Greedy Recovery of Sparse Signals with Dynamically Varying Support

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***Abstract*—In this paper, we propose a low-complexity greedy recovery algorithm which can recover sparse signals with time-varying support. We consider the scenario where the support of the signal (i.e., the indices of nonzero elements) varies smoothly with certain temporal correlation. We model the indices of support as discrete-state Markov random process. Then, we formulate the signal recovery problem as joint estimation of the set of the support indices and the amplitude of nonzero entries based on the multiple measurement vectors. We successively identify the element of the support based on the maximum a posteriori (MAP) criteria and subtract the reconstructed signal component for detection of the next element of the support. Our numerical evaluation shows that the proposed algorithm achieves satisfactory recovery performance at low computational complexity.**

I. Introduction

Sparsity is one of the useful properties used to process signals arising in many engineering applications. Basically, a sparse signal can be represented by a small number of coefficients in an appropriate basis. A decade of research on compressed sensing (CS) revealed that it is possible to recover the unknown signals from the measurements obtained from under-Nyquist sampling if the sparsity is properly exploited by the recovery algorithm. The theory on CS is well established in the literature [1] and various recovery algorithms have been developed for many applications including medical imaging, wireless localization, channel estimation, Radar signal processing, cognitive radio, and so on [2].

Among a variety of CS recovery algorithms, the l_1-norm minimization methods apply convex relaxation to the computationally demanding l_0-norm minimization for finding a sparse solution. This approach includes basis pursuit (BP) and BP denoising (BPDN) [3]. These algorithms still require relatively high computational complexity so that greedy recovery algorithms have been proposed. The greedy algorithms iteratively search for the locally optimal estimate of the signal support (i.e., the indices of nonzero elements in signal vector) in an iterative fashion so that their computational complexity can be reduced. The well known greedy algorithms include orthogonal matching pursuit (OMP) [4], subspace pursuit (SP) [5], and CoSaMP [6].

While the above methods were developed for the scenarios where the individual measurement vector is processed independently, we often encounter the scenarios where the measurement vector is sequentially acquired and the signal vector to be recovered is time-varying. The assumption widely adopted in MMV setup is that the support of the signal does not change over N measurement vectors while the amplitude of the signal does. The recovery algorithms developed for this setup include simultaneous OMP (SOMP) [7], convex relaxation [8], MSBL [9] and sKTS [10]. Unfortunately, the above algorithms are not suitable for the scenarios where the support of the signal changes in time continuously. In this setup, the assumption of common support might be too restrictive so that the recovery algorithms developed under the assumption of the common support does not yield good performance due to the model mismatch. Processing each measurement vector separately would not be a good choice either in that it does not fully exploit the temporal structure appearing in the multiple signal vectors. A few CS recovery algorithms have been proposed to address this problem [11]–[14] but these algorithms tend to require high computational complexity. To our best knowledge, the greedy algorithm for handing this setup has not been developed yet.

In this paper, we propose a new greedy recovery algorithm which produces the joint estimate of time-varying support and amplitude of a sparse signal sequentially at low complexity. We first formulate the signal recovery as statistical estimation of two random processes; 1) the set of the support indices and 2) the amplitude of nonzero entries of the signal vector. We model the temporal structure of the signal support using the discrete state Markov process and derive the recovery algorithm, which successively calculates the joint maximum a posteriori (MAP) estimate of each element of the support and amplitudes given the residual signal. Note that the residual signal is obtained by subtracting the effect of all signal elements detected from the measurement vectors and such greedy signal recovery is performed sequentially for each measurement vector. Our simulation results demonstrate that the proposed greedy recovery algorithm yields better recovery performance than the existing algorithms while achieving significant reduction in computational complexity.

II. Signal Model

In this section, we present the sparse recovery problem for dynamically changing sparse signal. The noisy measurement vector $\mathbf{y}_t \in \mathbb{C}^M$ can be represented as

$$\mathbf{y}_t = \Phi \mathbf{x}_t + \mathbf{w}_t \tag{1}$$

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where $\Phi \in \mathbb{C}^{M\times N}$ is the normalized sensing matrix, $w_t \in \mathbb{C}^M$ is the measurement noise and $\mathbf{x}_t \in \mathbb{C}^N$ is source signal. We assume that w_t is independent and identically distributed (i.i.d.) Gaussian $\sim \mathcal{CN}(0, \sigma_w^2 I)$ and $\mathbf{x}_t$ is the K-sparse signal, (i.e., $\mathbf{x}_t$ has K non-zero elements). The sparse signal $\mathbf{x}_t$ can be described by

$$\mathbf{x}_t = \Lambda_{\mathbf{c}_t}\mathbf{s}_t \tag{2}$$

where $\mathbf{c}_t$ denotes the set of indices in the support, $\Lambda_{\mathbf{c}_t} \in \mathbb{C}^{M\times K}$ is the matrix constructed by picking the columns specified by $\mathbf{c}_t$ from the identity matrix I, and $\mathbf{s}_t \in \mathbb{C}^K$ contains non-zero elements of $\mathbf{x}_t$. For example, a sparse vector $[0,3,0,1]^T$ can be described by $\mathbf{c}_t = \{2,4\}$, $\mathbf{s}_t = [3,1]$ and $\Lambda_{\mathbf{c}_t} = [e_1, e_2]$, where e_k is a vector which has 1 at kth entry and 0 at others. Hence, the recovery of the signal $\mathbf{x}_t$ is equivalent to estimating the two variables; the support set $\mathbf{c}_t$ and signal amplitude $\mathbf{s}_t$. We view these two variables $\mathbf{c}_t$ and $\mathbf{s}_t$ as stochastic process. We assume that the support of the signal changes smoothly in time with certain temporal correlation. Such behavior can be well modeled by Markov random process. Assuming that each element of $\mathbf{c}_t$ behaves independently, the transition probability $p(\mathbf{c}_t|\mathbf{c}_{t-1})$ for the support is given by

$$\mathbf{c}_t \sim p(\mathbf{c}_t|\mathbf{c}_{t-1}) = \prod_{k=1}^{K} p(\mathbf{c}_t^k|\mathbf{c}_{t-1}^k) \tag{3}$$

where $\mathbf{c}_t^k$ represents the kth element of $\mathbf{c}_t$. One example of the transition probability is the distribution which exponentially decays as the current state gets further from the previous state

$$p(\mathbf{c}_t^k = m|\mathbf{c}_{t-1}^k = n) = C_0\beta^{|m-n|} \tag{4}$$

where C_0 is a normalization constant and $\beta \in (0,1]$ is the model parameter. Note that β indicates the extent of variation of the support. Smaller value of β means slowly varying support. The signal amplitude $\mathbf{s}_t$ is modeled by i.i.d. Gaussian with zero mean and variance of σ_s^2.

III. Proposed Greedy Recovery Algorithm

Recovery of sparse signal $\mathbf{x}_t$ can be achieved by joint estimation of the support $\mathbf{c}_t$ and the signal amplitude $\mathbf{s}_t$ based on the sequence of measurement vectors $\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_t$. The MAP estimate of $\mathbf{c}_t$ and $\mathbf{s}_t$ can be obtained by

$$\hat{s}_t, \hat{c}_t = \arg\max_{\mathbf{s}_t,\mathbf{c}_t} p(\mathbf{s}_t, \mathbf{c}_t|\mathcal{Y}_t), \tag{5}$$

where $\mathcal{Y}_t = \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_t\}$. Next, we will present how to find the solution to (5) using a greedy algorithm.

A. Algorithm Derivation

Using $p(\mathbf{s}_t, \mathbf{c}_t|\mathcal{Y}) = p(\mathbf{s}_t|\mathbf{c}_t, \mathcal{Y})p(\mathbf{c}_t|\mathcal{Y}_t)$, we can write joint MAP estimation by

$$\begin{aligned}\hat{c}_t &= \arg\max_{\mathbf{c}_t}\max_{\mathbf{s}_t} \left(p(\mathbf{c}_t|\mathcal{Y}_t)p(\mathbf{s}_t|\mathbf{c}_t\mathcal{Y}_t)\right) \\ &= \arg\max_{\mathbf{c}_t}\left(p(\mathbf{c}_t|\mathcal{Y}_t)\left(\max_{\mathbf{s}_t} p(\mathbf{s}_t|\mathbf{c}_t, \mathcal{Y}_t)\right)\right).\end{aligned} \tag{6}$$

Given the support set $\mathbf{c}_t$, the signal amplitude $\mathbf{s}_t$ and measurements $\mathcal{Y}_t$ are jointly Gaussian. Hence, we can show that the conditional probability $p(\mathbf{s}_t|\mathbf{c}_t, \mathcal{Y}_t)$ is also Gaussian, i.e.,

$$p(\mathbf{s}_t|\mathbf{c}_t\mathcal{Y}_t) \sim \mathcal{CN}(\bar{s}_t, P_t) \tag{7}$$

where

$$\begin{aligned}\bar{s}_t &= \Sigma_{\mathbf{c}_t}^{-1}\Lambda_{\mathbf{c}_t}^H\Phi^H\mathbf{y}_t \\ P_t &= \sigma_w^2\Sigma_{\mathbf{c}_t}^{-1} \\ \Sigma_{\mathbf{c}_t} &= \Lambda_{\mathbf{c}_t}^H\Phi^H\Phi\Lambda_{\mathbf{c}_t} + \frac{\sigma_w^2}{\sigma_s^2}I.\end{aligned} \tag{8}$$

Since the maximizer of $p(\mathbf{s}_t|\mathbf{c}_t, \mathcal{Y}_t)$ is $\bar{s}_t$, (6) becomes

$$\hat{c}_t = \arg\max_{\mathbf{c}_t} p(\mathbf{c}_t|\mathcal{Y}_t)p(\bar{s}_t|\mathbf{c}_t, \mathcal{Y}_t). \tag{9}$$

If you use the logarithm function, (9) can be rewritten by

$$\hat{c}_t = \arg\max_{\mathbf{c}_t}\left[\log p(\mathbf{c}_t|\mathcal{Y}_t) + \log p(\bar{s}_t|\mathbf{c}_t, \mathcal{Y}_t)\right]. \tag{10}$$

Using Bayes' rule, $\log p(\mathbf{c}_t|\mathcal{Y}_t)$ in (10) can be expressed as

$$\log p(\mathbf{c}_t|\mathcal{Y}_t) = \log\frac{p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})p(\mathbf{c}_t|\mathcal{Y}_{t-1})}{p(\mathbf{y}_t|\mathcal{Y}_{t-1})}. \tag{11}$$

From (9) and (11), the MAP estimate $\hat{c}_t$ is given by

$$\begin{aligned}\hat{c}_t =& \arg\max_{\mathbf{c}_t}\left[\log p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1}) + \log p(\mathbf{c}_t|\mathcal{Y}_{t-1})\right. \\ &\left. + \log p(\bar{s}_t|\mathbf{c}_t, \mathcal{Y}_t)\right] + C_1,\end{aligned} \tag{12}$$

where the terms not related to $\mathbf{c}_t$ is included in the constant C_1.

Now, we aim to find the expression for $\log p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})$ and $\log p(\mathbf{c}_t|\mathcal{Y}_{t-1})$ used in (12). Using the marginalization over $\mathbf{s}_t$, the conditional probability $p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})$ can be expressed as

$$\begin{aligned}p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1}) &= \int_{\mathbf{s}_t} p(\mathbf{y}_t, \mathbf{s}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})d\mathbf{s}_t \\ &= \int_{\mathbf{s}_t} p(\mathbf{y}_t|\mathbf{c}_t, \mathbf{s}_t, \mathcal{Y}_{t-1})p(\mathbf{s}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})d\mathbf{s}_t \\ &= \int_{\mathbf{s}_t} p(\mathbf{y}_t|\mathbf{c}_t, \mathbf{s}_t)p(\mathbf{s}_t)d\mathbf{s}_t\end{aligned} \tag{13}$$

Since $\mathbf{s}_t$ and w_t are i.i.d. Gaussian, we have

$$p(\mathbf{y}_t|\mathbf{c}_t, \mathbf{s}_t) = \frac{1}{(\pi\sigma_w^2)^M}\exp\left(-\frac{\|\mathbf{y}_t - \Phi\Lambda_{\mathbf{c}_t}\mathbf{s}_t\|^2}{\sigma_w^2}\right) \tag{14}$$

$$p(\mathbf{s}_t) = \frac{1}{(\pi\sigma_s^2)^K}\exp\left(-\frac{\|\mathbf{s}_t\|^2}{\sigma_s^2}\right). \tag{15}$$

Then, $p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1})$ can be obtained as

$$p(\mathbf{y}_t|\mathbf{c}_t, \mathcal{Y}_{t-1}) = \mathbf{c}_2\exp\left(\frac{1}{\sigma_w^2}(\mathbf{y}_t^H\Phi\Lambda_{\mathbf{c}_t}\Sigma_{\mathbf{c}_t}^{-1}\Lambda_{\mathbf{c}_t}^H\Phi^H\mathbf{y}_t)\right) \tag{16}$$

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where $\mathbf{c}_2$ is the term indepedent to $\mathbf{c}_t$. The conditional probability $p(\mathbf{c}_t|\mathcal{Y}_{t-1})$ can also be obtained from

$$p(\mathbf{c}_t|\mathcal{Y}_{t-1}) = \sum_{\mathbf{c}_{t-1}} p(\mathbf{c}_t, \mathbf{c}_{t-1}|\mathcal{Y}_{t-1}) = \sum_{\mathbf{c}_{t-1}} p(\mathbf{c}_t|\mathbf{c}_{t-1}) p(\mathbf{c}_{t-1}|\mathcal{Y}_{t-1}) \quad (17)$$

Using (11), (16) and (17) and , we can express $p(\mathbf{c}_t|\mathbf{y}_t)$ as

$$\log p(\mathbf{c}_t|\mathcal{Y}_t) = \log \mathbf{c}_3 + \left(\frac{1}{\sigma_w^2}(\mathbf{y}_t^H \Phi \Lambda_{\mathbf{c}_t} \Sigma_{\mathbf{c}_t}^{-1} \Lambda_{\mathbf{c}_t}^H \Phi^H \mathbf{y}_t)\right) + \log \sum_{\mathbf{c}_{t-1}} p(\mathbf{c}_t|\mathbf{c}_{t-1}) p(\mathbf{c}_{t-1}|\mathcal{Y}_{t-1}) \quad (18)$$

where $\log C_3 = \log C_2 + C_1$. Finally, from (10) and (18) the MAP estimate $\hat{c}_t$ is given by

$$\hat{c}_t = \arg\max_{\mathbf{c}_t} \left[\log \mathbf{c}_3 + \frac{1}{\sigma_w^2}(\mathbf{y}_t^H \Phi \Lambda_{\mathbf{c}_t} \Sigma_{\mathbf{c}_t}^{-1} \Lambda_{\mathbf{c}_t}^H \Phi^H \mathbf{y}_t) + \log \sum_{\mathbf{c}_{t-1}} p(\mathbf{c}_t|\mathbf{c}_{t-1}) p(\mathbf{c}_{t-1}|\mathcal{Y}_{t-1}) + \log p(\bar{s}_t|\mathbf{c}_t, \mathcal{Y}_t)\right] \quad (19)$$

Once $\hat{c}_t$ is obtained from (19), we can obtain the MAP estimate of $\mathbf{s}_t$ with respect to $\hat{c}_t$

$$\hat{s}_t = \arg\max_{\mathbf{s}_t} p(\mathbf{s}_t|\hat{c}_t, \mathcal{Y}_t) = \Sigma_{\hat{c}_t}^{-1} \Lambda(\hat{c}_t)^H \Phi^H \mathbf{y}_t \quad (20)$$

We take a close look at the objective function in (19). The second and fourth terms in the right hand side can be easily calculated for the given candidate for $\mathbf{c}_t$. The third term needs summation over all possible states of $\mathbf{c}_{t-1}$, which requires computationally intensive operations. Note that the term $p(\mathbf{c}_t|\mathcal{Y}_t)$ can be recursively updated from $p(\mathbf{c}_{t-1}|\mathcal{Y}_{t-1})$ using (18). Even through we do not know the value of $\mathbf{c}_3$, the distribution $p(\mathbf{c}_t|\mathcal{Y}_t)$ can be obtained by normalizing it such that the distribution is summed to one. In order to find the maximizer $\hat{c}_t$ in (19), we have to search over all possible combinations of $\mathbf{c}_t$. For sparse-K signal $\mathbf{x}_t$, the total number of all possible combinations for $\mathbf{c}_t$ is $\binom{N}{K}$, which can require huge search complexity for large value of N. Hence, we employ the suboptimal search strategy that finds the element of $\mathbf{c}_t$ iteratively.

B. Greedy Recovery of Support

As mentioned, the complexity for solving the equation (19) is infeasible. Thus, we use the greedy algorithm which finds the element of $\mathbf{c}_t$ maximizing the objective function one by one. The proposed search strategy is inspired by that of the OMP algorithm. The OMP identifies each element of support using the approximation that the residual signal contains the signal with only single nonzero entry. That is, the correlating and projection operations performed in OMP are optimal if we consider the support of the size one.

Adopting the spirit of the OMP, we iteratively find each element of the support $\mathbf{c}_t$ assuming that the support size is one. Whenever we detect the element of support, we will construct the residual signal by subtracting the estimated signal component from the measurement vector. In order to select the dominant support index one by one, we assume that the residual signal vector contain the signal vector with single nonzero entry. Then, we can show that the second term in the right hand side in (19) becomes

$$\frac{1}{\sigma_w^2}((r_t^{(l)})^H \Phi \Lambda_{\mathbf{c}_t} \Sigma_{\mathbf{c}_t}^{-1} \Lambda_{\mathbf{c}_t}^H \Phi^H r_t^{(l)}) = \frac{1}{\sigma_w^2}\left(\phi_{\mathbf{c}_t^k}^H \phi_{\mathbf{c}_t^k} + \frac{\sigma_w^2}{\sigma_x^2}\right)^{-1} \left\|\phi_{\mathbf{c}_t^k}^H r_t^{(l)}\right\|^2, \quad (21)$$

where $r_t^{(l)}$ is the residual signal at the lth iteration and $\phi_{\mathbf{c}_t^k}$ is $\mathbf{c}_t^k$th column of Φ. In addition, the third term can be evaluated only over single element of $\mathbf{c}_k$. Thus we get

$$\log \sum_{\mathbf{c}_{t-1}} p(\mathbf{c}_t|\mathbf{c}_{t-1}) p(\mathbf{c}_{t-1}|\mathcal{Y}_{t-1}) = \log \sum_{\mathbf{c}_{t-1}^k} p(\mathbf{c}_t^k|\mathbf{c}_{t-1}^k) p(\mathbf{c}_{t-1}^k|\mathcal{Y}_{t-1}) \quad (22)$$

We can also show that the last term is equal to $\left(-\log \pi^k |P_t|\right)$. Finally, the objective function for our greedy recovery algorithm is given by

$$\psi(\mathbf{c}_t^k) = \frac{1}{\sigma_w^2}\left(\phi_{\mathbf{c}_t^k}^H \phi_{\mathbf{c}_t^k} + \frac{\sigma_w^2}{\sigma_x^2}\right)^{-1} \left\|\phi_{\mathbf{c}_t^k}^H r_t^{(l)}\right\|^2 + \log \sum_{\mathbf{c}_{t-1}^k} p(\mathbf{c}_t^k|\mathbf{c}_{t-1}^k) p(\mathbf{c}_{t-1}^k|\mathcal{Y}_{t-1}) + \log p(\bar{s}_t|\mathbf{c}_t, \mathcal{Y}_t) \quad (23)$$

Algorithm 1 Proposed greedy recovery algorithm

Initialize : $r_t^{(l)} = \mathbf{y}_t, \Omega = \{\}$

1: **for** $i = 1$ to K... **do**

2: Select the support index and the corresponding prior

$$\hat{c}_t^i = \arg\max_{\mathbf{c}_t^i} \psi(\mathbf{c}_t^i)$$

3: Update the support set Ω, $\hat{s}_t^i$ and residual $r_t^{(i)}$

$$\Omega = \Omega \cup \hat{c}_t^i$$
$$\hat{s}_t^{(i)} = \Sigma_\Omega^{-1} \Lambda(\Omega)^H \Phi^H y$$
$$r_t^{(l)} = \mathbf{y}_t - \Phi \Lambda_{\hat{c}_k^{(l)}} \hat{s}_k^{(l)}$$

4: Save the $p(\mathbf{c}_t^i|\mathbf{y}_t)$ to exploit in next time step as

$$p(\mathbf{c}_t^i|\mathcal{Y}_t) = \mathbf{c}_3 \exp\left[\psi(\mathbf{c}_t^i)\right]$$

5: **end for**

6: Selection step: Find approximate MAP estimate as

$$\hat{x}_t = \Lambda(\Omega) \hat{s}_t^{(K)}$$

7: set $t = t + 1$ and go to step 1

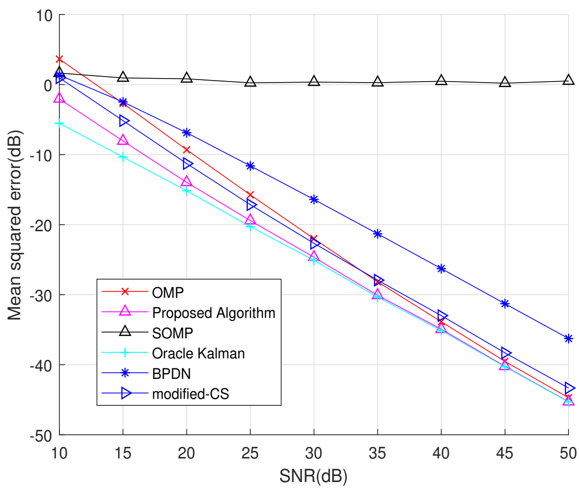

Fig. 1. MSE performance

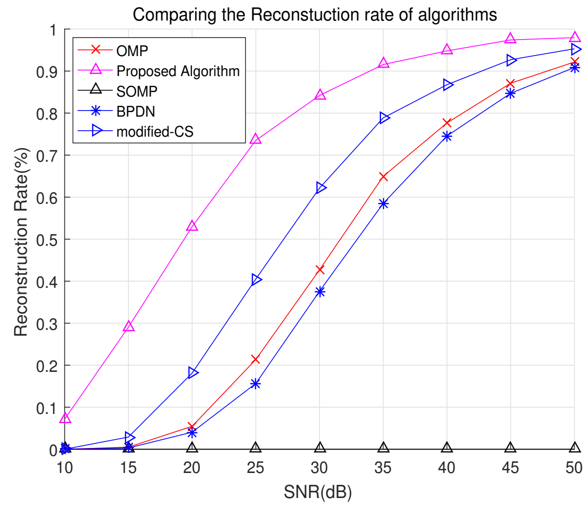

Fig. 2. Reconstruction rate performance

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY

	Runtime(s)	Complexity order
Proposed Algorithm	1.64	$\mathcal{O}(KN^2)$
OMP	0.54	$\mathcal{O}(KMN)$
BPDN	473.8	$\mathcal{O}(M^2N^3)$ [?]
modified CS	507.1	$\mathcal{O}(M^2N^3)$

Note that the first term of (23) can be found in the objective function of OMP. Two additional terms are added to reflect the temporal correlation underlying in the support. Note also that the update for $p(\mathbf{c}_{t-1}^k|\mathcal{Y}_{t-1})$ can be obtained by modifying (18) into

$$\log p(\mathbf{c}_t^k|\mathcal{Y}_t) = \log \mathbf{c}_3 + \psi(\mathbf{c}_t^k). \tag{24}$$

The proposed greedy algorithm is summarized in Algorithm 1. In the each iteration, the proposed method evaluates the objective function in (23) for each element of $\mathbf{c}_t$ which has not been selected until the current iteration. The effect of the detected element of the support is subtracted from the residual signal to generate the updated residual signal, i.e.,

$$r_t^{(l)} = \mathbf{y}_t - \Phi\Lambda_{\hat{c}_k^{(l)}}\hat{s}_k^{(l)}, \tag{25}$$

where $\hat{c}_k^{(l)}$ and $\hat{s}_k^{(l)}$ are the support set and signal amplitudes found until the lth iteration.

IV. SIMULATIONS

We evaluate the recovery performance of the proposed algorithm using simulations. We use the 128×256 normalized i.i.d. Gaussian matrix as a sensing matrix. We adopt the signal transition model $p(\mathbf{c}_t^k = m|\mathbf{c}_{t-1}^k = n) = C_0\beta^{|m-n|}$ with $\beta = 0.1$. The signal sparsity K is set to 20, the support transition parameter. Table. I provides the complexity of the several recovery algorithms of interest. We measure the processing time needed to perform 1000 repetitions of recovery from the single measurement vector. We compare the complexity of the proposed algorithm with that of OMP, BPDN, and modified CS [11]. Note that the complexity of the proposed greedy algorithm is much faster than the BPDN and modified CS. Though the proposed scheme is three times slower than OMP, it can handle dynamically changing sparse signal.

Fig. 1 and Fig. 2 provide the accuracy of the signal recovery algorithms. We conduct Monte Carlo simulations where 10,000 measurement vectors are randomly generated and the number of perfect recovery of support set is counted. MSE is calculated by $E\left[|x-\hat{x}|^2\right]$ and our reconstruction rate is the rate that the algorithm find the correct support set among 10,000 signal vectors. The MSE and recovery rate are provided as a function of SNR. The proposed algorithm achieves the significant performance gain over the existing recovery algorithms including OMP, SOMP [7], BPDN, modified CS [11]. Note that SOMP is the algorithm developed for MMV setup and the modified CS is the sequential recovery algorithms used for the dynamically changing sparse signal. In particular, we observe that the performance gap increases for low SNR range. This seems to be why the proposed algorithm exploits the temporal correlations in the multiple measurement vectors for recovery effectively.

V. CONCLUSION

In this paper, we have proposed the greedy recovery algorithm for time-varying sparse model. We provided the dynamically changing sparse signal model where the time-varying support is described by discrete-state Markov process. We present the greedy recovery algorithm which detects each element of support set in MAP criterion. Numerical evaluation show that the proposed algorithm achieves the significant gain in recovery performance over the existing recovery algorithms.

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