Robustifying Sequential Multiple Hypothesis Tests in Distributed Sensor Networks

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***Abstract*—We show how to robustify the Consensus + Innovations Matrix Sequential Probability Ratio Test against distributional uncertainties using robust estimators. Furthermore, we propose four distributed sequential tests for multiple hypotheses based on the median, the Hodges-Lehmann estimator, the M-estimator, and the sample myriad. Simulations verify the competitive performance of the proposed approach in comparison to an alternative method based on least favorable densities.**

***Index Terms*—sequential detection, multiple hypothesis testing, distributed detection, robustness, distributional uncertainties**

I. Introduction

Many modern real-time applications such as intelligent traffic control, smart homes, or video surveillance require sequential detectors that work reliably in distributed setups [1]. The goal is to make a decision for one out of two or more hypotheses based on as few measurements as possible. We consider the extension of distributed sequential detectors to multiple hypotheses. Moreover, we are interested in robust solutions that are insensitive to distributional uncertainties—a common phenomenon in real-life applications that can be due to outlying measurements, insufficient knowledge about the observed process, or model mismatches.

We show how to robustify the Consensus + Innovations Matrix Sequential Probability Ratio Test ($\mathcal{CI}$MSPRT) from [2] by leveraging neighborhood communication along with robust estimators. We propose four algorithms of this kind based on the median, the Hodges-Lehmann estimator [3], the M-estimator [4], [5], and the sample myriad [6]. Finally, we compare the performance of the proposed M-$\mathcal{CI}$MSPRT to the LFD-$\mathcal{CI}$MSPRT from [2], which uses least favorable densities for robustification.

The paper is structured as follows. Section II formulates the problem of multiple hypothesis testing in Gaussian and non-Gaussian environments. In Sections III and IV, we review the state-of-the-art in sequential binary, and sequential multiple hypothesis testing in a distributed sensor network. Section V details the concept of robustifying the $\mathcal{CI}$MSPRT using robust estimators in the test statistic update. In Section VI, we present selected results comparing the proposed M-$\mathcal{CI}$MSPRT with the alternative LFD-$\mathcal{CI}$MSPRT. Conclusions are drawn in Section VII.

II. Problem Formulation

Let $(Y_k(1), \dots, Y_k(t)), k = 1, ..., N$ be sequences of independent and identically distributed random variables. Their common distribution P admits a continuous density p. We consider a network of N agents modeled as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V}$ and $\mathcal{E}$ denoting the sets of agents and edges. The closed neighborhood of agent k is given by $\mathcal{N}_k = \{l \in \mathcal{V} \mid (k,l) \in \mathcal{E}\} \cup \{k\}$. A convenient way to define the neighborhood is by considering a communication radius $d_{\max}$.

When testing multiple simple hypotheses in a distributed setup, each agent decides between $M > 1$ hypotheses

$$\mathcal{H}_m \colon P = P_m, \quad m = 1, \dots, M.$$

We consider the following two test scenarios where each agent k should make a decision based on its measurement $y_k(t)$ at time instant t as well as information from its neighbors:

1) *Scenario 1:* Shift-in-Mean Test
The distributions P_m have different means μ_m. Assuming zero-mean Gaussian measurement noise with variance σ^2, the hypotheses become
$$\mathcal{H}_m : Y_k(t) \sim \mathcal{N}(\mu_m, \sigma^2), \; m = 1, \dots, M. \tag{1}$$
2) *Scenario 2:* Shift-in-Variance Test
P_m differ in variance σ_m^2. Hence, node k tests between
$$\mathcal{H}_m : Y_k(t) \sim \mathcal{N}(\mu, \sigma_m^2), \; m = 1, \dots, M. \tag{2}$$

In practice, there often is an uncertainty in the distribution of the data, causing the assumption of Gaussianity to be violated. Our goal is to develop sequential detectors that are robust and do not break down in the face of outliers. To this end, we consider M disjoint sets of feasible distributions $\mathcal{P}_m$ such that

$$\mathcal{H}_m \colon P \in \mathcal{P}_m, \quad m = 1, \dots, M.$$

Outliers can be modeled using the ε-contamination model, i.e., [4], [5]

$$P = (1-\varepsilon)P^\circ + \varepsilon H, \tag{3}$$

where P° denotes the nominal distribution, $0 \le \varepsilon < 0.5$ is the contamination coefficient, and H is the contaminating distribution.

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III. Distributed Sequential Binary Hypothesis Testing

A. The Sequential Probability Ratio Test (SPRT)

In the 1940s, Wald [7] proposed the single-sensor binary Sequential Probability Ratio Test (SPRT), where at each time step t a test statistic $S(t)$ is calculated according to

$$S(t) = \sum_{i=1}^{t} \log\left(\frac{p_1(y(i))}{p_0(y(i))}\right).$$

Subsequently, $S(t)$ is compared to a lower and an upper threshold $\gamma^l = \log \frac{\beta}{1-\alpha}$ and $\gamma^u = \log \frac{1-\beta}{\alpha}$, which are derived based on pre-specified bounds on the probabilities of false alarm α and misdetection β. The test stops as soon as one of the thresholds is crossed, and a decision is made according to

$$\begin{aligned}&\text{if } S(t) \geq \gamma^u: \text{ accept } \mathcal{H}_1\\ &\text{if } S(t) \leq \gamma^l: \text{ accept } \mathcal{H}_0\\ &\text{else}: \text{ continue sampling.}\end{aligned}$$

B. The Consensus + Innovations SPRT ($\mathcal{CI}$SPRT)

The Consensus + Innovations Sequential Probability Ratio Test ($\mathcal{CI}$SPRT) [8] is a distributed extension of the SPRT, which we generalized in [9]–[11] to be applicable in shift-in-mean as well as shift-in-variance tests. In the $\mathcal{CI}$SPRT, each node k obtains its log-likelihoood ratio (LLR) $\eta_k(t)$ as

$$\eta_k(t) = \log\left(\frac{p_1(y_k(t))}{p_0(y_k(t))}\right).$$

Its test statistic $S_k(t)$ is calculated and updated using neighbor information as

$$S_k(t) = \sum_{l\in\mathcal{N}_k} w_{kl} S_l(t-1) + \sum_{l\in\mathcal{N}_k} w_{kl}\eta_l(t),$$

where w_{kl} denote combination weights that are collected in matrix $\boldsymbol{W}$. For simplicity we set

$$w_{kl} = \begin{cases}\frac{1}{|\mathcal{N}_k|} & \text{if } l \in \mathcal{N}_k\\ 0 & \text{otherwise.}\end{cases}$$

$S_k(t)$ is compared to an upper and a lower threshold given by [9]

$$\gamma^u_{\mathcal{CI}} \geq \frac{4}{7}\frac{c\,\sigma^2_{\eta,0}}{\mu_{\eta,0}}\left(\log\left(\frac{\alpha}{2}\right) + \log\left(1 - e^{-\frac{1}{2}\frac{\mu^2_{\eta,0}}{c\sigma^2_{\eta,0}}}\right)\right) \tag{4}$$

$$\gamma^l_{\mathcal{CI}} \leq \frac{4}{7}\frac{c\,\sigma^2_{\eta,1}}{\mu_{\eta,1}}\left(\log\left(\frac{\beta}{2}\right) + \log\left(1 - e^{-\frac{1}{2}\frac{\mu^2_{\eta,1}}{c\sigma^2_{\eta,1}}}\right)\right). \tag{5}$$

Here, $\mu_{\eta,0}, \sigma^2_{\eta,0}$ and $\mu_{\eta,1}, \sigma^2_{\eta,1}$ denote the respective mean and variance of the LLR under $\mathcal{H}_0$ and $\mathcal{H}_1$. The constant $c = r^2 + \frac{1}{N}$ depends only on the network, with $r = \|\boldsymbol{W} - \frac{1}{N}\mathbf{1}\mathbf{1}^\top\|$ representing the network information flow. Furthermore, $\|\cdot\|$ is the Euclidean norm and $\mathbf{1}$ is the one-vector of length N. Node k makes a decision according to

$$\begin{aligned}&\text{if } S_k(t) \geq \gamma^u_{\mathcal{CI}}: \text{ accept } \mathcal{H}_1\\ &\text{if } S_k(t) \leq \gamma^l_{\mathcal{CI}}: \text{ accept } \mathcal{H}_0\\ &\text{else}: \text{ continue sampling.}\end{aligned}$$

IV. Sequential Multiple Hypothesis Testing in a Distributed Sensor Network

A. The Matrix SPRT (MSPRT)

The single-sensor SPRT can be extended to multiple hypotheses by computing the pairwise test statistics

$$S_{mn}(t) = \sum_{i=1}^{t} \log\left(\frac{p_m(y(i))}{p_n(y(i))}\right)$$

for all possible pairs $\mathcal{H}_m, \mathcal{H}_n$ with $m, n = 1, \ldots, M$ [1, Chapter 4]. In this Matrix SPRT (MSPRT), all $S_{mn}(t)$ are collected in a matrix $\boldsymbol{S}$ and an entrywise comparison to threshold matrix $\boldsymbol{\gamma}^u$ with entries $\gamma^u_{mn} = \log\left(\frac{1-\beta_{mn}}{\alpha_{mn}}\right) \approx \log\left(\frac{1}{\alpha_{mn}}\right)$ is performed. Note that the bounds on the probabilities of false alarm and misdetection of the pairwise hypothesis test are denoted by α_{mn} and β_{mn}.

In an acceptance test, we stop and decide for $\mathcal{H}_m$ once all entries in the mth row of $\boldsymbol{S}$—excluding the $(m, m)th$ entry—cross the corresponding thresholds, i.e.,

$$\begin{aligned}&\text{if there exists } m \in \{1, \ldots, M\} \text{ such that}\\ &S_{mn}(t) \geq \gamma^u_{mn} \quad \forall\, n \in \{1, \ldots, M\} \setminus \{m\}: \text{ accept } \mathcal{H}_m\\ &\text{else}: \text{ continue sampling.}\end{aligned}$$

This corresponds to performing $M(M-1)$ one-sided pairwise tests in parallel. Inverting the LLRs and comparing to lower thresholds γ^l_{mn} would lead to the alternative rejection test.

B. The Consensus + Innovations MSPRT ($\mathcal{CI}$MSPRT)

In [2], we proposed the Consensus + Innovations MSPRT ($\mathcal{CI}$MSPRT) as a fusion of the $\mathcal{CI}$SPRT and the MSPRT. Here, each node k computes the LLRs for all pairs $\mathcal{H}_m, \mathcal{H}_n$ as

$$\eta^k_{mn}(t) = \log\left(\frac{p_m(y_k(t))}{p_n(y_k(t))}\right). \tag{6}$$

Pairwise test statistics $S^k_{mn}(t)$ are calculated for all pairs as

$$S^k_{mn}(t) = \sum_{l\in\mathcal{N}_k} w_{kl} S^l_{mn}(t-1) + \sum_{l\in\mathcal{N}_k} w_{kl}\eta^l_{mn}(t). \tag{7}$$

An acceptance test is performed at each node according to

$$\begin{aligned}&\text{if there exists } m \in \{1, \ldots, M\} \text{ such that}\\ &S^k_{mn}(t) \geq \gamma^u_{mn} \quad \forall\, n \in \{1, \ldots, M\} \setminus \{m\}: \text{ accept } \mathcal{H}_m\\ &\text{else}: \text{ continue sampling,}\end{aligned}$$

with γ^u_{mn} denoting the upper threshold for the hypothesis pair $\mathcal{H}_m, \mathcal{H}_n$, which is calculated using (5).

V. Robustifying the $\mathcal{CI}$MSPRT

In [9], [11], we showed how to leverage neighborhood information together with robust estimators to robustify the binary $\mathcal{CI}$SPRT. In the sequel, we extend this method to the $\mathcal{CI}$MSPRT for multiple hypotheses.

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A. Reformulating the Update Equation

We reformulate (7) as

$$S_{mn}^{k}(t) = \sum_{l\in\mathcal{N}_k} w_{kl} S_{mn}^{l}(t-1) + \hat{\eta}_{mn}^{k}(t), \tag{8}$$

where $\hat{\eta}_{mn}^{k}(t)$ denotes the weighted average of the collective innovations of node k and its neighborhood for the hypothesis pair $\mathcal{H}_m, \mathcal{H}_n$ at time instant t. Without any a priori knowledge on the reliability of the neighbors, it is common to weight the information of all nodes equally. $\hat{\eta}_k(t)$ then becomes the sample mean with

$$\hat{\eta}_{mn}^{k,\text{mean}}(t) = \frac{1}{|\mathcal{N}_k|} \sum_{l\in\mathcal{N}_k} \eta_{mn}^{l}(t), \tag{9}$$

which is a non-robust estimator [5]. Due to the recursive nature of the update equation, using a robust estimator for $\hat{\eta}_{mn}^{k}(t)$ automatically robustifies the entire test statistic update.

In contrast to the alternative approach from [2], which is based on the concept of least favorable densities, the proposed robustification method censors the innovations at a later stage. More precisely, the use of a robust estimator in (7) limits the effect of outliers in the innovations. Since the log-likelihood ratios computed by each node stay intact, the original thresholds and decision rules of the $\mathcal{CI}$MSPRT remain valid.

B. Robust Estimators for the $\mathcal{CI}$MSPRT

One of the simplest robust alternatives to the sample mean is the median $\hat{\eta}_{mn}^{k,\text{median}}(t)$, which is calculated as

$$\hat{\eta}_{mn}^{k,\text{median}}(t) = \begin{cases} \boldsymbol{\eta}_{mn}^{k}(\frac{|\mathcal{N}_k|}{2}) & , |\mathcal{N}_k| \text{ even} \\ \frac{1}{2}\left(\boldsymbol{\eta}_{mn}^{k}(\frac{|\mathcal{N}_k|}{2}) + \boldsymbol{\eta}_{mn}^{k}(\frac{|\mathcal{N}_k|}{2}+1)\right) & , |\mathcal{N}_k| \text{ odd} \end{cases} \tag{10}$$

Here, vector $\boldsymbol{\eta}_{mn}^{k}$ contains the log-likelihood ratios of node k and its neighbors for the hypothesis pair $\mathcal{H}_m, \mathcal{H}_n$ in ascending order. Replacing $\hat{\eta}_k(t)$ in (7) with $\hat{\eta}_{mn}^{k,\text{median}}(t)$ leads to the Median-$\mathcal{CI}$MSPRT.

In [9], we showed that the median is not suitable for shift-in-variance tests due to the skewness contained in the LLR's probability density function. An alternative is to use the Hodges-Lehmann estimator, which calculates the median of the sample mean of all possible combinations of data points [3]. For the problem at hand, we obtain

$$\begin{aligned} &\hat{\eta}_{mn}^{k,\text{HL}}(t) = \text{median}\big(\boldsymbol{\eta}_{mn}^{k,\text{HL}}\big) \\ &\text{with } \boldsymbol{\eta}_{mn}^{k,\text{HL}} = \big[\text{mean}\big(\eta_{mn}^{i}(t), \eta_{mn}^{j}(t)\big)\big] \; \forall\, i,j \in \mathcal{N}_k, i \le j. \end{aligned} \tag{11}$$

Note that the median is calculated as in (10) and the mean as in (9). Using $\hat{\eta}_{mn}^{k,\text{HL}}(t)$ in (7) yields the HL-$\mathcal{CI}$MSPRT.

An important class of robust estimators are M-estimators [4], [5]. We obtain the M-$\mathcal{CI}$MSPRT by replacing $\hat{\eta}_k(t)$ in (7) with an M-estimate. This yields a weighted average, with weights

$$W(x) = \begin{cases} \frac{\psi(x)}{x} & , x \neq 0 \\ \psi'(0) & , x = 0 \end{cases},$$

where $\psi(x)$ is a score function and $\psi'(x)$ its first derivative. A popular choice of $\psi(x)$ was introduced by Huber [4] as

$$\psi_{\text{Hub}}(x) = \begin{cases} x & , |x| \le c_{\text{Hub}} \\ c_{\text{Hub}}\text{sign}(x) & , |x| > c_{\text{Hub}} \end{cases},$$

for some positive constant c_{Hub}. The M-estimate $\hat{\eta}_{mn}^{k,\text{M}}(t)$ is obtained by iterating

$$\begin{aligned} w_{mn}^{k}(t,i) &= W\left(\frac{\eta_{mn}^{k}(t) - \hat{\eta}_{mn}^{k,\text{M}}(t,i)}{\hat{\sigma}(\boldsymbol{\eta}_{mn}^{k})}\right) \\ \hat{\eta}_{mn}^{k,\text{M}}(t,i+1) &= \frac{\sum_{l\in\mathcal{N}_k} w_{mn}^{l}(i)\eta_{mn}^{l}(t,i)}{\sum_{l\in\mathcal{N}_k} w_{mn}^{l}(i)} \end{aligned} \tag{12}$$

until $\frac{|\hat{\eta}_{mn}^{k,\text{M}}(t,i+1) - \hat{\eta}_{mn}^{k,\text{M}}(t,i)|}{\hat{\sigma}(\boldsymbol{\eta}_{mn}^{k})} < \epsilon$ for a small, positive constant ϵ. We initialize the algorithm by setting $\hat{\eta}_{mn}^{k,\text{M}}(t,0) = \hat{\eta}_{mn}^{k,\text{median}}(t)$. For the scale estimation, we use the normalized median standard deviation $\hat{\sigma}_{\text{mad}}(\boldsymbol{\eta}_{mn}^{k}) = 1.483 \cdot \text{median}\left(|\boldsymbol{\eta}_{mn}^{k} - \hat{\eta}_{mn}^{k,\text{median}}(t)|\right)$.

Finally, we consider the sample myriad [6]

$$\hat{\eta}_{mn}^{k,\text{myriad}}(t) = \arg\min_{\eta} \prod_{l\in\mathcal{N}_k} \left[q^2 + \left(\eta_{mn}^{l}(t) - \eta\right)^2\right] \tag{13}$$

where commonly $q = \hat{\sigma}_{\text{mad}}(\boldsymbol{\eta}_{mn}^{k})$. Using $\hat{\eta}_{mn}^{k,\text{myriad}}(t)$ in (7) results in the Myriad-$\mathcal{CI}$MSPRT.

VI. Simulations

We consider two network sizes, $N \in \{15, 30\}$, and connectivities, $d_{\text{max}} \in \{0.3, 0.6\}$. The networks are shown in Fig. 1. We perform a shift-in-mean $\left(\mu_m \in \{-2,-1,1,2\}, \sigma^2 = 4\right)$ and a shift-in-variance test $\left(\sigma_m^2 + \sigma^2 \in \{1,2,4,16\}\right)$. The required probability of false alarm is fixed to $\alpha_{mn} = 0.01$, and all hypotheses are contaminated with $h_m = \mathcal{N}(0, 81)$. ε is swept over $[0, 0.3]$. For each hypothesis 1 000 Monte Carlo runs are performed. We consider the ratio of correct detection and the average stopping time (AST) as performance metrics. Due to space constraints, we only show results for the M-$\mathcal{CI}$MSRT ($c_{\text{hub}} = 1.8$), which were the best in all test cases, and compare them with those of the LFD-$\mathcal{CI}$MSPRT from [2].

A. Simulation Results

Rows 1 and 2 in Fig. 2 show the results for the shift-in-mean test using the M-$\mathcal{CI}$MSRT under $\mathcal{H}_1, \ldots, \mathcal{H}_4$. Rows 3 and 4 pertain to the LFD-$\mathcal{CI}$MSPRT. While the LFD-$\mathcal{CI}$MSPRT delivers perfect detection results under all hypotheses independently of the contamination ratio, this comes at the cost of a much higher AST. The M-$\mathcal{CI}$MSPRT obtains a constant ratio of correct detection of 1 only for $\mathcal{H}_2$ and $\mathcal{H}_3$. In this case, the AST actually drops with increasing contamination since—for the two hypotheses in the middle—outliers help in making a correct decision. Under $\mathcal{H}_1$ and $\mathcal{H}_4$, about 25-30 % contamination can be tolerated. Here, the AST increases with contamination and spikes just before the performance drop. Network size and connectivity have the same effect on both algorithms, with the connectivity having a large, and the

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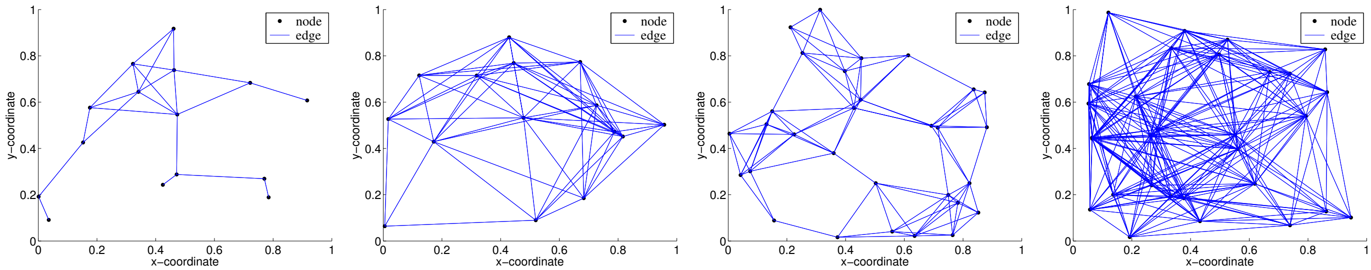

(a) $N = 15$, $d_{\max} = 0.3$. (b) $N = 15$, $d_{\max} = 0.6$. (c) $N = 30$, $d_{\max} = 0.3$. (d) $N = 30$, $d_{\max} = 0.6$.

Fig. 1. Sample networks obtained by randomly generating simple, connected and undirected graphs.

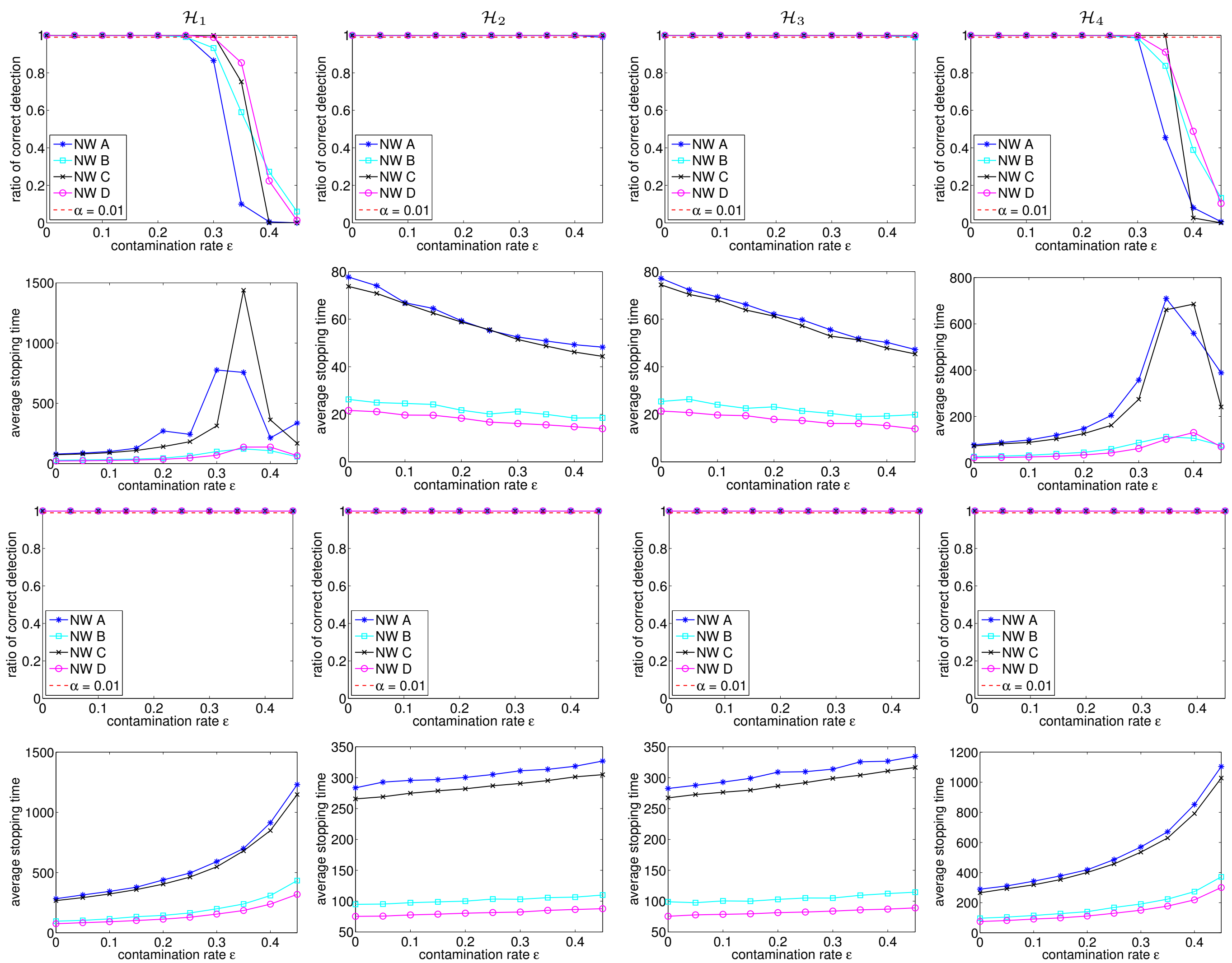

Fig. 2. Simulations results for a shift-in-mean test using the M-$\mathcal{CI}$MSPRT (row 1 and 2) and the LFD-$\mathcal{CI}$MSPRT (row 3 and 4) under $\mathcal{H}_1, \ldots \mathcal{H}_4$ (left to right). The upper row always depicts the ratio of correct detection while the lower row shows the average stopping time.

network size a small impact on the AST. For the M-$\mathcal{CI}$MSRT, a higher connectivity also slightly increases the amount of tolerable contamination.

Figure 3 shows the results for the shift-in-variance test. Both algorithms perform pretty similar with the tolerable amount of contamination depending on the hypothesis, i.e., on the assumed nominal variance. For the M-$\mathcal{CI}$MSRT, we observe an additional dependency on the network properties with the connectivity having a larger impact than the network size. Under $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$, the AST of both algorithms increases with contamination. $\mathcal{H}_4$ is the easiest test case, where outliers help to make a correct decision. Hence, the AST decreases

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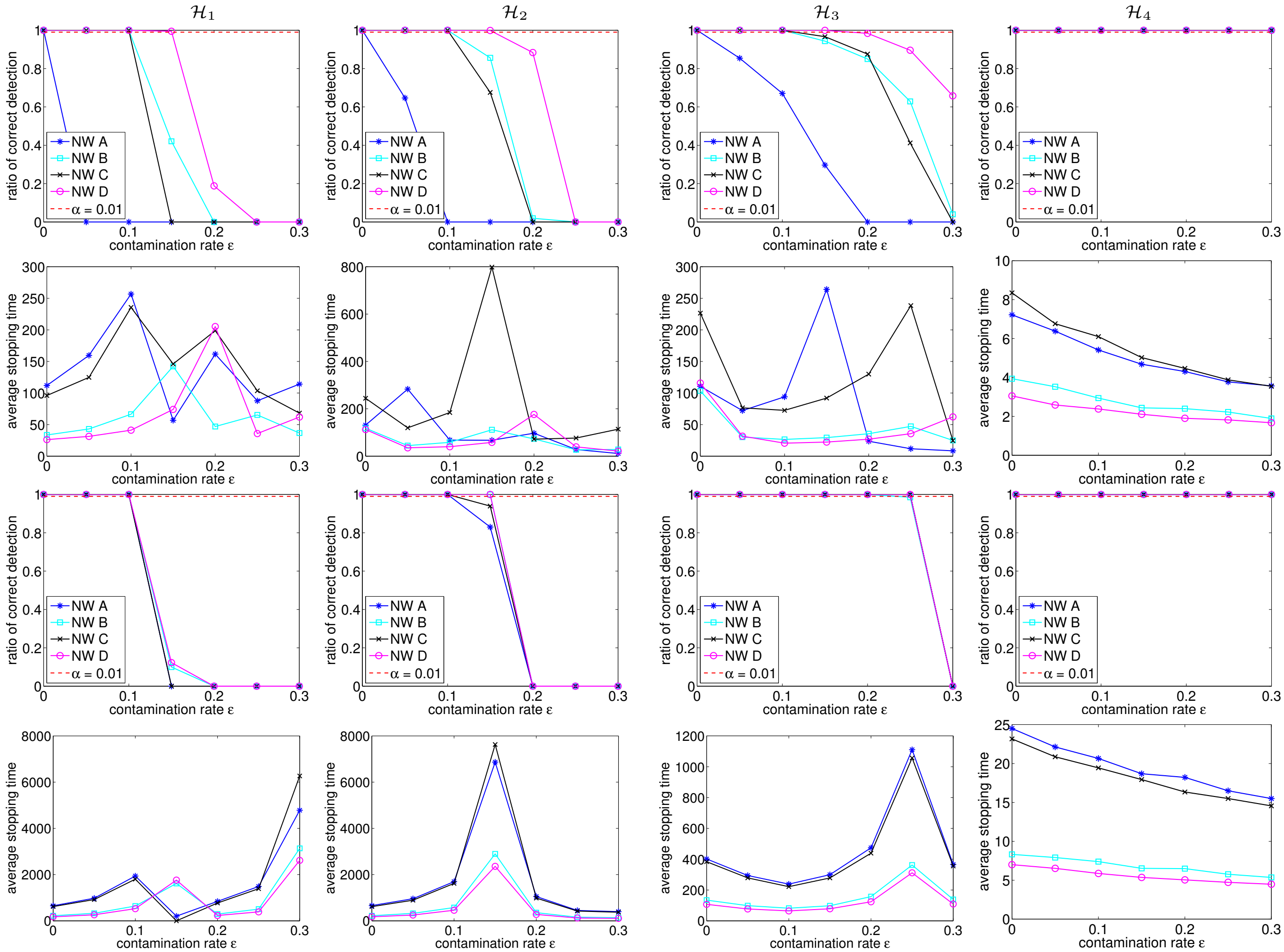

Fig. 3. Simulations results for a shift-in-variance test using the M-$\mathcal{CI}$MSPRT (row 1 and 2) and the LFD-$\mathcal{CI}$MSPRT (row 3 and 4) under $\mathcal{H}_1, \ldots \mathcal{H}_4$ (left to right). The upper row always depicts the ratio of correct detection while the lower row shows the average stopping time.

with increasing contamination. As before, the M-$\mathcal{CI}$MSRT exhibits a considerably lower AST overall than the LFD-$\mathcal{CI}$MSPRT.

In summary, the M-$\mathcal{CI}$MSRT is a good alternative to the LFD-$\mathcal{CI}$MSPRT, delivering accurate detection results at a much lower AST. In the shift-in-variance case, it might even outperform the LFD-$\mathcal{CI}$MSPRT.

VII. Conclusion

We showed how to robustify the Consensus+Innovations Matrix Sequential Probability Ratio Test with robust estimators. We proposed four robust algorithms of this kind and showed that the best-performing solution based on the M-estimator is comparable to the Least-Favorable-Density Consensus+Innovations Matrix Sequential Probability Ratio Test. Moreover, it has a much lower average stopping time and can outperform the competitor under certain network conditions.

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