Robust Subspace Clustering for Radar Detection

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***Abstract*—Target detection embedded in a complex interference background such as jamming or strong clutter is an important problem in signal processing. Traditionally, statistical adaptive detection processes are built from a binary hypothesis test performing on a grid of steering vectors. This usually involves the estimation of the noise-plus-interference covariance matrix using i.i.d. samples assumed to be target-free. Moving from this paradigm, we exploit the fact that the interference (clutter and/or jammers) lies in a union of low-dimensional subspaces. Hence, the matrix of concatenated samples can be modeled as a sum of low-rank matrices (union of subspaces containing interferences) plus a sparse matrix times a dictionary of steering-vectors (representing the targets contribution). Recovering this factorization from the observation matrix allows to build detection maps for each sample. To perform such recovery, we propose a generalized version of the *robust subspace recovery via bi-sparsity pursuit* algorithm [1]. Experimental results on a real data set highlight the interest of the approach.**

I. Introduction

Adaptive detection of targets embedded in a complex environment (strong clutter, jammers, etc.) is an important issue in array processing. Following the classical statistical paradigm, the detection problem can be formulated as a binary hypothesis test (target present or not), with unknown statistical parameters (e.g., the disturbance covariance matrix). This topic has been extensively studied in the signal processing literature for a plethora of signal models and assumed noise distributions. For example, detection in Gaussian context was investigated in [2–5]. Generalizations to the complex elliptically symmetric distributions was studied in [6–8]. In order to deal with low sample support, the use of relaxations [9] and regularization methods [10, 11] have been investigated and are still an active topic of research.

Most of the aforementioned derivations have been built upon the availability of a homogeneous secondary data set, i.e., independent identically distributed (i.i.d.), and target-free samples, that are used to learn the unknown statistical parameters. From a practical point of view, the scanned environment can indeed be assumed stationary for a given amount of observations. However embedded systems encounter non-stationarity due to varying environment and/or switching jammers. Dealing with change points upstream is not a trivial task, which often leads to heterogeneous secondary data sets. Moreover, the secondary data are also potentially corrupted by outliers, such as targets. Generally speaking, statistical-based methods may suffer from an important performance loss if the assumed hypothesis are not met, or in mismatched situations. While recent works have robustness to heterogeneity/corruption issues in mind, it seems interesting to explore new methodologies, such as geometrical formulations, in order to alleviate this problem.

In this study we move from the statistical paradigm and explore the use of recent advances in robust union-of-subspaces learning for detection purposes. This is motived by the fact that the radar clutter (and/or jamming) interference is contained in a subspace of low dimension compared to the size of the data [12, 13]. Hence, the background of a piecewise stationary environment can be modeled as a union-of-subspaces. Additionally, the present sources can be modeled as a known dictionary of steering-vectors times a sparse matrix of power times phase-shifts coefficients. Recovering these two components from a noisy observation (the sample set) is referred to a robust subspace clustering problem [1, 14, 15].

Specifically, in order to tackle such recovery problem, we propose to extend the algorithm introduced in [1], referred to as *robust subspace recovery via bi-sparsity pursuit* algorithm (RoSuRe), to account for the dictionary. Eventually, the considered methodology can be applied for two purposes: either performing the detection itself by looking at the recovered sparse error matrix (revealing present targets), and/or for doing a first step clustering of homogeneous samples, that can be then used in a traditional statistical detection process. Note that this second application is not investigated here, as this paper focuses on the single-step detection problem. Eventually, this approach offers an interesting alternative due to its simplicity and minimal parameterization: a simultaneous estimation/detection process can be performed on the whole data without specifying noise distribution, change points, subspaces ranks, etc., avoiding then statistical-based methods limitations.

The paper is organized as follows: Section II introduces the data model. Section III presents a brief review of the statistical detection paradigm. Section IV builds upon the data model to recast detection as a robust subspace clustering problem, and the RoSuRe algorithm is introduced to solve this problem. Section V presents an application of the proposed method to perform detection on a real data set.

II. Data Model

A radar receiver consists in an array of Q antenna elements processing P pulses in a coherent processing interval which acquires a set of samples. For presentation convenience in this section, it is assumed that these samples are homogeneous, i.e., that the disturbance is i.i.d. distributed. The extension to the heterogeneous case is covered in section IV. Consider a

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cluster of K samples $\{\mathbf{z}_k\}_{k\in[\![1,K]\!]} \in \mathbb{C}^M$, these samples can be modeled (see e.g., [9, 16]) as:

$$\mathbf{z}_k = \mathbf{v}_k + \mathbf{c}_k + \mathbf{n}_k \tag{1}$$

where
• $\mathbf{v}_k$ is the sum of target responses, expressed as:

$$\mathbf{v}_k = \sum_{i=1}^{I} \alpha_i^k \mathbf{d}_i \tag{2}$$

where $\mathbf{d}_i$'s are known steering vectors (model of targets we seek to detect) and I is the size of the dictionary. The coefficients α_i^k are power times phase-shifts coefficients. We denote in matrix form:

$$\mathbf{v}_k = [\ \mathbf{d}_1\ \dots\ \mathbf{d}_I\ ]\,\boldsymbol{\alpha}_k = \mathbf{D}\boldsymbol{\alpha}_k \tag{3}$$

with $\boldsymbol{\alpha}_k = \left[\alpha_1^k, \dots, \alpha_I^k\right]^T$. Under the realistic assumption that there are few targets to be detected, only $\tilde{I}_k \ll I$ entries in $\boldsymbol{\alpha}_k$ are non-zero ($\tilde{I}_k$ denotes the number of present targets in the k'th sample). Therefore, the vectors $\boldsymbol{\alpha}_k$ are sparse.
• $\mathbf{c}_k$ represents the interference, such as ground clutter (response of the scanned environment) and/or jammers. Such response are commonly considered in the literature as zero mean with an assumed existing covariance matrix $\boldsymbol{\Sigma}_c$ and following an given (possibly heavy-tailed [8]) distribution. In this work, the underlying distribution will be considered unknown and unspecified. A crucial point is that, from physical considerations on the system [12, 13], we known that the interference covariance matrix is of low rank $R \ll M$, i.e., its singular value decomposition (SVD) reads

$$\boldsymbol{\Sigma}_c = \sum_{r=1}^{R} c_r \mathbf{u}_r \mathbf{u}_r^H \tag{4}$$

• Finally, $\mathbf{n}_k$ represent the thermal noise, assumed to be white Gaussian with a covariance matrix $\sigma^2\mathbf{I}$.

III. Statistical Detection

A. Adaptive detection

In the classical detection paradigm, the following binary hypothesis test is considered:

$$\begin{cases} H_0: \ \mathbf{z}_0 = \mathbf{c}_0 + \mathbf{n}_0 & ;\ \mathbf{z}_k = \mathbf{c}_k + \mathbf{n}_k\ ,\ \forall k \in [\![1,K]\!] \\ H_1: \ \mathbf{z}_0 = \mathbf{D}\boldsymbol{\alpha}_k + \mathbf{c}_0 + \mathbf{n}_0 & ;\ \mathbf{z}_k = \mathbf{c}_k + \mathbf{n}_k\ ,\ \forall k \in [\![1,K]\!] \end{cases}$$

where $\mathbf{z}_0$ is referred to as primary sample (tested cell) and the $\mathbf{z}_k, k \in [\![1,K]\!]$ are the secondary sample, assumed to be i.i.d. and target-free. Depending on the noise model, various detection schemes and likelihood ratios can be envisioned. From a robust and practical point of view, one can rely on the adaptive coherence estimator (ACE) [17, 18], also referred to as ANMF detector, which is defined as:

$$\hat{\Lambda}(\hat{\boldsymbol{\Sigma}}) = \frac{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{z}_0|^2}{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p}||\mathbf{z}_0^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{z}_0|} \underset{H_0}{\overset{H_1}{\gtrless}} \delta_{\hat{\boldsymbol{\Sigma}}}, \tag{5}$$

for a given "plug-in" estimator $\boldsymbol{\Sigma}$ of the noise-plus-interference covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_c + \sigma^2\mathbf{I}$, computed from the secondary data $\{\mathbf{z}_k\}_{k\in[\![1,K]\!]}$ (excluding $\mathbf{z}_0$). The detection test is performed on the grid of $\mathbf{p} = \mathbf{d}_i$ for $i \in [\![1,I]\!]$ (the whole dictionary). To sum up, a classical 2-step detection process is performed as follows:

$$\text{Step 1:}\ \ \{\mathbf{z}_k\}_{k\in[\![1,K]\!]} \xrightarrow{\text{statistical learning}} \hat{\boldsymbol{\Sigma}}$$
$$\text{Step 2:}\ \ \left\{\mathbf{z}_0, \hat{\boldsymbol{\Sigma}}\right\} \xrightarrow{\text{plug-in detector}} \text{detected } \{\mathbf{d}_i\}_{i\in[\![1,\tilde{I}_0]\!]}$$

In order to improve the performance of this detection process, the estimation of the interference plus noise covariance matrix represents a crucial step. This problem drives a lot of current research, notably for dealing with the problems of robustness and low sample supports. Recent advances includes the use of robust M-estimators [8], regularization methods [11] and the introduction of prior information on the CM structure [19, 20]. In the considered context, the noise is known to be low-rank structured, consequently, the estimator $\hat{\boldsymbol{\Sigma}}^{-1}$ in the detector may be also replaced by $\hat{\boldsymbol{\Pi}}^{\perp}$ the orthogonal projection onto the dominant eigensubspace of the interference complement. This process is similar to interference cancellation, and is refered to as "Low Rank" (LR) adaptive detection [9]. This process is known to be asymptotically sub-optimal but to provide better results at low sample support. Estimators $\hat{\boldsymbol{\Pi}}^{\perp}$ can be derived from the SVD of a covariance matrix estimator, or from MLE/Bayesian approaches [16, 21].

B. Some limitations

The 2-step detection process described previously relies on two major hypotheses. First, it is assumed that the secondary data are homogeneous, meaning that we know a priori the partition the whole sample set in clusters of i.i.d. secondary data. Second, it is assumed that the tested sample has been isolated from a set of target-free samples, used for the covariance learning step. The sample selection/partition can be performed and checked using a more complex estimation chain. Though, efficient in practice, this process may be tedious and computationally expensive as it involves numerous unknown parameters. Motivated by this issue, the present work proposes a novel formulation and an algorithm that perform a simultaneous estimation/detection process on the whole data set without aforementioned limiting requirements.

IV. Subspace clustering for detection

A. Union of subspaces model for heterogeneous data

Now, consider that the whole collected sample set is not necessarily homogeneous: the interference (clutter and/or jammers) covariance and distribution may change at certain points of the acquisition, with J unknown homogeneous sub-partitions (or clusters). Denote the partitioned sample set $\{\mathbf{z}_k^j\}$ with $j \in [\![1,J]\!]$, $k \in [\![1,K_j]\!]$ and $\sum_{j=1}^{J} K_j = K$. These samples are drawn as

$$\mathbf{z}_k^j = \mathbf{v}_k^j + \mathbf{c}_k^j + \mathbf{n}_k^j \tag{6}$$

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that still follows the model described in section II, except that the covariance matrix $\mathbf{\Sigma}_c^j$ of the interference in each cluster , reads

$$\mathbf{\Sigma}_c^j = \sum_{r=1}^{R_j} c_r^j \mathbf{u}_r^j \mathbf{u}_r^{jH} \tag{7}$$

with unspecified interference distribution (possibly different in each cluster). Note that, as evoked previously, this model is complex to deal with in a single step since the number of partitions J, the different distributions of interferences, their corresponding covariance matrices $\mathbf{\Sigma}_c^j$ and ranks R_j, the index of target-free samples, etc.) are unknown. From a practical point of view, this model involves too many parameters for deriving an efficient estimation procedure (such as Expectation-Maximization) under the statistical paradigm.

Now, denote the orthogonal projector onto the j-th interference subspace, of rank R_j, as $\mathbf{\Pi}_c^j = \sum_{r=1}^{R_j} \mathbf{u}_k^j \mathbf{u}_k^{jH}$. Consequently, we have a low rank representation satisfied by the interference:

$$\mathbf{c}_k^j = \mathbf{\Pi}_c^j \mathbf{c}_k^j \quad \forall\, k \in [\![1, K_j]\!] \tag{8}$$

In a given class $j \in [\![1, J]\!]$, with a sufficient number of samples (actually $K_j > R_j$ with probability one in our context), each interference realization living in the hyperplane spanned by $\mathbf{\Pi}_c^j$ can be obtained as a linear combination of the others:

$$\mathbf{c}_k^j = \sum_{p \neq k} \gamma_p^j \mathbf{c}_p^j \tag{9}$$

or in a matrix form

$$\mathbf{C}_j = \mathbf{C}_j \mathbf{W}_j, \quad [\mathbf{W}_j]_{i,i} = 0 \tag{10}$$

where $\mathbf{W}_j$ is the matrix containing the coefficients γ_p^j. This formulation is also refereed to as a *self-representative* property of the data. Denote the concatenation operator $\Cup$ and the corresponding concatenation $M \times K$ of all the vectors defined previously:

$$\begin{cases} \mathbf{Z} = \Cup\{\mathbf{z}_k^j\}, & \mathbf{C} = \Cup\{\mathbf{c}_k^j\} = \Cup\{\mathbf{C}_j\},\ \mathbf{V} = \Cup\{\mathbf{v}_k^j\}, \\ \mathbf{N} = \Cup\{\mathbf{n}_k^j\}, & \mathbf{A} = \Cup\{\boldsymbol{\alpha}_k^j\},\ \mathbf{W} = \Cup\{\mathbf{W}_j\}. \end{cases}$$

we obtain the formulation of the data matrix as:

$$\mathbf{Z} = \mathbf{V} + \mathbf{C} + \mathbf{N} \tag{11}$$

with relations

$$\begin{cases} \mathbf{C} = \mathbf{C}\mathbf{W}, & [\mathbf{W}]_{i,i} = 0 \\ \mathbf{V} = \mathbf{D}\mathbf{A} \end{cases} \tag{12}$$

To sum up on this formulation, most of the power of the samples is contained in the union of unknown subspace $\mathbf{\Pi}_c^j$ and the matrix $\mathbf{A}$ is a sparse matrix that contain the information about the present targets.

B. Robust subspace recovery for detection

From the point of view of a simultaneous estimation and detection, the problem is to recover an union of low rank subspace (interferences) and a sparse matrix (targets responses) from a noisy observation of the matrix $\mathbf{DA} + \mathbf{C}$. Recent advances allow to infer such recovery, as this problem is currently intensively investigated in the computer vision and machine learning community [1, 14, 15]. In this work, we consider the use of the algorithm from [1] (extended to include a dictionary), that solves:

$$\begin{array}{ll} \min\limits_{\mathbf{W},\mathbf{A}} & \|\mathbf{W}\|_1 + \lambda \|\mathbf{A}\|_1 \\ \text{s.t} & \mathbf{Z} = \mathbf{C} + \mathbf{DA} \\ & \mathbf{C} = \mathbf{CW}, \quad [\mathbf{W}]_{i,i} = 0 \end{array} \tag{13}$$

to perform a one step estimation/detection process. The optimization is carried out using ADMM and Linearized-ADMM (see [1] for implementation details). Eventually the obtained sparse matrix $\mathbf{A}$ allows to build detection maps for each samples, as non-zero coefficients of this matrix indicates the presence of targets. This work mainly focuses on the single-step detection problem, however, it is worth mentioning that the obtained matrix $\mathbf{W}$ is (up to a sorting permutation) block diagonal, which also allows to infer a partition of samples in homogeneous clusters.

V. Experimental results

A. STAP for airborne radar

In this section, we present an experimental validation of the proposed approach to perform radar detection. Space Time Adaptive Processing (STAP) [22] is a technique used in airborne radar to perform moving target detection. Typically, the radar receiver consists in an array of Q sensors processing P pulses in a coherent processing interval. Following the model in [22], the steering vector of a target $\mathbf{d}_i$ is function of the unknown angle of arrival (AoA) θ_i and the unknown target velocity v_i. Hence a dictionary matrix $\mathbf{D}$ can be build by concatenation, using a grid over these two parameters. From the Brennan Rule [12] we known that the ground clutter interference is contained in a low-dimensional subspace.

B. Experimental data

The data are provided by the French agency DGA/MI: the clutter is real but the targets are synthetic. The number of sensors is $Q = 4$ and the number of coherent pulses is $P = 64$, the size of the data is then $M = QP = 256$. The center frequency and the bandwidth are respectively equal to $f_0 =$ 10GHz and the bandwidth $B = 5MHz$. The radar celerity is $V =$ 100m/s. The inter-element spacing is $d = 0,3$m and the pulse repetition frequency is $f_r =$ 1kHz. The clutter rank, computed from Brennan Rule [12], is $R = 45$ and the CNR is evaluated around 20dB. For present targets, the Signal to Clutter Ratio (SCR) is evaluated around -5dB. We consider the following data matrix $\mathbf{Z} = \Cup\{\mathbf{z}_k\}$ set built as follows: $\mathbf{z}_1$ is under H_1 with 10 targets at various speed/angle with additional clutter plus noise. $\mathbf{z}_2$ is under H_1 and contains a target at (-4 m/s, 0 deg) with additional clutter plus noise. $\mathbf{z}_3$ is under H_1 and contains a target at (4 m/s, 0 deg) with additional clutter plus noise. $\mathbf{Z}_k$ for $k \in [\![4, K]\!]$, are under H_0 with only clutter plus noise.

C. Compared methods

We compare the following detection process:

- The LR-ANMF detector (cf. section III.A.) using an estimator $\hat{\mathbf{\Pi}}_{RTy}^{\perp}$ build from the SVD of regularized Tyler's estimator [23]. In order to provide a benchmark, $\hat{\mathbf{\Pi}}_{RTy}^{\perp}$ is computed using $K = 400$ target-free secondary data and the regularization tuned to give the best visual results. The detection is performed on each 3 first samples over a 256×256 grid of steering-vectors, resulting in 3 detection maps.
- The LR-ANMF detector using an estimator $\hat{\mathbf{\Pi}}_{SCM}^{\perp}$ build from the SVD of the sample covariance matrix [9]. First, $\hat{\mathbf{\Pi}}_{SCM}^{\perp}$ is computed using $K = 120$ target-free secondary data. Second, $\hat{\mathbf{\Pi}}_{SCM}$ is computed with added 3 samples under H_1 in the secondary data set ($K = 123$). The detection is performed as previously, resulting in 3 detection maps.
- The proposed RoSuRe-Detector, that solves problem (13) with the given data matrix $\mathbf{Z}$ (composed of the same $K = 123$ samples as the second detector) and the dictionary of 256×256 tested steering vectors. The the 3 first columns of recovered matrix $\mathbf{A}$ ($\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and $\boldsymbol{\alpha}_3$) are reshaped so that they can be displayed as a detection map.

D. Results

Figure 1 displays the results of the considered detection process for the 3 samples under H_1. LR-ANMF with $\hat{\mathbf{\Pi}}_{RTy}^{\perp}$ and a large sample set offers a benchmark for the ideal case. The standard LR-ANMF build with $\hat{\mathbf{\Pi}}_{SCM}^{\perp}$ using less secondary data (a more realistic scenario) is still satisfactory as target detection is possible, with an apparently low false alarm rate on the clutter ridge. Third line show a robustness issue of this process: performance of this detector are dramatically impacted by presence of targets in the secondary data set. Conversely, the proposed RoSuRe-Detector appears robust to this effect, as it allows to perform an accurate one-step detection on the whole sample set. Also note that targets are recovered with low side-lobes effect, which is an interesting property to distinguish them. The last line shows a limitation of this process when the RoSuRe parameter λ is not properly selected. Here, the targets can still be detected, but the ground response is also absorbed in the matrix $\mathbf{A}$ of the recovery, leading to false alarms on the clutter ridge diagonal.

VI. Conclusions

The robust subspace clustering methodology has been investigated for the purpose of radar detection in complex interference environments. The major interest of the approach is that it allows to perform a simultaneous estimation/detection on all samples in a single process, without assuming the interference statistical properties (e.g., distribution or rank). It also offers a way to process all available data at once, without assumption on homogeneity of the interference, nor the absence of targets in a training set. The acknowledged trade-off is that this process performs a sparse regression rather as a statistical detection test. Therefore, there is currently a lack of theoretical characterization (distribution, CFAR properties, etc.) so the algorithm has to be properly tuned to ensure, e.g., a given probability of false alarm. However, experimental results showed that this approach provides an interesting perspective.

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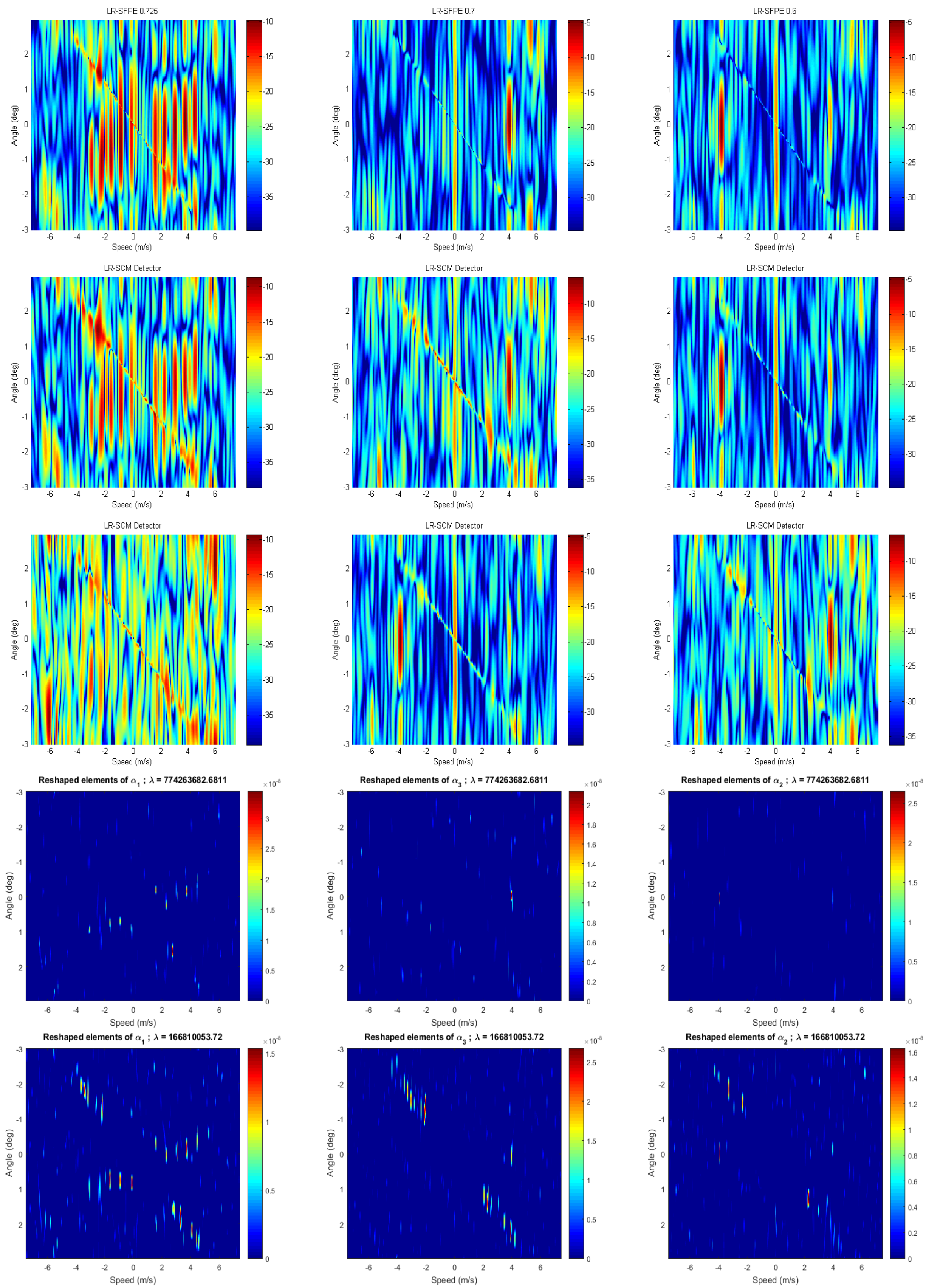

Fig. 1. From top to bottom: LR-ANMF detection maps for $\hat{\mathbf{\Pi}}_{RTy}$ and $K = 400$ target-free samples. LR-ANMF detection maps for $\hat{\mathbf{\Pi}}_{SCM}$ and $K = 120$ target-free samples. LR-ANMF detection maps for $\hat{\mathbf{\Pi}}_{RTy}$ and $K = 120$ target-free samples plus 3 samples under H_1. RoSuRe-Detector using the same $K = 123$ samples and for two different coefficients γ (two lines).