Incoherent Projection Matrix Design for Compressed Sensing Using Alternating Optimization

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***Abstract*—In this paper we address the design of projection matrix for compressed sensing. In most compressed sensing applications, random projection matrices have been used but it has been shown that optimizing these projections can greatly improve the sparse signal reconstruction performance. An incoherent projection matrix can greatly reduce the recovery error for sparse signal reconstruction. With this motivation, we propose an algorithm for the construction of an incoherent projection matrix with respect to the designed equiangular tight frame (ETF) for reducing pairwise mutual coherence. The designed frame consists of a set of column vectors in a finite dimensional Hilbert space with the desired norm and reduced pairwise mutual coherence. The proposed method is based on updating ETF with inertial force and constructing incoherent frame and projection matrix using alternating minimization. We compare the performance of the proposed algorithm with state-of-the-art projection matrix design algorithms via numerical experiments and the results show that the proposed algorithm outperforms the other algorithms.**

***Index Terms*—Compressed sensing, projection matrix, mutual coherence, equiangular tight frame.**

I. Introduction

Compressed sensing (CS) has generated a lot of research interest in the signal and image processing communities since its introduction [1]–[3]. CS provides an alternative to the Shannon-Nyquist sampling theorem via a single step compression and sampling scheme. It has gained popularity due to its ability to recover a high dimensional signal from significantly fewer measurements than the number of ambient signal measurements required in conventional schemes. Compressed sensing allows us to exploit the sparse structure of the signal or underlying phenomena for capturing incoherent measurements using a projection (or sensing) matrix [4]. CS provides the mathematical framework for reconstructing a signal $\boldsymbol{x} \in \Re^{N\times 1}$ from linear measurements $\boldsymbol{y} \in \Re^{M\times 1}$ acquired through a projection matrix $\boldsymbol{\Phi} \in \Re^{M\times N}$ [5]:

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} \tag{1}$$

where $M \ll N$. We would like to reconstruct the signal $\boldsymbol{x}$ from the projections $\boldsymbol{y}$, however since (1) is an underdetermined system, there are infinite number of participant signals $\boldsymbol{x}$ which satisfy (1). We need to introduce additional constraint on $\boldsymbol{x}$ to be able to solve (1) for a unique $\boldsymbol{x}$. The CS system exploits the sparse structure of the underlying phenomena:

$$\boldsymbol{x} = \sum_{i=1}^{L} \theta_i \psi_i = \boldsymbol{\Psi}\boldsymbol{\theta} \tag{2}$$

where $\boldsymbol{\Psi} \in \Re^{N\times L}$ is the (sparsifying) transform basis and $\boldsymbol{\theta}$ is the vector of sparse coefficients. Using (1) and (2), the measurement vector can be expressed as:

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\theta} = \boldsymbol{D}\boldsymbol{\theta} \tag{3}$$

where $\boldsymbol{D} \triangleq [\boldsymbol{d_1}, \boldsymbol{d_2}, \ldots, \boldsymbol{d_L}] \in \Re^{M\times L}$ is an overcomplete frame or dictionary with $L \gg M$. With an overcomplete frame $\boldsymbol{D}$, the vector $\boldsymbol{\theta}$ is typically not unique for a given measurement vector $\boldsymbol{y}$ [6]. The additional sparsity constraint on $\boldsymbol{x}$ thus plays an important role. The signal $\boldsymbol{x}$ is said to be sparse if most of the coefficients of θ and the sparse signal is said to be K-sparse if the number of non-zero coefficients is K, also known as the sparsity level of the signal. With the sparsity assumption, the reconstruction problem can be formulated as:

$$\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_0 \quad \text{s.t.} \quad \boldsymbol{y} = \boldsymbol{D}\boldsymbol{\theta} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\theta} \tag{4}$$

which is NP-hard. Greedy algorithms, such as orthogonal matching pursuit (OMP) [7], matching pursuit (MP) [8], generalized OMP [9], and others [10] can under certain conditions solve for θ (with theoretical guarantees) and recover $\boldsymbol{x}$. In CS, the early work relied on random projection matrices but it has been shown that an appropriately designed projection matrix offers better signal recovery from the under-sampled measurements [4]. The spark of a matrix is defined as the smallest number of linearly dependent columns and this yields a guarantee for uniqueness of the sparse solution provided $\|\theta\|_0 \geq \text{spark}(\boldsymbol{D})/2$ [11]. In other words, a larger value of spark results in a larger signal space for the exact sparse recovery, which in turn implies the need to design a projection matrix with maximized spark. However, this is a computationally intensive task and hence CS systems rely on projection matrix design with reduced pairwise mutual coherence $\mu(\boldsymbol{D})$, which will be introduced in the next section.

In this paper, we focus on designing incoherent projection matrix for improved recovery performance in CS systems.

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This is achieved by designing equiangular tight frame (ETF) for the corresponding Gram matrix and updating the frame so as to reduce the mutual coherence. The matrices are updated iteratively through the target ETF using the weighted distance between the previous iterations as inertial force. Numerical experiments were performed to evaluate the recovery performance of the proposed algorithm and the results demonstrate that our method has a better overall performance compared to the state-of-the-art projection matrix design algorithms.

The rest of the paper is organized as follows. In Section II, we present some preliminaries including the metrics used for evaluating the CS performance followed by a discussion of related work. Section III presents the proposed incoherent frame and projection matrix design method. Simulation results are presented in Section IV demonstrating the performance of the proposed algorithm. Finally, concluding remarks are presented in Section V.

II. Background

A. Preliminaries

The mutual coherence of a frame $\boldsymbol{D}$, represented by $\mu(\boldsymbol{D})$, is defined as the largest absolute and normalized inner product between the different columns of the frame [11], [12]:

$$\mu(\boldsymbol{D}) = \max_{\substack{i\neq j \\ 1\leq i,j\leq L}} \frac{|\boldsymbol{d_i}^T \boldsymbol{d_j}|}{\|\boldsymbol{d_i}\|_2 \|\boldsymbol{d_j}\|_2} \tag{5}$$

Given a frame $\boldsymbol{D}$, a K-sparse signal $\boldsymbol{x}$ can be recovered from the measurements $\boldsymbol{y}$ using (4) provided the following is satisfied [11]:

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\boldsymbol{D})}\right) \tag{6}$$

Mutual coherence as a metric considers sparse representation and recovery performance from a worst-case perspective as it reflects the extreme pair-wise correlation in the frame which can be misleading. However, it has the capability to capture the behavior of uniform dictionaries and is easy to compute [11]. In the Gram matrix $\boldsymbol{G} = \boldsymbol{D}^T\boldsymbol{D} = \boldsymbol{\Psi}^T\boldsymbol{\Phi}^T\boldsymbol{\Phi}\boldsymbol{\Psi}$, the $(i,j)^{\text{th}}$ element is represented as $g_{ij} = \boldsymbol{d_i}^T\boldsymbol{d_j}$ where we assume $\boldsymbol{D}$ to be the normalized effective frame. In addition to the mutual coherence defined in (5), an alternative coherence metric that can be used to evaluate the recovery efficiency of the measurement matrix is the t-averaged mutual coherence μ_t for a given coherence threshold t, defined in (7). μ and μ_t represent the maximum and averaged values of the off-diagonal elements of the Gram matrix $\boldsymbol{G}$, respectively. These two coherence parameters will be used as performance measures for the projection matrix design in this paper.

$$\mu_t(\boldsymbol{D}) = \frac{\sum\limits_{1\leq i,j\leq L, i\neq j} (|g_{ij}| \geq t)\cdot |g_{ij}|}{\sum\limits_{1\leq i,j\leq L, i\neq j} (|g_{ij}| \geq t)}. \tag{7}$$

We employ the concept of tight frames in order to design projection matrix while minimizing the mutual coherence. A finite frame for the Hilbert space $\Re^{M\times 1}$ is defined as a set of atoms (or columns $\boldsymbol{d_k}$ of matrix $\boldsymbol{D} \in \Re^{M\times L}$) that satisfies the Parseval's condition $\alpha\|v\|_2^2 \leq \|\boldsymbol{D}^T v\|_2^2 \leq \beta\|v\|_2^2, \ \forall \ v \in \Re^M$ where α and β are the positive constants. If these constants are equal i.e., $\alpha = \beta$ then $\boldsymbol{D}$ is known as α-tight frame, and if $\alpha = \beta = 1$ then $\boldsymbol{D}$ is known as unit norm tight frame (UNTF). Welch bound on mutual coherence μ_W, given in (8), is a lower bound on the maximum pairwise correlation between the frame atoms and can be achieved with an ETF as it has the minimum mutual coherence. The UNTF with the minimum mutual coherence among all frames having the same dimension is called a Grassmannian frame [13].

$$\mu(\boldsymbol{D}) \geq \mu_W \triangleq \sqrt{\frac{(L-M)}{M(L-1)}} \tag{8}$$

B. Related Work

Next we discuss in brief some of the key frameworks in the literature for the design and optimization of the projection matrix. The first work on optimization of the projection matrix was the shrinkage scheme proposed by Elad [4]. CS recovery based on mutual coherence (5) does not reflect the average signal reconstruction performance. Elad proposed optimizing the projection matrix based on the t-averaged mutual coherence (7). However, the method in [4] is computationally intensive and the shrinkage function introduces some large values as off-diagonal elements of the Gram matrix that were not present earlier. Due to these large magnitude off-diagonal elements in the Gram matrix the worst case guarantees of the recovery algorithms no longer hold. A different shrinkage function was introduced in [14] for projecting the Gram matrix onto a convex non-empty set by reducing the off-diagonal elements towards the Welch bound on mutual coherence.

Authors in [15] propose a method for constructing $\boldsymbol{D}$ by making any subset of its columns as orthogonal as possible, or equivalently minimizing the difference between $\boldsymbol{G}$ and the identity matrix (the simplest ETF). The sensing matrix $\boldsymbol{\Phi}$ for a fixed $\boldsymbol{\Psi}$ is computed by reducing the largest M components of the error matrix. This method is non-iterative with significant computational improvements compared to Elad's method but with only a slight reduction in the reconstruction error. Zelnik-Manor et al. introduced optimized measurement matrix design based on block-sparse representations and its application to block-sparse decoding. They obtained a weighted surrogate function given in (9).

$$\begin{aligned} &\|\boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{I}\|_F^2 \\ &= \sum_{j=1}^{B}\sum_{i\neq j} \|\boldsymbol{D}[i]^T\boldsymbol{D}[j]\|_F^2 + \sum_{j=1}^{B} \|\boldsymbol{D}[j]^T\boldsymbol{D}[j] - \boldsymbol{I}\|_F^2 \end{aligned} \tag{9}$$

where $\boldsymbol{D}$ is represented as a concatenation of B column-blocks $\boldsymbol{D}[j]$. The first term in the right hand side (RHS) of (9) is the total interblock coherence and the second term is diagonal penalty [16]. In [17], randomly initialized sensing matrix is optimized using gradient descent based alternating minimization resulting in a matrix with reduced coherence than the initial one.

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III. Problem Formulation

An incoherent frame $\boldsymbol{D}$ is designed w.r.t. the updated ETF in each iteration for the given or learned sparsifying basis $\boldsymbol{\Psi}$, which is similar to minimizing the off-diagonal elements of Gram matrix $\boldsymbol{G}$. Hence, the cost function for projection matrix design (10) aims to reduce the mutual coherence by designing an ETF and then updating the $\boldsymbol{D}$ and $\boldsymbol{\Phi}$ at each iteration based on the designed ETF.

$$\|\boldsymbol{D}^T\boldsymbol{D}-\boldsymbol{I}\|_F^2 = \|\boldsymbol{\Psi}^T\boldsymbol{\Phi}^T\boldsymbol{\Phi}\boldsymbol{\Psi}-\boldsymbol{I}\|_F^2 \tag{10}$$

In this paper, Gram matrix $\boldsymbol{G} = \boldsymbol{D}^T\boldsymbol{D}$ is not optimized directly with respect to the identity matrix but with a designed ETF at each iteration. Here, updated $\boldsymbol{G}$ will be close to the corresponding ETF designed by the algorithm and contained in the convex set H_μ:

$$H_\mu \triangleq \{\boldsymbol{H} \in \Re^{L\times L} : \boldsymbol{H} = \boldsymbol{H}^T, H_{ii} = 1, \max_{i\neq j}|H_{ij}| \leq \mu_W\}$$

For designing an incoherent frame $\boldsymbol{D}$, cost function in (10) is reformulated and posed as the following minimization:

$$\min_{\boldsymbol{D}}\|\boldsymbol{E}\|_F^2 + P_\Pi(\boldsymbol{D}) \quad \text{s.t.} \quad \boldsymbol{E} = \boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{H} \tag{11}$$

where $\boldsymbol{H} \in H_\mu$, and $P_\Pi(\boldsymbol{D})$ defines the projection of $\boldsymbol{D}$ onto a convex set Π which regulates its column norm, given by:

$$P_\Pi(\boldsymbol{D}) : \{\boldsymbol{d_i}\}_{i=1}^L = \begin{cases} \boldsymbol{d_i} & \|\boldsymbol{d_i}\|_2 < 1 \\ \frac{\boldsymbol{d_i}}{\|\boldsymbol{d_i}\|_2} & \text{otherwise.} \end{cases}$$

For updating the ETF $\boldsymbol{H}$, we reduce the larger off-diagonal elements by projecting $\boldsymbol{G}$ onto H_μ at each iteration and then updating $\boldsymbol{D}$. We use μ_W as the threshold for updating $\boldsymbol{H}$. Let $\boldsymbol{E_p} = \boldsymbol{D_k^T}\boldsymbol{D_k} - \boldsymbol{I}$ at the p^{th} outer iteration, then after projecting $\boldsymbol{E}$ onto H_μ we obtain the ETF. This is achieved by constraining the off-diagonal elements of $\boldsymbol{E}$ using the shrinkage function S_{Ω_μ}.

$$S_{\Omega_\mu}(\boldsymbol{E}) : \underset{(i\neq j)}{E_{ij}} = \begin{cases} E_{ij} & |Eij| \leq \mu_W \\ \text{sign}(E_{ij}) \cdot \mu_W & \text{otherwise} \end{cases} \tag{12}$$

At each iteration this ensures that the large off-diagonal elements will be reduced in magnitude. Unlike other algorithms which project $\boldsymbol{E}$ onto the set of ETF via a shrinkage function, here we apply shrinkage on the weighted difference of the off-diagonal elements which is an estimate of the distance between the updated $\boldsymbol{E}$ and the corresponding ETF. The update scheme can be accelerated by incorporating an inertial force. The inertial force is computed as the weighted difference of the estimates of $\boldsymbol{E}$ from iterations p and $(p-1)$.

$$\begin{aligned} \boldsymbol{E_p} &= \boldsymbol{D_k^T}\boldsymbol{D_k} - \boldsymbol{I} \\ \boldsymbol{H_p} &= \boldsymbol{E_p} + w_1(\boldsymbol{E_p} - \boldsymbol{E_{p-1}}) \end{aligned} \tag{13}$$

where $w_1 \geq 0$ is a weighting parameter. At each iteration, $\boldsymbol{E_p}$ is computed using the current distance between the Gram matrix and ETF along with the inertial force.

$$\boldsymbol{H_{p+1}} = S_{\Omega_\mu}(\boldsymbol{H_p}) + \boldsymbol{I} \tag{14}$$

A. *Incoherent Frame Design*

For updating $\boldsymbol{D}$ and $\boldsymbol{\Phi}$, we apply the alternating minimization method to the updated H. In [18], authors have shown different methods for designing incoherent projection matrix corresponding to an ETF using alternating minimization. At each iteration, $\boldsymbol{D}$ is updated corresponding to the ETF $\boldsymbol{H}$ via the gradient method with inertial force. Let us denote the objective function for $\boldsymbol{D}$ by $J = \min_{\boldsymbol{D}}\|\boldsymbol{D'D} - \boldsymbol{H}\|_F^2$ and the derivative of J with respect to $\boldsymbol{D}$ as $\nabla_{\boldsymbol{D}}(J)$:

$$\begin{aligned} \nabla_{\boldsymbol{D}}(J) &= 4\boldsymbol{D}\left(\boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{H}\right) \\ \boldsymbol{D_{k+1}} &= \boldsymbol{D_k} - \eta\boldsymbol{D}\left(\boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{H_{p+1}}\right) \end{aligned} \tag{15}$$

Similar to (13), the update step for $\boldsymbol{D}$ in (15) can be modified to use an inertial force by taking a weighted difference of the estimates of $\boldsymbol{D}$ from iterations k and $(k-1)$:

$$\begin{aligned} \boldsymbol{D_{k+1/2}} &= \boldsymbol{D_k} - \eta\boldsymbol{D}\left(\boldsymbol{D}^T\boldsymbol{D} - \boldsymbol{H_{p+1}}\right) \\ \boldsymbol{D_{k+1}} &= P_\Pi\left(\boldsymbol{D_{k+1/2}}\right) + w_2\left(\boldsymbol{D_k} - \boldsymbol{D_{k-1}}\right) \end{aligned} \tag{16}$$

where η is the step-size and $w_2 \geq 0$ is a weighting parameter. Using the updated $\boldsymbol{D}$, we update $\boldsymbol{\Phi}$ by solving:

$$\boldsymbol{\Phi_{k+1}} = \min_{\boldsymbol{\Phi}}\|\boldsymbol{D_{k+1}} - \boldsymbol{\Phi}\boldsymbol{\Psi}\|_F^2 \tag{17}$$

In each iteration, the algorithm alternately updates $\boldsymbol{H_{p+1}}$ using (14) followed by updating $\boldsymbol{D_{k+1}}$, $\boldsymbol{\Phi_{k+1}}$ using (15) and (17), respectively. The alternating minimization is repeated for few iterations until convergence is achieved. The projection matrix design algorithm is summarized in Algorithm 1 below. In the proposed incoherent projection matrix design (IPMD) algorithm, shrinkage function S_{Ω_μ} is applied only on the off-diagonal elements and updated with an inertial force. Diagonal elements of $\boldsymbol{H}$ remain unity during the update step (14) due to which a normalization step is not required. Step size η and

Algorithm 1: Incoherent projection matrix design (IPMD) using alternating optimization

Objective: Design incoherent projection matrix.
Input: Sparsifying basis $\boldsymbol{\Psi}$, weighting parameters w_1, w_2, step size η, number of iterations N_{total}, N_{outer}, and N_{inner}.
Initialization: Initialize $\boldsymbol{\Phi}$ to a random matrix.
while $i < N_{total}$ **do**
    for $p = 1 : N_{outer}$ **do**
        Update ETF $\boldsymbol{H_{p+1}}$ using (14)
        for $k = 1 : N_{inner}$ **do**
            Update $\boldsymbol{D_{k+1}}$ using (15)
            Update $\boldsymbol{\Phi_{k+1}}$ using (17)
        end
        $\boldsymbol{\Phi_{k+1}} = \boldsymbol{\Phi_{N_{inner}}}$
    end
    $i = i + 1$
end
Output: Projection matrix $\boldsymbol{\Phi_{output}}$

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weighting parameters w_1, w_2 were determined empirically. With $w_1 = w_2 = 0.80$ incoherent frames were obtained and a step-size $\eta = 0.09$ is used to avoid divergence. For updating Φ, $N_{inner} = 3$ gives good results. We set the number of iterations $N_{total} = 100$ for stopping the update process. However, other stopping criteria can also be used. For example, the algorithm may iterate until the change in cost function value is less than a certain threshold or until $\boldsymbol{D}$ achieves a predefined coherence threshold.

IV. Simulation Results

In this section, we illustrate the performance of the proposed IPMD algorithm and compare it with some of the key projection matrix design methods from literature including random Gaussian matrix, Elad [4], method in [16] referred to as "Zelnik-Manor-Rosenblum-Eldar" (ZMRE), gradient descent based algorithm in [17] referred to as "Abolghasemi-Ferdowsi-Sanei" (AFS), and [15] referred to as "Duarte-Carvajalino-Sapiro" (DCS). We compare these methods via extensive set of numerical experiments for 100 iterations each and for IPMD, $N_{outer} = 15$. The algorithm parameters are kept fixed across the entire set of experiments. The initialized random matrix is generated for $N = 80$, $L = 100$, $M = 25$ with sparsity level $K = 15$. For Elad's algorithm, the other parameters are: $t = 0.4$ and $\gamma = 0.95$. In Fig. 1, histogram of the absolute correlation between atoms of the frame is presented. It illustrates the distribution of the absolute off-diagonal values of the normalized Gram matrix for each algorithm. It is noted that the histogram for the IPMD algorithm displays a shift towards the left (or origin), which indicates reduction in the pairwise correlation. Next we consider the averaged mutual coherence μ_t and cumulative coherence as a performance metrics since these are better measure of average recovery performance than the mutual coherence μ. Cumulative coherence measures the maximum total mutual coherence between a fixed set of atoms and a collection of other atoms in the dictionary for better insight. The evolution of these two performance parameters is shown in Fig. 2. It is seen that the proposed IPMD algorithm

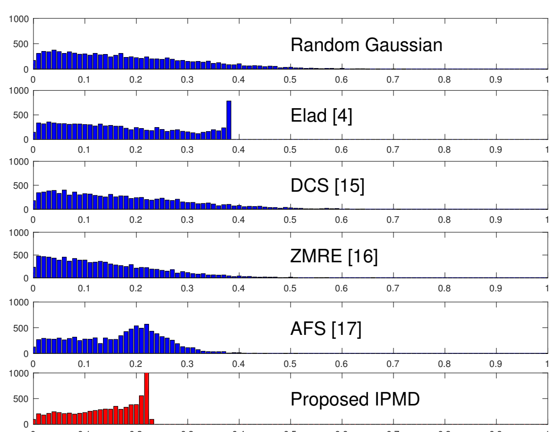
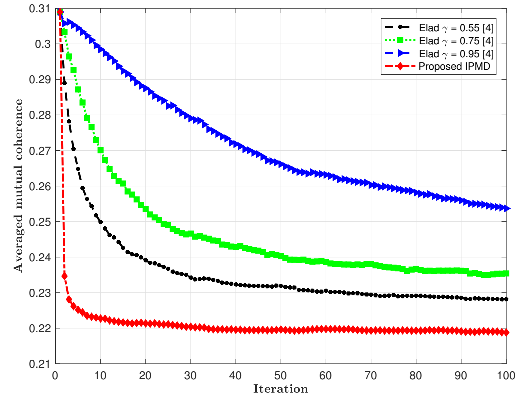
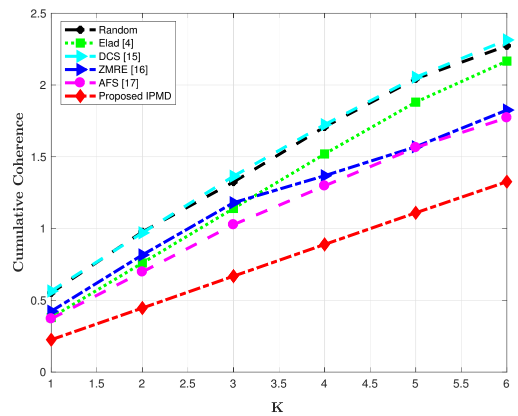


Fig. 1: Histogram of the absolute off-diagonal values of the optimized Gram matrix ($N = 80$, $L = 100$, and $M = 25$).



(a) Evolution of t-averaged mutual coherence (μ_t)



(b) Evolution of cumulative coherence

Fig. 2: Evolution of coherence parameters for $N = 80$, $L = 100$, $M = 25$, and $t = \mu_W = 0.4$.

results in significant smaller values for μ_t and cumulative coherence.

As seen from the Table I, CS_{IPMD} has lowest averaged mutual coherence but $||I - G||_F$ is comparable to CS_{ZMRE}. However, IPMD yields an improved performance in terms of the signal reconstruction accuracy. For evaluating the recovery performance we performed two set of experiments. Using a learned or given transform basis $\boldsymbol{\Psi}$ we generate a set of $N_s = 1500$ signals with θ_j ($j = 1, \ldots, N_s$) which are K-sparse. Measurement vectors y_j are computed for these signals using $\boldsymbol{\Phi}$ designed by the above mentioned algorithms with a fixed sparsifying basis $\boldsymbol{\Psi}$. OMP algorithm is used for recovering the sparse vectors from the measurements using (4). The average reconstruction error is computed as:

$$e_r = \frac{1}{N_s} \sum_{j=1}^{N_s} \frac{\|x_j - \widehat{x_j}\|_2^2}{\|x_j\|_2^2} \tag{18}$$

where $\widehat{x_j}$ is the recovered sparse signal. We first study the reconstruction performance with the number of iterations. Fig. 3 shows that as the signal becomes less sparse (higher K), the reconstruction error increases gradually. Next we study the reconstruction performance as a function of the number of measurements. In Fig. 4, we see that the recovery performance

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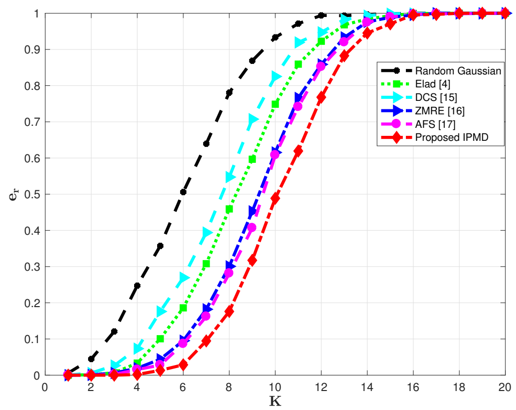

Fig. 3: Recovery performance error e_r along varying sparsity for $N = 80, L = 100, M = 25$ with optimized projection matrix

improves as the number of measurements (M) increases. Fig. 3 and Fig. 4 illustrate the improvement in recovery performance when using an optimized projection matrix $\mathbf{\Phi}$ using the IPMD. In addition, the performance of the proposed IPMD algorithm is consistently better compared to the other methods when using OMP algorithm for reconstruction. Algorithms such as OMP used for sparse reconstruction rely on the orthogonality of the columns of $\boldsymbol{D}$ and the IPMD algorithm achieves this to a greater extent than the other methods.

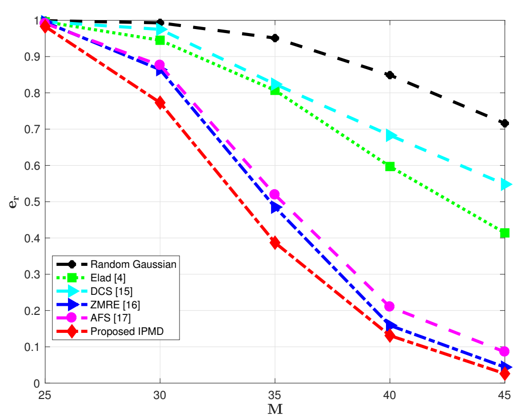

Fig. 4: Recovery performance error e_r along varying number of measurements for $N = 80, L = 100, K = 15$ with optimized projection matrix

TABLE I: Performance evaluation of various CS systems (M = 25, N = 80, L = 100 AND SNR = 20 dB)

	$\|\|I - G\|\|_F^2 \times 10^5$	μ_{avg}
CS_{randn}	4.465	0.3038
CS_{Elad}	0.00473	0.2875
CS_{DCS}	0.00492	0.2961
CS_{ZMRE}	0.00301	0.2647
CS_{AFS}	0.00341	0.2339
CS_{IPMD}	0.00307	0.2126

V. Conclusion

We presented the framework for the design of an incoherent projection matrix for CS applications. The proposed IPMD algorithm was shown to design an optimized projection matrix whose columns have reduced mutual coherence in order to achieve improved recovery performance. We design an ETF using inertial force update and the corresponding frame and projection matrix are updated using the alternating minimization method. The designed projection matrix was also shown to have reduced cumulative coherence. The proposed method achieves improved recovery performance (or achieves the same recovery performance with fewer measurements) as some of the state-of-the-art algorithms in the literature. The experiments illustrated that proposed method outperforms the other methods in recovery performance.

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