 6, pp. 947–952, 2014.

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