DQLC Optimization for Joint Source Channel Coding of Correlated Sources over Fading MAC

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***Abstract*—Distributed Quantizer Linear Coding (DQLC) is a joint source-channel coding scheme that encodes and transmits distributed Gaussian sources over a MAC under severe delay constraints, providing significant gains when compared to uncoded transmissions. DQLC, however, relies on the appropriate optimization of its parameters depending on source correlation, channel state and noise variance. In this work, we propose a parameter optimization strategy that relies on the lattice structure of the mapping, reduces the number of parameters to estimate, and exhibits lower computational complexity.**

I. Introduction

Fading Multiple Access Channel (MAC) is a channel model where several users transmit their individual information to a centralized receiver over the same wireless channel. Most works in the literature consider the digital encoding of the information and are designed under the assumption of source-channel separation. However, this approach is not optimal when users transmit correlated information and do not cooperate to encode their data [1]–[3]. In such cases, it is more appropriate to jointly consider the operations of source and channel coding and thus directly encode the source symbols into channel symbols by means of suitable mapping functions.

For several MAC scenarios, plain uncoded transmission, i.e. users simply send scaled versions of their source symbols, has been shown to provide optimal performance [2], [4]. This is not the case when transmitting correlated information over an orthogonal MAC with zero-delay where the optimal mappings resemble modulo functions [5]. Parametric modulo-like mappings have also been studied in [6], [7] and have been shown to provide good performance since the source correlation is efficiently exploited at decoding.

For non-orthogonal MAC, the optimal zero-delay mappings for two correlated users consist of a combination of uncoded transmission and a quantization-like scheme [8]. A hybrid discrete-analog scheme referred to as Scalar Quantizer Linear Coding (SQLC) based on a scalar quantizer and a linear continuous mapping was also proposed in [9] for the transmission of bivariate Gaussian sources over the Gaussian MAC. Finally, [9] extends these results to an arbitrary number of correlated users over the Gaussian MAC. The proposed mapping, called Distributed Quantizer Linear Coding (DQLC), outperforms the uncoded scheme for high Signal-to-Noise Ratios (SNRs).

In this work, we consider an arbitrary number of users transmit correlated information over a fading MAC. We present a transmission scheme based on the DQLC mapping, where the decoding operation is optimized to provide Minimum Mean Squared Error (MMSE) estimates with affordable complexity, even for a large number of users. In addition, we propose an alternative strategy for the parameter optimization with respect to [9] that allows optimizing the mapping for an arbitrary number of users and source correlation. In summary, the contributions of this work are:

- The extension of DQLC mappings to MACs with fading and an arbitrary number of users while proposing a decoding strategy that provides MMSE estimates of the source symbols with affordable complexity.
- A parameter optimization strategy with a practical computational cost. Such strategy relies on decoupling the DQLC parameters and searches for the optimal user power allocations. This approach does not require the exact computation of the expected DQLC distortion (see [9]).

II. System Model

Let us consider the transmission of correlated information over a fading MAC with K single-antenna users antenna and a single-antenna central node. Users are assumed to send discrete-time continuous-amplitude real-valued symbols which follow a zero-mean multivariate Gaussian distribution with covariance matrix $\mathbf{C_s}$. We assume that $[\boldsymbol{C_s}]_{k,k} = 1\ \forall k$, while $[\boldsymbol{C_s}]_{i,j},\ i \neq j$ represents the correlation between the source symbols of the i-th and j-th users.

Source symbols are individually encoded at each user with an appropriate mapping function and then transmitted over the MAC. Hence, the received signal is

$$y = \boldsymbol{h}^T \boldsymbol{f}(\boldsymbol{s}) + n, \quad (1)$$

where $\boldsymbol{h} \in \mathbb{R}^{K\times 1}$ is the fading MAC channel response, $n \sim \mathcal{N}(0, \sigma_n^2)$ is the additive white Gaussian noise, and $\boldsymbol{f}(\boldsymbol{s}) = [f_1(s_1), \ldots, f_K(s_K)]^T$ is an element-wise encoding function satisfying the individual power constraints $E\left[|f_k(s_k)|^2\right] \leq T_k$. Without loss of generality we assume $|h_1| \geq |h_2| \geq \ldots \geq |h_K|$, since users can be arbitrarily ordered.

We assume DQLC mappings are used to encode the source symbols [9]. In general, a subgroup of users transmit a quantized version of their information while the remaining

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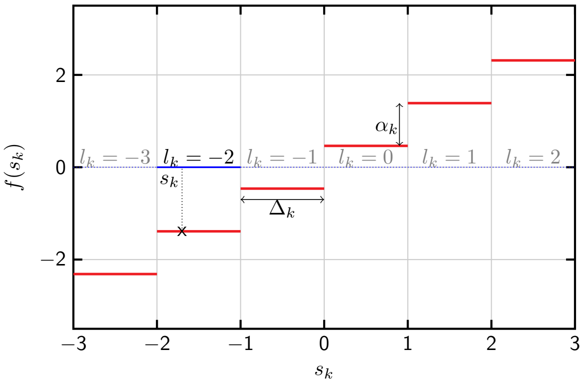

Fig. 1. Example of quantized mapping in DQLC with $\Delta_k = 1$. The source symbol $s_k = 1.7$ is mapped to the interval corresponding to $l_k = -2$.

users send a scaled version of their symbols. Therefore, the mapping function is

$$f_k(s_k) = \begin{cases} \alpha_k \left\lceil \frac{s_k}{\Delta_k} - \frac{1}{2} \right\rfloor + \frac{1}{2} & 1 \le k \le K_q \\ \alpha_k s_k & K_q < k \le K \end{cases}, \quad (2)$$

where α_k is a gain factor that ensures the encoding operation satisfies the power constraints, Δ_k is the quantization step for the k-th user, and K_q is the number of users that quantize their information. For the quantized users, the factor α_k can be computed as $\alpha_k \le \sqrt{\frac{T_k}{\Gamma(\Delta_k)}}$, $\forall k,\ 1 \le k \le K_q$ with

$$\Gamma(\Delta_k) = 2 \sum_{l=0}^{\infty} (l + 1/2)^2 \left(Q\left(\Delta_k (l+1)\right) - Q\left(\Delta_k l\right) \right), \quad (3)$$

where $Q(\cdot)$ is the error function. For the uncoded users, $\alpha_k < \sqrt{T_k}$, $\forall k,\ K_q + 1 < k \le K$. Other works propose clipping functions to improve performance at low correlations [9], but we will not consider them to simplify the model.

The non-linear mapping function for the k-th quantized user is conveniently rewritten using the following auxiliary function

$$f_{l_k}(s_k) = \begin{cases} \alpha_k \left(l_k + \frac{1}{2} \right), & s_k \in [\Delta_k l_k, \Delta_k (l_k + 1)] \\ 0 & \text{otherwise} \end{cases}, \quad (4)$$

where l_k is an integer-valued variable which indexes the quantizer interval where the source symbol s_k falls into. Fig. 1 shows a mapping example for a quantized user with parameter $\Delta_k = 1$. As observed, each quantizer interval is indexed for its corresponding l_k value such that the above function is only defined for the l_k value corresponding to the interval where the user symbol falls into.

From (4), and also considering the uncoded users, we define $\boldsymbol{f_l}(\boldsymbol{s}) = [f_{l_1}(s_1), \ldots, f_{l_{K_q}}(s_{K_q}), \alpha_{K_q+1} s_{K_q+1}, \ldots, \alpha_K s_K]^T$, which can also be expressed as

$$\boldsymbol{f_l}(\boldsymbol{s}) = \begin{cases} \boldsymbol{A} \boldsymbol{q_l}, & \boldsymbol{s} \in [\boldsymbol{a_l}, \boldsymbol{b_l}] \\ 0 & \text{otherwise} \end{cases}, \quad (5)$$

with $\boldsymbol{q_l} = \left[l_1 + \frac{1}{2}, \ldots, l_{K_q} + \frac{1}{2}, s_{K_q+1}, \ldots, s_K \right]^T$, $\boldsymbol{A} = \mathrm{diag}\{\alpha_1, \ldots, \alpha_K\}$, and $\mathbf{l} = [l_1, \ldots, l_{K_q}]^T$ the vector that stacks the interval indexes for the K_q quantized users. Note that, in general, each vector $\mathbf{l}$ comprises a feasible combination of K_q interval indexes. The interval limits are given by $\boldsymbol{a_l} = [\Delta_1 l_1, \ldots, \Delta_{K_q} l_{K_q}, -\infty, \ldots, -\infty]^T$ and $\boldsymbol{b_l} = [\Delta_1 (l_1 + 1), \ldots, \Delta_{K_q}(l_{K_q}+1), \infty, \ldots, \infty]^T$. Finally, the mapping function corresponds to the sum of all functions $\boldsymbol{f_l}$, i.e.

$$\boldsymbol{f}(\boldsymbol{s}) = \sum_{\boldsymbol{l} \in \mathbb{Z}^{K_q}} \boldsymbol{f_l}(\boldsymbol{s}). \quad (6)$$

At the receiver, an estimate of the source symbols is determined from the received signal, y. In this work, we seek to minimize the average Mean Squared Error (MSE) between the source and the estimated symbols. In this context, the optimal decoding is the MMSE estimator. However, the mapping function is non-linear and the calculation of the MMSE estimates requires to compute the resulting integrals numerically, which significantly increases the computational cost even for a small number of users. In the next section, we use sphere decoding to reduce the computational cost with the help of the alternative definition of DQLC mapping in (6).

Note that this system model considers real-valued variables but can be easily extended to the complex-valued case by treating the real and imaginary parts separately [7].

III. MMSE Sphere Decoder

At the receiver, the MMSE estimates are computed as

$$\hat{\mathbf{s}}_{\mathrm{MMSE}} = \mathbb{E}[\mathbf{s}|y] = \frac{\int \mathbf{s} p(y|\mathbf{s}) p(\mathbf{s}) d\mathbf{s}}{\int p(y|\mathbf{s}) p(\mathbf{s}) d\mathbf{s}}. \quad (7)$$

Using the alternative definition for the DQLC mapping in (6), the conditional probability is

$$p(y|\boldsymbol{s}) = \frac{1}{\sqrt{(2\pi\sigma_n^2)^K}} \exp\left(-\frac{1}{2\sigma_n^2} \| y - \boldsymbol{h}^T \boldsymbol{f}(\boldsymbol{s}) \|^2 \right) \quad (8)$$

$$= \sum_{\boldsymbol{l} \in \mathbb{Z}^{K_q}} T\left(y, \boldsymbol{h}^T \boldsymbol{A} \boldsymbol{q_l}, \sigma_n^2, \boldsymbol{a_l}, \boldsymbol{b_l} \right), \quad (9)$$

where $T(\boldsymbol{s}, \boldsymbol{\mu}, \boldsymbol{C}, \boldsymbol{a}, \boldsymbol{b})$ represents a truncated Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{C}$, in the interval $[\boldsymbol{a}, \boldsymbol{b}]$. Hence, the posterior probability is

$$\begin{aligned} p(\boldsymbol{s}|y) &\propto p(y|\boldsymbol{s}) p(\boldsymbol{s}) \\ &\propto \sum_{\boldsymbol{l} \in \mathbb{Z}^{K_q}} T\left(y, \boldsymbol{h}^T \boldsymbol{A} \boldsymbol{q_l}, \sigma_n^2, \boldsymbol{a_l}, \boldsymbol{b_l} \right) p(\boldsymbol{s}), \\ &\propto \sum_{\boldsymbol{l} \in \mathbb{Z}^{K_q}} T_{\boldsymbol{l}}(\boldsymbol{s}), \end{aligned} \quad (10)$$

where $T_{\boldsymbol{l}}(\boldsymbol{s}) = T(\boldsymbol{s}, \hat{\boldsymbol{s}}_{\boldsymbol{l}}, \boldsymbol{C}_e, \boldsymbol{a_l}, \boldsymbol{b_l})$, $\hat{\boldsymbol{s}}_{\boldsymbol{l}}$ is the linear MMSE estimator for the uncoded users assuming that the quantized users are correctly decoded, and $\boldsymbol{C}_e$ is its corresponding error covariance matrix, i.e.

$$\hat{\boldsymbol{s}}_{\boldsymbol{l}} = \frac{1}{\sigma_n^2} \left(\frac{1}{\sigma_n^2} \boldsymbol{G}_u^T \boldsymbol{h} \boldsymbol{h}^T \boldsymbol{G}_u + \boldsymbol{C}_s^{-1} \right)^{-1} \boldsymbol{G}_u^T \boldsymbol{h} \left(\boldsymbol{y} - \boldsymbol{h}^T \boldsymbol{G}_q \boldsymbol{q_l} \right) \quad (11)$$

$$\boldsymbol{C}_e = \left(\frac{1}{\sigma_n^2} \boldsymbol{G}_u^T \boldsymbol{h} \boldsymbol{h}^T \boldsymbol{G}_u + \boldsymbol{C}_s^{-1} \right)^{-1}, \quad (12)$$

where $\boldsymbol{G}_q = \mathrm{diag}\{\alpha_1, \ldots, \alpha_{K_q}, 0, \ldots, 0\}$ and $\boldsymbol{G}_u = \mathrm{diag}\{0, \ldots, 0, \alpha_{K_q+1}, \ldots, \alpha_K\}$.

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The posterior probability (10) can be used to obtain MMSE or Maximum A Posteriori (MAP) estimates of the received symbols after DQLC encoding, but the number of feasible vectors $\boldsymbol{l}$ is arbitrarily large. Hence, it is important to find a set of feasible vectors $\boldsymbol{l}$ as small as possible.

An ideal approach to determine the relevant vectors $\boldsymbol{l}$ would be to have a closed-form expression of the maximum of the truncated Gaussian as a function of $\boldsymbol{l}$. However, this is not possible, and we circumvent this limitation by evaluating the truncated Gaussian functions in (10) in the middle point of the intervals defined for the quantized users, while the uncoded users are evaluated on their corresponding linear MMSE decoding, i.e. $\tilde{\boldsymbol{s}}_{\boldsymbol{l}} = \left[\boldsymbol{D}\left(\boldsymbol{l}+\frac{1}{2}\right), [\hat{\boldsymbol{s}}_{\boldsymbol{l}}]_{K_q+1:K}\right]$ with $\boldsymbol{D} = \operatorname{diag}\left\{\Delta_1, \ldots, \Delta_{K_q}\right\}$, assuming that this provides large values for the likely vectors $\boldsymbol{l}$. Since the last components of $\tilde{\boldsymbol{s}}_{\boldsymbol{l}}$ are equal to the linear MMSE estimates in (11), in the exponential part of the truncated Gaussian $T_{\boldsymbol{l}}(\tilde{\boldsymbol{s}})$ given by

$$\exp\left(-\frac{1}{2}(\tilde{\boldsymbol{s}}_{\boldsymbol{l}}-\hat{\boldsymbol{s}}_{\boldsymbol{l}})^T\boldsymbol{C}_e^{-1}(\tilde{\boldsymbol{s}}_{\boldsymbol{l}}-\hat{\boldsymbol{s}}_{\boldsymbol{l}})\right),$$

the term $\tilde{\boldsymbol{s}}_{\boldsymbol{l}} - \hat{\boldsymbol{s}}_{\boldsymbol{l}}$ is zero in the last $K - K_q$ components corresponding to the uncoded users. This allows to express the exponent of each truncated Gaussian $T_{\boldsymbol{l}}$ only as a function of $\boldsymbol{l}$ in such a manner that the search of the relevant vectors $\boldsymbol{l}$ reduces to finding those vectors whose corresponding exponent is above a given threshold.

First, we partition the covariance matrix $\boldsymbol{C}_s$ as

$$\boldsymbol{C}_s = \left(\begin{array}{c|c}\boldsymbol{Q} & \boldsymbol{v}\\ \hline \boldsymbol{v}^T & \boldsymbol{U}\end{array}\right) \tag{13}$$

where $\boldsymbol{U}$ corresponds to the uncoded users. Using this formulation, the exponent in (13) can be rewritten in a lattice form and the search for the relevant $\boldsymbol{l}$ are the vectors that satisfy

$$\exp\left(-\left(\boldsymbol{l}-\boldsymbol{l}_o\right)^T\boldsymbol{\Lambda}\left(\boldsymbol{l}-\boldsymbol{l}_o\right)\right) > R \tag{14}$$

where

$$\boldsymbol{B} = \boldsymbol{D} + \boldsymbol{v}\left(\boldsymbol{A}_u^T\boldsymbol{h}_u\boldsymbol{h}_u^T\boldsymbol{A}_u\boldsymbol{U}+\sigma_n^2\boldsymbol{I}\right)^{-1}\boldsymbol{A}_u^T\boldsymbol{h}_u\boldsymbol{h}_q^T\boldsymbol{A}_q \tag{15}$$

$$\boldsymbol{\Lambda} = \frac{1}{2}\boldsymbol{B}^T\left(\boldsymbol{Q}-\boldsymbol{v}\boldsymbol{U}^{-1}\boldsymbol{v}^T\right)^{-1}\boldsymbol{B} \tag{16}$$

$$\boldsymbol{l}_o = \boldsymbol{B}^{-1}\left(\boldsymbol{v}\left(\boldsymbol{A}_u^T\boldsymbol{h}_u\boldsymbol{h}_u^T\boldsymbol{A}_u\boldsymbol{U}+\sigma_n^2\boldsymbol{I}\right)^{-1}\times \boldsymbol{A}_u^T\boldsymbol{h}_u\left(\boldsymbol{y}-\frac{1}{2}\boldsymbol{h}_q^T\boldsymbol{A}_q\right)-\frac{1}{2}\boldsymbol{D}\right) \tag{17}$$

where the channel matrix was decomposed as $\boldsymbol{h} = [\boldsymbol{h}_q, \boldsymbol{h}_u]$, $\boldsymbol{A}_q = \operatorname{diag}\left\{\alpha_1, \ldots, \alpha_{K_q}\right\}$ and $\boldsymbol{A}_u = \operatorname{diag}\left\{\alpha_{K_q+1}, \ldots, \alpha_K\right\}$.

Hence, it is possible to build a set of feasible vectors $\boldsymbol{l}$ such that the following metric is below a threshold R', i.e.

$$\left(\boldsymbol{l}-\boldsymbol{l}_o\right)^T\boldsymbol{\Lambda}\left(\boldsymbol{l}-\boldsymbol{l}_o\right) < R'. \tag{18}$$

After decomposing $\boldsymbol{\Lambda} = \boldsymbol{L}^T\boldsymbol{L}$ where $\boldsymbol{L}$ is a low triangular matrix, a sphere decoder will be used to efficiently determine the $\boldsymbol{l}$ points in a lattice that fall into a sphere of radius R' centered in $\boldsymbol{l}_o$. Once the above search criterion is defined as a function of the lattice $\boldsymbol{\Lambda}$, we apply the same decoding strategy with the sphere decoder proposed for modulo mappings [7]. Solving (7) directly requires techniques such as Monte Carlo integration, that is even infeasible for moderate number of users. Sphere decoding helps to reduce computational complexity when correlation is larger than zero, by exploiting the structure of the mapping, and narrowing the integration space by reducing the number of relevant terms in (10).

As shown in [7], the size of the feasible intervals for each component of $\boldsymbol{l}$ is inversely proportional to the diagonal elements of $\boldsymbol{L}$. Therefore, a necessary condition that $\boldsymbol{L}$ must satisfy to make those intervals as small as possible is that the diagonal elements are larger than a certain value. In other words, low values for the diagonal elements would imply large intervals for all components of $\boldsymbol{l}$, hence generating ambiguities in the decoding process. As explained in the ensuing section, this will play an important role in the parameter optimization.

IV. Parameter Optimization

In order to improve the performance of DQLC in fading MAC, the mapping parameters in $\boldsymbol{A}$ and $\boldsymbol{D}$ have to be optimized. Since an exhaustive search over the parameter space becomes prohibitive as the number of users increases, we propose the following constrained optimization problem

$$\begin{aligned} &\arg\min_{\boldsymbol{A},\boldsymbol{D}} \; E\left[\left|\boldsymbol{s}-\hat{\boldsymbol{s}}_{\text{MMSE}}\right|^2\right] \qquad (19)\\ &s.t.\;\; 0 \le \alpha_k \le \sqrt{\frac{T_k}{\Gamma(\Delta_k)}}, \qquad \forall k, 1 \le k \le K_q\\ &\qquad\; 0 \le \alpha_k \le \sqrt{T_k}, \qquad \forall k, K_q < k \le K \end{aligned}$$

This metric simplifies if we assume the quantized users are correctly decoded. This can be accomplished with two additional constraints to the above problem. First, a necessary condition that the quantized users must satisfy is that the diagonal elements of the matrix resulting from the Cholesky decomposition of $\boldsymbol{\Lambda}$ be larger than a certain value.

Another important issue is the fact that allocating more power to a user in DQLC implies, in general, to increase the Δ_k parameters for previous users and, consequently, the α_k parameters to avoid ambiguities in the received channel symbols (see [9]). However, the maximum value for α_k is upper bounded by the available power. Hence, if this bound is reached by some users, allocating more power to others can cause ambiguities in the decoding process.

In order to avoid this situation, we introduce a constraint over the maximum achievable values for the α_k of the quantized users. Since $\lim_{\Delta_k\to\infty}\Gamma(\Delta_k) \approx \frac{1}{2}$, an upper bound for such α_k values is $\sqrt{2T_k}$, and we can detect this situation by determining the distance of α_k to that upper bound. Hence, (19) is approximated as

$$\begin{aligned} &\arg\min_{\boldsymbol{A},\boldsymbol{D}} \; e(\boldsymbol{D},\boldsymbol{A}) \qquad (20)\\ &s.t.\;\; 0 \le \alpha_k \le \sqrt{\frac{T_k}{\Gamma(\Delta_k)}}, \qquad \forall k, 1 \le k \le K_q \end{aligned}$$

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$$0 \leq \alpha_k \leq \sqrt{T_k}, \qquad \forall k, K_q < k \leq K$$
$$0 \leq \alpha_k \leq \sqrt{2T_k} - \mu, \quad \forall k, 1 \leq k \leq K_q \quad (21)$$
$$[\boldsymbol{L}]_{k,k} \geq S, \quad (22)$$

where $e(\boldsymbol{D}, \boldsymbol{A})$ is the error assuming that the intervals of the quantized symbols are correctly guessed at the receiver, S is a constant to ensure the diagonal elements are above some threshold and is directly related to the chosen radius in the sphere decoder, and μ avoids the α_k's of the quantized users achieve their maximum value for large Δ_k.

We first obtain an upper bound for the error of the quantized users as

$$e_q(\Delta_k) = \sum_{i=1}^{\infty} \int_{\Delta_k i}^{\Delta_k (i+1)} (s - \delta_i)^2 p(s) ds, \quad (23)$$

where δ_i is the decoded value corresponding to the user transmitted in the i-th interval and is given by

$$\delta_i = \int_{\Delta_k i}^{\Delta_k (i+1)} s p(s) ds = \sqrt{\frac{2\sigma_s^2}{\pi}} \frac{\exp(-a_i^2) - \exp(-b_i^2)}{Q(b_i) - Q(a_i)} \quad (24)$$

where $a_i = \frac{\Delta_k i}{\sqrt{2\sigma_s^2}}$ and $b_i = \frac{\Delta_k (i+1)}{\sqrt{2\sigma_s^2}}$. Hence, the above bound can be expressed as

$$\begin{aligned} e_q(\Delta_k) = & \sigma_s^2 + \frac{1}{2} \sum_{i=1}^{\infty} \delta_i^2 (Q(b_i) - Q(a_i)) \\ & - \sqrt{\frac{2\sigma_s^2}{\pi}} \sum_{i=1}^{\infty} \delta_i (\exp(-a_i^2) - \exp(-b_i^2)). \end{aligned} \quad (25)$$

This is an upper bound because it computes the error ignoring the source correlation. Then, an upper bound on the error of the uncoded users is computed as $e_{u,k}(\boldsymbol{A}_u) = [\boldsymbol{C}_e]_{k,k}$. Thus, an upper bound on the overall MMSE assuming the quantized users are correctly decoded is

$$e(\boldsymbol{D}, \boldsymbol{A}_u) = \sum_{k=1}^{Q} e_q(\Delta_k) + \sum_{k=Q+1}^{K} e_{u,k}. \quad (26)$$

We now address the rewriting of the constraint (21). As observed in (16), the elements of $\boldsymbol{\Lambda}$ depend on $\mathbf{D}, \boldsymbol{A}_q$ and $\boldsymbol{A}_u$. Let us decompose $\boldsymbol{A}_q$ as $\boldsymbol{A}_q = \boldsymbol{P}_q \bar{\boldsymbol{A}}_q$, with $\boldsymbol{P}_q = \text{diag}\left\{\sqrt{P_1}, \ldots, \sqrt{P_{K_q}}\right\}$ and $\bar{\boldsymbol{A}}_q = \text{diag}\left\{\sqrt{\frac{1}{\Gamma_1}}, \ldots, \sqrt{\frac{1}{\Gamma_{K_q}}}\right\}$. For low Δ_k values, $\Delta_k \approx \sqrt{\frac{1}{\Gamma_k}}$ and, therefore, we can approximate $\boldsymbol{A}_q \approx \boldsymbol{P}_q \boldsymbol{D}$. Replacing this approximation in (15), we obtain

$$\boldsymbol{B} \approx \left(\mathbf{I} + \boldsymbol{v} \left(\boldsymbol{A}_u^T \boldsymbol{h}_u \boldsymbol{h}_u^T \boldsymbol{A}_u \boldsymbol{U} + \sigma_n^2 \boldsymbol{I} \right)^{-1} \boldsymbol{A}_u^T \boldsymbol{h}_u \boldsymbol{h}_q^T \boldsymbol{P}_q \right) \boldsymbol{D}.$$

We now define the lattice

$$\bar{\boldsymbol{\Lambda}} = \boldsymbol{D}^{-1} \boldsymbol{\Lambda} \boldsymbol{D}^{-1} = \boldsymbol{D}^{-1} \boldsymbol{L}^T \boldsymbol{L} \boldsymbol{D}^{-1},$$

that only depends on $\boldsymbol{P}_q$ and $\boldsymbol{A}_u$. Hence, for given $\boldsymbol{P}_q$ and $\boldsymbol{A}_u$, we look for the minimum Δ_k that ensures the diagonal elements of $\boldsymbol{L}$ are above some threshold S. This can be computed with the help of the decomposition $\bar{\boldsymbol{\Lambda}} = \bar{\boldsymbol{L}}^T \bar{\boldsymbol{L}}$ and making

$$\Delta_k = \frac{S}{\left[\bar{\boldsymbol{L}}\right]_{k,k}}, \ \forall k, 1 \leq k \leq K_q. \quad (27)$$

Finally, we replace the constraint (22) in the optimization problem by this expression.

Next, let us define the power allocation vector $\boldsymbol{p} = [p_1, \ldots, p_{K_q}, \alpha_{K_{q+1}}, \ldots, \alpha_K]$, where p_k are the diagonal elements of $\boldsymbol{P}_q$. Replacing $\alpha_k = \frac{p_k}{\sqrt{\Gamma(\Delta_k)}}, \forall k, 1 \leq k \leq K_q$, the problem (19) is transformed into

$$\begin{aligned} \arg\min_{\boldsymbol{p}} \quad & \sum_{k=1}^{K_q} e_q(\Delta_k) + \sum_{k=K_q+1}^{K} e_{u,k} \quad (28) \\ s.t. \quad & 0 \leq p_k \leq \sqrt{T_k}, \qquad \forall k \\ & \frac{p_k}{\sqrt{\Gamma(\Delta_k)}} \leq \sqrt{2T_k} - \mu, \qquad \forall k, 1 \leq k \leq K_q, \\ & \Delta_k = \frac{S}{[\bar{\boldsymbol{L}}]_{k,k}}, \qquad \forall k, 1 \leq k \leq K_q. \end{aligned}$$

This is a non-linear optimization problem that must be solved numerically, but the operations involved in the computation of the cost function and the constraints have a lower computational complexity than the exact computation of the expected distortion [9]. Also, it removes assumptions on the structure of the source covariance matrix. Finally, the search space is reduced since the Δ_k quantization steps are estimated as a function of the user power allocation. In the ensuing section, we resorted to the Matlab function `fmincon` to numerically solve this problem.

V. Simulation Results

In this section, the performance of the DQLC mapping is assessed by computer simulations for different fading MAC scenarios. At each time instant, a vector of K source symbols is generated from a zero-mean multivariate Gaussian distribution with a covariance matrix whose elements are given by $[\boldsymbol{C_s}]_{k,k} = 1$ and $[\boldsymbol{C_s}]_{i,j} = \rho, \forall i \neq j$. The source symbols are individually encoded at each user using the described DQLC mapping with the parameters optimized as explained in Section IV. We focus on the case of $K_q = K - 1$. Thus, only one user will transmit a scaled version of its source symbol. The radius of the sphere decoder was set to $R = 5$. The parameter optimization was carried out with $S = 5$ and $\mu = 0.03$. We also assume that the power constraints are equal for all users, i.e. $T_k = T, \forall k$, and hence the SNR is $\eta = T/\sigma_n^2$. Channel vectors $\boldsymbol{h}$ remain constant during the transmission of blocks of 200 user symbols, and the channel realizations are generated according to a Rayleigh distribution. Performance upper bounds assuming collaboration among all users are also plotted. Such bounds are obtained by equating the rate-distortion function of correlated sources to the single user MISO capacity with K transmit antennas.

The encoded symbols are then transmitted over the MAC and the received symbol are employed to obtain an estimate of

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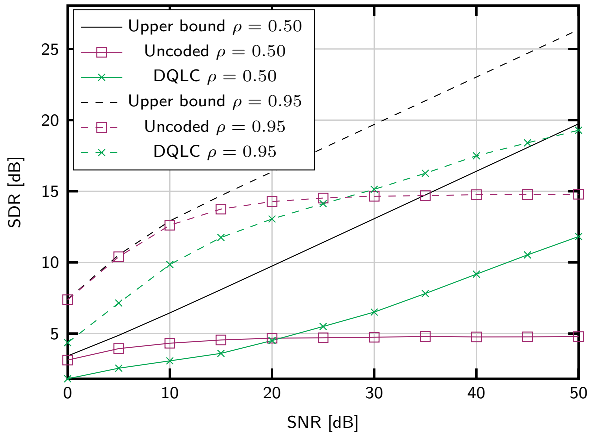

Fig. 2. SDR vs SNR for $K = 3$ and different correlation factors.

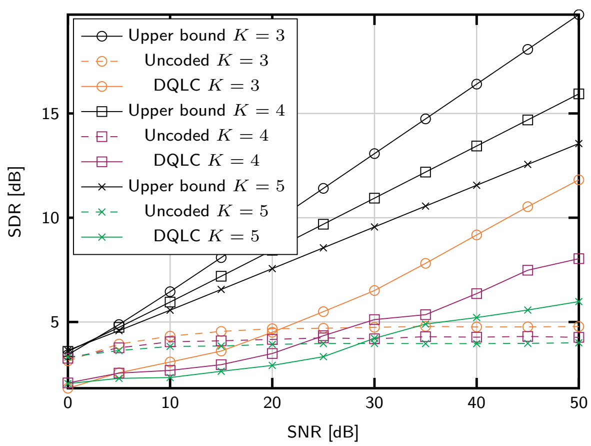

Fig. 3. SDR vs SNR for $\rho = 0.5$ and different number of users.

the transmitted source symbols. Finally, the distortion between the source and estimated symbols is measured according to the MSE, and averaged over L independent transmissions

$$\hat{\xi} = \frac{1}{KL} \sum_{l=1}^{L} \sum_{k=1}^{K} |s_k - \hat{s}_k|^2. \tag{29}$$

The system performance is measured in terms of Signal-to-Distortion Ratio (SDR) defined as $\text{SDR[dB]} = 10 \log_{10}(1/\hat{\xi})$.

Fig. 2 shows the SDR obtained with the optimized DQLC mapping for $K = 3$ users and correlation factors $\rho = 0.5$ and $\rho = 0.95$, when the SNR ranges from 0 to 50 dB. The curve corresponding to the performance obtained with uncoded transmission and to the performance upper bound assuming collaboration is also plotted for comparison. As observed, for low SNR values, the uncoded scheme performs better than DQLC, but its performance saturates above some SNR threshold ($\eta \geq 10$ dB for $\rho = 0.5$ and $\eta \geq 20$ dB for $\rho = 0.9$). Beyond that point, DQLC provides superior performance to the uncoded scheme, and this performance gain also increases with the SNR. Uncoded transmission achieves the upper bound for low SNRs, and there is a gap around 6-8 dB with respect to DQLC in large SNRs, depending on the source correlation.

Fig. 3 compares the performance for different number of users with $\rho = 0.5$. The same behavior is observed, where the DQLC is able to improve the performance of uncoded transmission only for large SNR values. Moreover, the SDR decreases when the number of users increases, a behavior also observed in the upper bound, hence these gains are lower when K is large.

VI. Conclusion

Parameter optimization for DQLC joint source-channel coding in fading MAC with correlated sources has been considered. Optimization is based on an upper bound of the MSE assuming that the quantized users are correctly decoded. The optimal parameters are obtained by solving a non-linear constrained optimization problem that minimizes the likelihood that the interval of the quantized users is missed at the receiver. Results show that this approach provides good solutions for different numbers of users, and allows to beat the performance of uncoded transmission for large SNR values.

Acknowledgements

This work has been funded by Office of Naval Research Global of United States (N62909-15-1-2014), the Xunta de Galicia (ED431C 2016-045, ED341D R2016/012, ED431G/01), the Agencia Estatal de Investigación of Spain (TEC2015-69648-REDC, TEC2016-75067-C4-1-R) and ERDF funds of the EU (AEI/FEDER, UE).

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