

# Denoising Phonocardiogram signals with Non-negative Matrix Factorization informed by synchronous Electrocardiogram

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**Abstract**—The phonocardiographic signals (PCG) are of interest for the analysis of the cardiac mechanical function. However, they are not always directly exploitable because of ambient interference (gastric noises, breathing noises, etc.). We aim to denoise PCG signals using another cardiac modality, the electrocardiographic (ECG) signal. In this paper, we investigate an informed non-negative matrix factorization to extract signal components out of the noisy PCG signal, considering synchronous ECG information. Our approach is applied and evaluated on a database consisting of real and artificially noisy PCG signals.

## I. INTRODUCTION

Cardiac sounds are non-stationary signals recorded and known as phonocardiogram (PCG) signals. Among these sounds, two are particularly audible (Fig. 1):  $S_1$  (which corresponds to the closure of the atrial-ventricular valves and marks the beginning of the ventricular systole) and  $S_2$  (which corresponds to the closure of aortic and pulmonary valves and marks the onset of the ventricular diastole). The analysis of these types of sounds brings information on mechanical function of the heart. However, the raw signals are roughly exploitable because of ambient interference (voice, cough, gastric noise, etc.). It is therefore necessary to remove noise components before interpretation.

Several methods of PCG denoising have been proposed based on principles of adaptive filtering [1], Kalman filtering [2], wavelets [3], or Empirical Modal Decomposition (EMD) [4]. In [1], authors have proposed an application of modified Adaptive Line Enhancement filtering to noisy PCG. This filtering is based on a recursive least squares algorithm. Authors of [2] have looked for removing noise from a PCG signal by considering respiratory sounds as interferences. A Reduced-Order Kalman Filter based on a second order autoregressive process is used. A decomposition in wavelets has been implemented in [3]: a threshold on transform coefficients allows to separate the useful signal from noise. A new non-linear Empirical Denoising Algorithm approach based on EMD has been proposed in [4] and allows denoising PCG signals without changing the sounds position. Although these methods have shown their interest, they do not exploit, or a little, the important propriety of quasi-periodicity of PCG

signals.

In this paper, in order to denoise PCG, we aim to exploit the non-negative matrix factorization (NMF) [5][6] and some derivatives [7][8] in a multimodal context coupling PCG and the electrocardiogram (ECG) sensor (Fig. 1), giving access to the electrical cardiac activity.

It is now common to record physiological signals with different types of sensors leading to multimodal recordings [9][10][11]. Information redundancy and complementarity between modalities can be used for processing and/or interpretation of physiological behaviors. In cardiac analysis, ECG and PCG signals are quasi-periodic and quasi-synchronous. There is always a delay named  $RS_1$ , which varies from one heartbeat to another one, between the occurrence of the R peak, corresponding to the positive wave of the ventricular depolarization complex (QRS) of the ECG signal, and the first sound  $S_1$  (Fig. 1). This delay and the associate jitter have to be considered for joint processing of ECG and PCG.

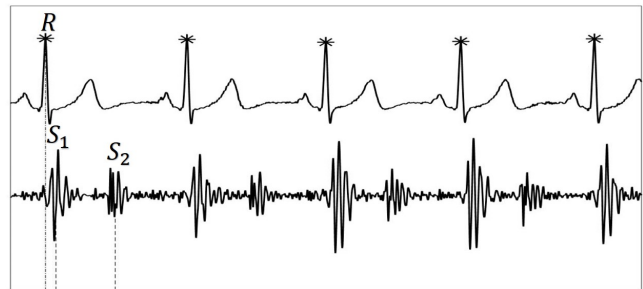


Fig. 1. Synchronous ECG and PCG signals. PCG is a quasi-periodic succession of two bumps  $S_1$  and  $S_2$ , following the  $R$  peak of ECG signal.

The NMF method decomposes a signal in components. It approximates an observation matrix  $X \in \mathbb{R}^{m \times n}$  of positive or zero coefficients by a product of two matrices with non-negative values  $W \in \mathbb{R}^{m \times k}$  and  $H \in \mathbb{R}^{k \times n}$  with  $k$  the number of estimated components:  $X = V + N$  where  $V = WH$  and  $N$  is the residual error from the approximation. With no more constraint, the NMF is not necessarily unique, but under some conditions, it may become unique (for example

with a sparsity constraint) [12]. For physiological signals, the use of NMF is of interest in time-frequency representations [13]. Indeed, the NMF algorithm applied to the spectrogram is well-suited to identify events with particular spectrum and temporal regularity (quasi-periodicity). In this case,  $W$  is the matrix of frequency patterns and  $H$ , the matrix of time activations.

Preliminary tests of NMF for PCG denoising have been carried out considering an ECG reference [14]. Satisfactory results were obtained but the multimodal aspect was just exploited as a post-processing for the decomposition achieved by the NMF. Indeed, the ECG modality is only used to select a posteriori the components related to the cardiac signal based on a rhythm analysis. It may be of interest to take into account multimodality all along the NMF processing. Hence, we propose now to adapt some existing derivative methodologies [7][8] to explore joint NMF. For one speaker identification during a discussion, [7] proposed to take advantage of both modalities, video and audio. The proposed NMF approach aims to minimize the distance between the activation signals of both modalities. In [8], a general deformation model has been proposed for audio sources separation. This model allows to constraint data according to the processing or interpretation. We aim to investigate these two existing methods and adapt the algorithms in order to propose a NMF solution for PCG decomposition, informed by ECG reference. The proposed method consists then of denoising PCG, using Wiener filtering as in [14]. Evaluation is carried out on a real signals database previously proposed to *Signal Separation Evaluation Campaign* in 2016 (SiSEC 2016) [15].

## II. SIGNALS DATABASE

Acquisitions were performed on three healthy volunteers at TIMC-IMAG laboratory as part of *MAPO-RCVQ* protocol (CHU of Grenoble promoter). PCG were recorded with a cardiac microphone (*MLT201*) put on the skin in front of the heart and maintained by a thoracic belt. *D2* lead ECG signals were simultaneously conditioned with a BioAmp amplifier. All signals were synchronously acquired with a PowerLab data acquisition system (ADInstruments) and sampled at  $1kHz$ . PCG signals were band-pass filtered between 15 and  $300Hz$ . Sixteen samples with a duration varying from 10 to 70 *seconds* have been artificially created from the filtered PCG and different real interference signals (radio, cough, pseudo-periodic noises of breath type, etc.), recorded separately. The database is composed of sixteen samples, each composed of the filtered PCG  $s(t)$ , the interference signal  $n(t)$ , the noisy PCG  $x(t) = s(t) + n(t)$  and the synchronous ECG  $ecg(t)$ .

## III. PROPOSED METHODOLOGY

In this section, before presenting our work related to our application in Section III-B, the basics of NMF are recalled

(Section III-A). Finally, the proposed informed NMF is introduced in Section III-C.

### A. NMF based on beta divergences

The derivative NMF [7][8] are based on computation of cost functions better known as  $\beta$ -divergences [16]. Such functions quantify the dissimilarity or divergence between the observation (the spectrogram in our case)  $X$  and its approximated decomposition  $V = WH$  under the constraints of positivity, leading to the following cost function

$$C(W, H) = D_\beta(X | V), \quad (1)$$

$D_\beta$  is the  $\beta$ -divergence.

To identify the components  $W$  and  $H$ , multiplicative updates are considered. To ensure the decrease of the  $\beta$ -divergence, an alternative algorithm based on Minimization-Maximization (MM) is applied [16]. In what follows,  $\beta$  is chosen equal to 2, *i.e.* the  $\beta$ -divergence is simply the Euclidian distance. This leads to estimate  $W$  knowing  $H$  as :

$$W \leftarrow W \otimes \frac{[(WH) \otimes V]H^T}{[(WH)]H^T}, \quad (2)$$

and  $H$  knowing  $W$  as :

$$H \leftarrow H \otimes \frac{W^T[(WH) \otimes V]}{W^T[(WH)]}, \quad (3)$$

where  $\otimes$  and the division are the element-wise multiplication and division, respectively and  $^T$  is the transpose operator.

The spectrogram of the noisy PCG  $x(t)$  is modeled as  $X = WH + N = W_1H_1 + W_2H_2 + N$ , where  $W_1H_1$  (*resp.*  $W_2H_2$ ) corresponds to the signal part, *i.e.* essentially clean PCG, (*resp.* the noise part, *i.e.* composed of interferences).

Compared to a classical NMF for which  $W$  and  $H$  are first estimated by (2) and (3), respectively and then sorted to identify  $W_1$ ,  $W_2$  and  $H_1$ ,  $H_2$  based on prior knowledge, the proposed methodology aims at using another modality (here the ECG) as reference to inform the NMF so that  $W_1$ ,  $W_2$  and  $H_1$ ,  $H_2$  are directly identified. To this end, some constraints on  $H_1$  based on a NMF of the ECG signal are used since the PCG signal  $s(t)$  and the ECG one share the same origin: the heart beats.

In our approach, the ECG components out of NMF will not be modified but used as reference for updating the components estimated from the noisy PCG. We will thus speak of "informed" NMF instead of joint-NMF.

### B. Related work: penalised NMF

The spectrogram of the ECG reference is modeled as :  $X_{ref} = W_{ref}H_{ref} + N_{ref}$ . The idea in this section, as in [7], is to constraint the activation signals  $H_1$  to be like the  $H_{ref}$  ones. Indeed, the ECG and PCG share the same origin so that activation patterns  $H_{ref}$  and  $H_1$  should also share the same behavior.

Following [7], one can optimize the following cost function

$$C_1(W_1, H_1, W_2, H_2) = D_2(X | W_1H_1 + W_2H_2) + \delta P(W_{ref}, H_{ref}, W_1, H_1), \quad (4)$$

where  $P$  is the penalization function defined by

$$P = \| \Lambda_{ref} H_{ref} - S \Lambda_1 H_1 \|_F^2, \quad (5)$$

with  $\Lambda_{ref}$  and  $\Lambda_1$  two diagonal matrices whose diagonal elements are the sum of each column of  $W_{ref}$  and  $W_1$ .  $S$  is a diagonal matrix for scaling  $H_1$  in order to compare it with  $H_{ref}$ .  $W_{ref}$ ,  $H_{ref}$  and initial values of  $W_1$ ,  $H_1$ ,  $W_2$  and  $H_2$  are given by the classic NMF [5] on the ECG and the noisy PCG spectrograms. The number of templates for ECG and PCG are mandatorily the same ( $k_1 = k_{ref}$ ). The penalization term  $P$  forces the activation signals  $H_1$  to be superimposed on the  $H_{ref}$  ones (up to some scaling factors).

However, this approach has several drawbacks for the considered application. First, the physiological delay between events in both modalities ECG and PCG is not considered in this method. Nevertheless, this may be corrected by integrating a mean delay to readjust positions. In addition, an ECG beat is mainly modeled by the R-peak while a PCG beat is mainly modeled by  $S_1$  and  $S_2$  sounds. Consequently, a direct comparison between  $H_{ref}$  and  $H_1$  will surely fail. Therefore the shapes of PCG time activations  $H_1$  are not accurate. Finally, there is a lack of flexibility as the approach imposes to work with the same number of templates for both modalities. This constraint is not suitable with the physiological properties of ECG and PCG signals.

### C. Proposed Informed NMF with ECG reference and transformation matrix

To bypass these limits, we investigate the proposition described in [8] in our context. Thus, the considered model is the joint equations:

$$X_{ref} = W_{ref} H_{ref} + N_{ref} \quad (6)$$

$$X = W_1 H_{ref} T_1 + W_2 H_2 + N, \quad (7)$$

where  $T_1$  is a transformation matrix that aims at transforming the activation patterns  $H_{ref}$  of the ECG into the activation ones  $H_1 \simeq H_{ref} T_1$  of the PCG by constraining these latter ones. Indeed, in particular, we intend to consider the succession of two bumps  $S_1$ ,  $S_2$  of the PCG in a quasi-periodic manner, as well as the delays between  $R$  peaks and  $S_1$  and  $S_2$  sounds (named  $RS_1$  and  $RS_2$  intervals) Fig. 1. To this end,  $T_1$  is defined as a “bidiagonal” and sparse matrix where the first diagonal is centered at  $RS_1$  value and the second diagonal at  $RS_2$  value (Fig. 2). It is worth noting that the “diagonals” should have some widths to take into account the fact that the delays between  $R$  peaks and sounds  $S_1$  and  $S_2$  are not constant.  $W_{ref}$ ,  $H_{ref}$  and initial values of  $W$  and  $H$  are computed by NMF considering  $k_{ref}$  and  $k = k_1 + k_2$  numbers of templates, with  $k_1 = k_{ref}$ . The estimation of the components is thus achieved by the minimization of:

$$C_2(W_1, T_1, W_2, H_2) = \| X - W_1 H_{ref} T_1 - W_2 H_2 \|_F^2 \quad (8)$$

Finally, based on MM algorithm, components are iteratively computed by multiplicative updates. We propose two types of structure for the transformation matrix  $T_1$ , illustrated in

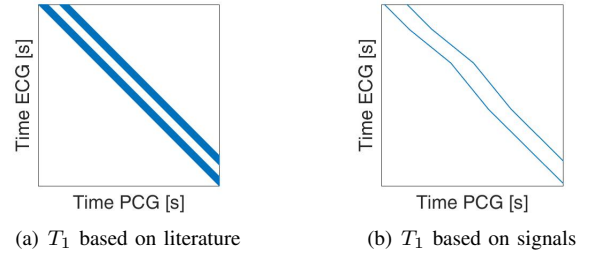


Fig. 2. Illustration of  $T_1$  structures

Fig. 2 and described in the following.

1)  $T_1$  based on the literature ( $T_{1,lit}$ ): With this choice, the “diagonals” defining  $T_1$  are chosen based on prior knowledge of average  $RS_1$  and  $S_1 S_2$  intervals. According to [17],  $RS_1$  is well modeled by a Gaussian distribution

$$RS_1 \sim \mathcal{N}(\mu_1, \sigma) = \mathcal{N}(70, 20)[ms]. \quad (9)$$

Moreover, in [18], the  $S_1 S_2$  interval is estimated to about  $300ms$ . The combination of these results leads to the following distribution of  $RS_2$

$$RS_2 \sim \mathcal{N}(\mu_2, \sigma) = \mathcal{N}(370, 20)[ms]. \quad (10)$$

The bandwidths  $RS_1 \pm \sigma$  and  $RS_2 \pm \sigma$  are chosen large enough to take into account the intra- and inter-subject variability; this introduces a too large number of degrees of freedom leading thus to a lack of constraints between  $H_{ref}$  and  $H_1 \simeq H_{ref} T_1$ .

2)  $T_1$  based on the signals themselves ( $T_{1,sig}$ ): With this choice, to overcome the lack of constraints of the prior choice described in the previous paragraph, the two “diagonals” of  $T_1$  will be defined from the signal themselves.

$RS_1$  and  $RS_2$  intervals are estimated beat by beat for each sample based on two standard NMF [5] with 12 components on noisy PCG and one component on ECG noted  $H_{ecg}$ . Let  $H^*$  denotes the PCG component that is the most correlated to  $H_{ecg}$ .  $R$  peaks are first detected on the ECG time activation  $H_{ecg}$ , then  $S_1$  and  $S_2$  sounds are detected as local maxima on  $H^*$ . The  $RS_1$  and  $RS_2$  values are used to initialize the transformation matrix putting thus more constraints on signal components structure since the width of each diagonal is smaller than in the method described in the previous paragraph.

Finally, the proposed algorithm is summarized in Algorithm 1.

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#### Algorithm 1 Proposed informed NMF

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Inputs:  $H_{ref}$ , structure of  $T_1$

**while** stopping criterion not reach **do**

    Update  $W_1$  and  $W_2$  assuming  $T_1$  and  $H_2$  based on (2)

    Update  $T_1$  and  $H_2$  assuming  $W_1$  and  $W_2$  based on (3)

Outputs:  $W_1$ ,  $W_2$ ,  $T_1$  and  $H_2$

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#### D. PCG Reconstruction with Wiener filtering

Considering the signal and noise components obtained from the proposed algorithm, an estimated PCG reconstruction  $\hat{s}(t)$ , is provided by applying a Wiener filtering. This classic filter is based on the spectral densities  $W_1 H_{ref} T_1$ , denoted  $P_s(t, f)$ , and  $W_2 H_2$ , denoted  $P_n(t, f)$ . The impulse response of the filter is :

$$H_{wiener}(t, f) = \frac{P_s(t, f)}{P_s(t, f) + P_n(t, f)}. \quad (11)$$

The estimated short term Fourier transform (STFT) of the estimated PCG  $\hat{S}(t, f)$  is thus expressed as

$$\hat{S}(t, f) = H_{wiener}(t, f) X(t, f), \quad (12)$$

where  $X(t, f)$  is the STFT of the noisy PCG  $x(t)$ . The denoised PCG  $\hat{s}(t)$  is computed as the inverse STFT of  $\hat{S}(t, f)$ .

#### IV. RESULTS

In this section, we analyze the performances of our proposed solution by comparison with a standard NMF approach. Evaluation was carried out on the PCG database proposed to SiSEC 2016 [15]. ECG and PCG spectrograms are computed using Hamming window of length 64ms and a shift of one ms, with zero-padding of 512 samples.

##### A. Evaluation settings

As we focus on two representative physiological events ( $S_1$  and  $S_2$  sounds) in noisy PCG signals, we consider  $k_1 = 2$  for the signal components. Moreover, we choose a limited number of noise components ( $k_2 = 2$ ) to force the constraints on the signal time activations. Therefore  $k_{ref} = 2$  and  $k = 4$ .

The results of our informed NMF are compared with a standard NMF [5], using the same number of components. The standard NMF is computed on ECG signal  $ecg(t)$  with  $k_{ref}$  components and on noisy PCG  $x(t)$  with  $k$  components. We consider as signal the  $k_{ref}$  PCG components whose time activations are the most correlated to the time activation of the first ECG component. The remaining PCG components are considered as noise. The Wiener filtering is applied, as in Section III-D, to reconstruct a PCG denoised estimation, noted  $\tilde{s}(t)$ .

##### B. Observation of denoised signals

Fig. 3(a) and 3(b) present examples of PCG denoising after NMF and Wiener filtering and compare results of standard NMF and informed NMF for 2 different samples of the database. The  $T_1$  structure is based on the signals themselves ( $T_{1,sig}$ ).

In these examples, informed NMF is able to remove specific noises better than standard NMF. Indeed, in Fig. 3 (a), the impulse noises around 12s and 14s do not appear anymore in the denoised PCG  $\hat{s}(t)$  with informed NMF, which is not the case for  $\tilde{s}(t)$  obtained after standard NMF. In Fig. 3 (b), the main noise is stochastic with a quasi-periodic power variation and representative of respiratory interference. Most of these interference are still present in  $\tilde{s}(t)$  whereas they have been

quite well removed in  $\hat{s}(t)$ . These two examples show that it is of interest to link the activation profiles  $H_1$  with the ones of the ECG to better estimate the PCG components from the noisy signal.

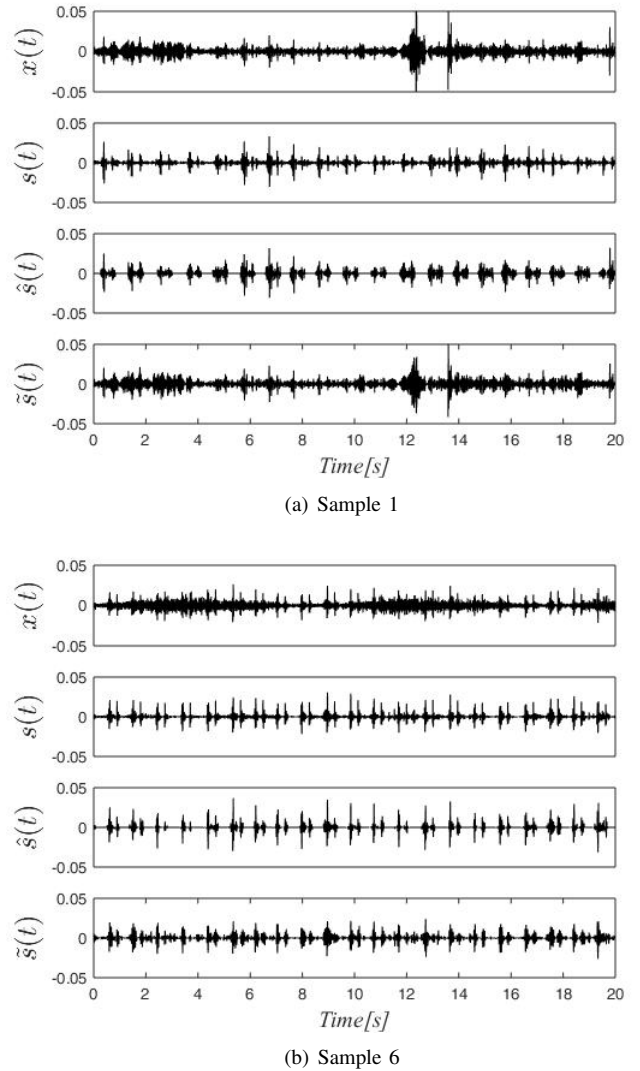


Fig. 3. Signals examples for samples 1 and 6.  $x(t)$ : noisy PCG,  $s(t)$ : original PCG,  $\hat{s}(t)$ : denoised PCG by informed NMF with  $T_{1,sig}$ ,  $\tilde{s}(t)$ : denoised PCG by standard NMF

##### C. Global performance of the proposed methodology

Quality of PCG estimation was quantified by using *BSS Eval Toolbox* [19]. We consider the Signal to Distortion Ratio (SDR) that globally estimates the Signal-to-Noise Ratio (SNR). The difference, expressed in *dB*, between the SDR computed on the estimation and on the noisy signal represents the performance gain. Boxplots represent the parameters distribution: the line inside the box corresponds to the median value, the boxlimits represent the first and the third quartiles and whiskers highlight extreme values.

The SDR gain is computed on all sixteen samples of the

SiSEC database for standard and informed NMF, for both  $T_1$  structures  $T_{1,lit}$  and  $T_{1,sig}$ .

One can see on Fig. 4 that the overall performances are comparable between the standard NMF and informed NMF with transformation matrix  $T_{1,lit}$ , and slightly better using the informed NMF with  $T_{1,sig}$  than using the standard NMF, as also shown in the previous paragraph. Indeed, the median values of SDR gain are 3.2 dB for standard NMF, 4.1 dB for informed NMF  $T_{1,lit}$  and 6.8 dB for informed NMF  $T_{1,sig}$ .

Moreover, as expected, the best performances are obtained with the  $T_1$  structure driven by the data. Limiting the 'diagonals' bandwidth of  $T_1$  improves the signal and noise components separation and therefore the PCG denoising.

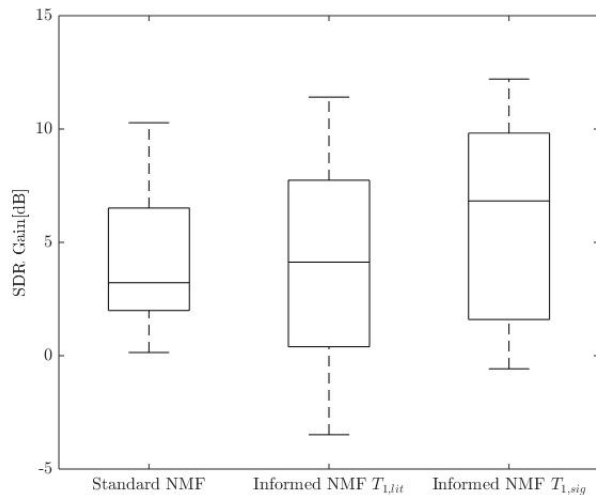


Fig. 4. SDR gain in dB for standard NMF, informed NMF with  $T_1$  based on literature and informed NMF with  $T_1$  based on the signals themselves.

## V. CONCLUSION

In this paper, we have proposed a method of PCG denoising based on NMF informed by constraining the activation profiles of the PCG to be similar to those of the ECG up to a limited transformation. This informed NMF exploits the multimodality better than in our previous work [14]. The results obtained from actual recordings are promising since they allow to remove some specific interference occurring in daily situations.

In perspective, an in-depth analysis of the parameter choices will be investigated. Some issues are noted. In particular it should be possible to constraint each signal component by a different transformation matrix considering thus the specific structure of the different components. Moreover, the quasi-periodic aspect of physiological signals is not taken into account in the informed NMF yet.

## ACKNOWLEDGMENT

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