A Frequency-Domain Band-MMSE Equalizer for Continuous Phase Modulation over Frequency-Selective Time-Varying Channels

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***Abstract*—In this paper, we consider single carrier continuous phase modulations (*CPM*) over frequency selective time-varying channels. In this context, we propose a new low-complexity frequency-domain equalizer based on the minimum mean square error (*MMSE*) criterion exploiting efficiently the band structure of the associated channel matrix in the frequency domain. Simulations show that this band-MMSE equalizer exhibits a good performance complexity trade-off compared to existing solutions.**

I. Introduction

Continuous Phase Modulations (*CPM*) are widely known for their good spectral occupancy, their low-energy consumption and their constant envelope, which make them robust to non-linearities as the ones introduced by embedded amplifiers. In the past few years, they have been considered for a wide range of applications, such as the Internet of Things, military communication (Ultra High Frequency band), 60Ghz communication or also aeronautical communication for Unmanned Aerial Systems (*UAS*).
Due to their non linear structure, making a CPM transmission over frequency selective channels is by nature a very challenging problem leading to advanced receiver techniques to mitigate intersymbol interference. Several papers address this issue in the case of time-invariant channels. The optimal approach consists in considering jointly channel equalization and CPM detection, using an extended trellis. However, this approach has a prohibitive complexity as the trellis size grows exponentially with the CPM memory and the channel length. Hence, a common approach is to separate the equalization from the CPM detection at the receiver side. In the case of Time-Invariant (*TIV*) channel, a viable strategy is to perform Frequency-Domain (*FD*) equalization, before feeding a CPM detector developed for AWGN channel. Low-complexity schemes can be achieved by capitalizing on the diagonalization of the channel matrix in the frequency-domain. In [1], the authors present a minimum mean square error (*MMSE*) based equalizer, a decision-feedback and also a turbo-equalizer. They all rely on the Laurent Decomposition [2], estimating its transmitted pseudo-symbols. However, this implies the use of a non-conventional detector as the one in [3]. In [4], the authors propose a slightly different version of the FD-MMSE equalizer based on a polyphase representation of the received signal. In this case, the output of the equalizer consists of an equalized signal which fed, in this case, a conventional CPM detector [5]. [6] shows that the equalizers proposed in [1] and [4] have the same performance if a proper post-processing of the equalizers outputs in [1] is done. In [7], [8], the authors present low-complexity Frequency-domain MMSE Equalizers by not taking into account the auto-correlation of CPM signals. Indeed, this allows them to obtain a ”one-tap” equalizer with an overall complexity dominated by the (Inverse) Discrete Fourier Transform. However, all those Frequency-Domain equalizers use the hypothesis of a TIV channel, which is not true in case of large Doppler Spread as, for example, in the aeronautical channel [9]. In this case, the channel is frequency-selective (multiple paths) and time-varying, leading to an even harder equalization problem. Surprisingly, to the authors’ knowledge, only few papers deal with equalization for CPM over Time-Varying (*TV*) channels.

In [10], the authors develop time-varying MMSE and Zero-Forcing (*ZF*) equalizers in the time-domain, as they deal with multipaths channels and large Doppler Spread. They represent the TV channel with the popular Basis Expansion Model (*BEM*) [11] and they capitalize on the Laurent representation for binary CPMs, which allows them to write their CPM (GMSK) signal as a sum of PAM waveforms and to reduce the complexity of their overall receiver. However, the computational complexity is still high as it requires full-matrix inversion, which has a cubic dependency on the time-dispersion of the channel.
In this paper, we present a new Frequency-Domain Equalizer for CPM over time-varying and frequency selective channels. As the channel matrix in the frequency-domain is no more diagonal, it is often argued that the complexity of such an equalizer can be prohibitive as it requires a full-matrix inversion, and thus, time domain equalization should be preferred. However, by enforcing, as an approximation, a band structure for both the channel matrix and the CPM signal pseudo-

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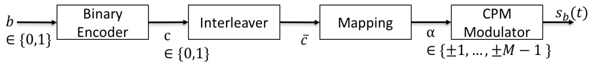

Fig. 1. CPM BICM Transmitter

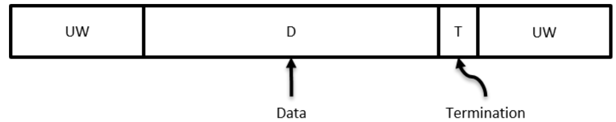

Fig. 2. Block-based structure of the CPM signal

symbols correlation matrix, we will show that it is possible to reduce drastically the overall complexity for FD equalization of CPM signals over TV multi-paths channels. The proposed approach enables a lower complexity than existing time-domain approaches and performs better at low signal to noise ratio (SNR) in the uncoded case, which is a good feature for the coded case when iterative detection and decoding is performed. However, this reduction of complexity comes at the price of a reasonable performance loss in the high SNR regime in the uncoded case.
The paper is organized as follows. In Section II, we present the system model for block-based CPM over TV channels. We present the proposed band-MMSE frequency-domain equalizer in Section III and show how to exploit the band structure of the equalizer matrix to reduce the computational complexity, which is discussed in Section IV. Some simulations results are given in Section V. Conclusions and perspectives are drawn in Section VI.

II. Block-based CPM representation

A. Notations

In the following, a vector will be represented by an underlined letter (e.g. $\underline{\boldsymbol{v}}$, $\underline{\boldsymbol{V}}$) and a matrix by a doubly underlined letter (e.g. $\underline{\underline{\boldsymbol{m}}}$, $\underline{\underline{\boldsymbol{M}}}$). Apart from transform matrices, small letters (resp. capital letters) for vectors or matrices will refer to as time domain quantities (resp. *frequency domain* quantities). We note $\underline{\underline{\boldsymbol{F}}}_N$ the Fourier transform matrix of size $N \times N$ which corresponds to a FFT of size N. The matrix $\underline{\underline{\boldsymbol{I}}}_N$ is the identity matrix of size $N \times N$.

B. Communication system description

We consider the general Bit Interleaved Coded Modulation (***BICM***) transmission scheme for CPM, as given in Fig 1. Let $\{\alpha_n\}_{0\leq n\leq N-1} \in \{\pm 1, \pm 3, \ldots, \pm M-1\}^N$ be a sequence of N symbols taken from the M-ary alphabet . The complex envelope $s_b(t)$ associated with the transmitted CPM signal is written as follows

$$s_b(t) = \sqrt{\frac{2E_s}{T}}\exp\left(j\theta(t,\boldsymbol{\alpha})\right) \tag{1}$$

where

$$\theta(t,\boldsymbol{\alpha}) = 2\pi h \sum_{i=0}^{N-1} \alpha_i q(t-iT)$$

and

$$q(t) = \begin{cases} \int_0^t g(\tau)d\tau, t \leq L_{\text{cpm}} \\ 1/2, t > L_{\text{cpm}} \end{cases}$$

E_s is the symbol energy, T is the symbol period, $\theta(t,\boldsymbol{\alpha})$ is the information phase, $q(t)$ is the phase response, h is the modulation index and L_{cpm} is the CPM memory.
Let us now consider a transmission over a TV channel $h_c(t,\tau)$. At the receiver, we assume ideal low-pass filtering using the front-end filter $\Psi(t)$ and ideal synchronization. Denoting $h(t,\tau) = \Psi * h_c(t,\tau)$, where $*$ is the convolution operator, the received signal can be written as:

$$r(t) = \sum_m s(m\frac{T}{k})h\big(t, t-m\frac{T}{k}\big) + w(t), \tag{2}$$

where $w(t)$ is a complex baseband additive white Gaussian noise with power spectral density $2N_0$, and k is the oversampling factor.
In order to perform a frequency-domain equalization, we need to *circularize* the channel. As for linear modulations, we can derive a block-based model by using a Cyclic Prefix (CP) or a known Unique Word (UW), also called training sequence. We will consider here the use of a UW. Indeed, even if the introduction of a UW brings a higher loss of spectral efficiency, compared to the introduction of a CP, this allows to increase the performance of the Block Decision Feedback Equalizer [12] and to perform some useful estimations, such as the carrier phase and frequency or the channel parameters [13]. However, obtained results can still be extended to a CP approach. Unlike for linear modulation, due to the CPM memory, we need to add some termination symbols at the end of the data block in order to ensure the phase continuity and the uniqueness of the UW, as illustrated in Fig.2. Moreover, the length of a UW must be larger than the time dispersion of the channel to avoid interference between CPM blocks.

C. Baseband representation

Using a Fractionnally-Spaced representation of the received signal, we have the following expression:

$$\begin{aligned} r[l] = r\left(\frac{lT}{k}\right) &= \sum_m s(m\frac{T}{k})h\Big(l\frac{T}{k}, (l-m)\frac{T}{k}\Big) + w\left(l\frac{T}{k}\right) \\ &= \sum_m s[m]h[l;l-m] + w[l] \end{aligned} \tag{3}$$

By defining the channel matrix $\underline{\underline{\boldsymbol{h}}}$ as given in Eq.(5) where L is the channel span, the signal has the following matrix-wise representation:

$$\begin{aligned}
&\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{h}}}\underline{\boldsymbol{s}} + \underline{\boldsymbol{w}} && (4)\\
&\quad \text{with } \underline{\boldsymbol{r}} = [r[0], r[1], \ldots, r[kN-1]]^T\\
&\quad \underline{\boldsymbol{s}} = [s[0], s[1], \ldots, s[kN-1]]^T\\
&\quad \text{and } \underline{\boldsymbol{w}} = [w[0], w[1], \ldots, w[kN-1]]^T
\end{aligned}$$

Then, the received signal in the FD is now:

$$\begin{aligned}
\underline{\boldsymbol{R}} &= \underline{\underline{\boldsymbol{F}}}_{kN}\underline{\boldsymbol{r}}\\
&= \underbrace{\underline{\underline{\boldsymbol{F}}}_{kN}\underline{\underline{\boldsymbol{h}}}\underline{\underline{\boldsymbol{F}}}^H_{kN}}_{=\underline{\underline{\boldsymbol{H}}}}\underbrace{\underline{\underline{\boldsymbol{F}}}_{kN}\underline{\boldsymbol{s}}}_{=\underline{\boldsymbol{S}}} + \underbrace{\underline{\underline{\boldsymbol{F}}}_{kN}\underline{\boldsymbol{w}}}_{=\underline{\boldsymbol{W}}} \qquad (6)
\end{aligned}$$

In the case where the channel is time-invariant, the matrix $\underline{\underline{\boldsymbol{h}}}$ is circulant. Therefore, $\underline{\underline{\boldsymbol{H}}}$ will be a diagonal matrix. For time-varying channels, $\underline{\underline{\boldsymbol{H}}}$ is not a diagonal matrix. However, it can be well approximated using a band matrix [14]. We note Q the number of lower and upper diagonals retained from $\underline{\underline{\boldsymbol{H}}}$. By defining the matrix $\underline{\underline{\boldsymbol{B}}}^{(Q)}$ as having only non-zeroes on the Q lower and upper diagonals, we approximate our matrix $\underline{\underline{\boldsymbol{H}}}$ by

$$\underline{\underline{\boldsymbol{H}}}_Q = \underline{\underline{\boldsymbol{B}}}^{(Q)} \circ \underline{\underline{\boldsymbol{H}}} \qquad (7)$$

where $\circ$ is the element-wise product operator. The approximated received signal can be finally written as

$$\underline{\boldsymbol{R}}_Q = \underline{\underline{\boldsymbol{H}}}_Q\underline{\boldsymbol{S}} + \underline{\boldsymbol{W}} \qquad (8)$$

III. Band FD-MMSE Equalizer

In this section, we present the proposed low-complexity band MMSE equalizer for CPM over TV channels. Let $\underline{\underline{\boldsymbol{J}}}_{\text{MMSE}}$ be the matrix of size $kN \times kN$ minimizing the following Mean Square Error (*MSE*) criterion:

$$\text{MSE} = \mathbb{E}\Big\{(\underline{\boldsymbol{S}} - \underline{\underline{\boldsymbol{J}}}_{\text{MMSE}}\underline{\boldsymbol{R}}_Q)^H(\underline{\boldsymbol{S}} - \underline{\underline{\boldsymbol{J}}}_{\text{MMSE}}\underline{\boldsymbol{R}}_Q)\Big\} \qquad (9)$$

The linear block MMSE equalizer is given by:

$$\underline{\underline{\boldsymbol{J}}}_{\text{MMSE}} = \underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H_Q\underline{\underline{\boldsymbol{K}}}^{-1} \qquad (10)$$

with $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{H}}}_Q\underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H_Q + N_0\underline{\underline{\boldsymbol{I}}}_{kN}$ and $\underline{\underline{\boldsymbol{R}}}_{\text{SS}}$ is the auto-correlation matrix of $\underline{\boldsymbol{S}}$. We can note that the correlation matrix $\underline{\underline{\boldsymbol{R}}}_{\text{BB}}$ of the pseudo-symbols vector $\underline{\boldsymbol{B}}$ and so the correlation matrix $\underline{\underline{\boldsymbol{R}}}_{\text{SS}}$ can be precomputed (see [4]) as $\underline{\underline{\boldsymbol{R}}}_{\text{SS}} = \underline{\underline{\boldsymbol{L}}}\underline{\underline{\boldsymbol{R}}}_{\text{BB}}\underline{\underline{\boldsymbol{L}}}^H$ where $\underline{\underline{\boldsymbol{L}}}$ is the Laurent Pulses matrix, using [2]. This can be also extended to the M-ary case. To keep a band structure for our equalizer, we have to substitute the autocorrelation matrix $\underline{\underline{\boldsymbol{R}}}_{SS}$ by a truncated band version defined as $\underline{\underline{\boldsymbol{R}}}_{SS,Q} = \underline{\underline{\boldsymbol{B}}}^{(Q)} \circ \underline{\underline{\boldsymbol{R}}}_{SS}$. Then, the proposed band-MMSE equalizer is explicitly written as follows:

$$\underline{\underline{\boldsymbol{J}}}_{\text{MMSE,Q}} = \underline{\underline{\boldsymbol{R}}}_{\text{SS},Q}\underline{\underline{\boldsymbol{H}_{\boldsymbol{Q}}}}^H[\underline{\underline{\boldsymbol{H}}}_Q\underline{\underline{\boldsymbol{R}}}_{\text{SS,Q}}\underline{\underline{\boldsymbol{H}}}^H_Q + N_0\underline{\underline{\boldsymbol{I}}}_{kN}]^{-1} \qquad (11)$$

which is now also a band-matrix by construction.

The computation of the inverse of the matrix $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{H}}}_Q\underline{\underline{\boldsymbol{R}}}_{\text{SS,Q}}\underline{\underline{\boldsymbol{H}}}^H_Q + N_0\underline{\underline{\boldsymbol{I}}}_{kN}$ can be seen at first sight as computationally prohibitive, which is often argued for FD equalization over TV channels. However, by enforcing this band structure, as done above, we can exploit this structure to perform low-complexity equalization. Indeed, following the idea of [15], the equalization can be computationally efficient using the LDL Decomposition. Thus, the following procedure can be applied to efficiently equalize the received signal:

- Compute the band matrix $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{H}}}_Q\underline{\underline{\boldsymbol{R}}}_{\text{SS,Q}}\underline{\underline{\boldsymbol{H}}}^H_Q + N_0\underline{\underline{\boldsymbol{I}}}_{kN}$,
- Compute the LDL decomposition of $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{L}\boldsymbol{D}\boldsymbol{L}}}^H$ where $\underline{\underline{\boldsymbol{L}}}$ is a lower triangular matrix and $\underline{\underline{\boldsymbol{D}}}$ a diagonal matrix following [15],
- Solve the triangular system $\underline{\underline{\boldsymbol{L}}}\underline{\boldsymbol{f}} = \underline{\boldsymbol{R}}$,
- Solve the diagonal system $\underline{\underline{\boldsymbol{D}}}\underline{\boldsymbol{g}} = \underline{\boldsymbol{f}}$,
- Solve the triangular system $\underline{\underline{\boldsymbol{L}}}^H\underline{\boldsymbol{d}} = \underline{\boldsymbol{g}}$,
- Compute $\widehat{\underline{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{R}}}_{\text{SS,Q}}\underline{\underline{\boldsymbol{H}}}^H_Q\underline{\boldsymbol{d}}$.

In the case of time-invariant channels, $\underline{\underline{\boldsymbol{H}}}$ is a diagonal matrix by DFT properties. Then, by choosing $Q = kN$, we obtain the following linear block MMSE equalizer :

$$\underline{\underline{\boldsymbol{J}}}_{\text{MMSE, TIV}} = \underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H[\underline{\underline{\boldsymbol{H}}}\underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H + N_0\underline{\underline{\boldsymbol{I}}}_{kN}]^{-1} \qquad (12)$$

This equalizer corresponds to the FD-MMSE equalizers for CPM of [1], [4] which are equivalent up to a proper post-processing [6].

Moreover, by approximating the autocorrelation matrix $\underline{\underline{\boldsymbol{R}}}_{\text{SS}}$ by the identity matrix and by choosing $Q = 0$, we obtain the following equalizer:

$$\underline{\underline{\boldsymbol{J}}}_{\text{approx, TIV}} = \underline{\underline{\boldsymbol{H}}}^H[\underline{\underline{\boldsymbol{H}}}\underline{\underline{\boldsymbol{H}}}^H + N_0\underline{\underline{\boldsymbol{I}}}_{kN}]^{-1} \qquad (13)$$

We remark that $\underline{\underline{\boldsymbol{J}}}_{\text{approx, TIV}}$ is a diagonal matrix:

$$J_{\text{approx, TIV}}[l] = \frac{H[l]^*}{|H[l]|^2 + N_0} \qquad (14)$$

which corresponds to the FD linear MMSE equalizers for TIV channels proposed in [7], [8].

IV. Complexity Analysis

In this section, we discuss the computational complexity of the proposed band MMSE equalizer. Due to the band structure of the matrix $\underline{\underline{\boldsymbol{K}}}$ and similarly to [15], only $(2Q^2 + 3Q)kN$ complex multiplications, $(2Q^2 + Q)kN$ complex additions and $2QkN$ complex divisions are required. Then, only one diagonal and two triangular systems have to be solved in order to equalize the received signal. They can be solved by band forward and backward substitution [16]. By taking into account the Fourier Transform, the overall complexity of the proposed equalizer is in the order of $O\big(kN(2Q^2 + Q + \log(kN)\big)$.

In the special case where $Q = 0$, the matrix $\underline{\underline{\boldsymbol{H}}}_Q$ and $\underline{\underline{\boldsymbol{R}}}_{\text{SS},Q}$ are both diagonal matrices. Hence, our equalizer does not require the LDL factorization of $\underline{\underline{\boldsymbol{K}}}$ as the inversion has a lower complexity complexity. Then, the overall complexity of our equalizer is dominated by the complexity of the Fast Fourier Transform, which is in $O\big(kN\log(kN)\big)$.

We also evaluate the computational complexity of the Time-Domain Equalizer [10]. For each symbols sent, this equalizer requires the inversion of a correlation matrix noted $\underline{\underline{\boldsymbol{R}}}_{\text{zz}}$ of size kL_e where k is the oversampling factor and L_e is a parameter

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$$\underline{\underline{\boldsymbol{h}}} = \begin{bmatrix} h[0,0] & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ h[L-1,L-1] & & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & 0 \\ 0 & \dots & h[kN-1,L-1] & \dots & h[kN-1,0] \end{bmatrix} + \begin{bmatrix} 0 & \dots & h[0,L-1] & \dots & h[0,1] \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & & \ddots & & h[L-2,L-1] \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} \quad (5)$$

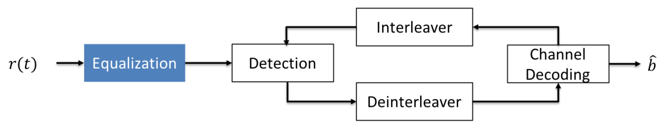

Fig. 3. Overall Receiver Structure

corresponding to the number of tap of the equalizer in number of symbols. Typical values of L_e used in [10] are in the set $\{2, 3, 4, 5\}$. Hence, the equalizer has a overall complexity of $O\big(N(kL_e)^3\big)$ which is of higher complexity compared to the proposed FD equalizer. However, the performance of the equalizer given in [10] does not suffer from the approximations we have done to ensure the band matrix structure. Thus, lower complexity is expected from our structure but at the price of some (hopefully) reasonable loss of performance.

V. Simulations results

In this section, we present some simulations results. We consider a binary CPM scheme with a raised-cosine (*RC*) pulse shape, a memory of $L_{\text{CPM}} = 3$ and a modulation index $h = 1/2$ in the C-band. The transmitted signal is composed by 9 block of 512 symbols, where a block is divided into a data block and a Unique Word of 36 symbols. As we consider a BICM scheme, as an outer coding scheme, we use a convolutional code with polynomial generator $(5, 7)_8$ given in octal. The overall structure of the receiver is illustrated in Fig.3.

The channel considered here is the "En Route" aeronautical channel by satellite with a $C/M = 5$dB and a Doppler Spread of 500Hz [9]. We assume that the channel is perfectly known at the receiver.

Fig.4 plots the obtained bit error rate as a function of E_s/N_0, for several values of Q and considering an iterative concatenated scheme using the proposed equalizer with 20 iterations between the CPM Detector and the MAP channel decoder. We can see that choosing $Q = 1$ instead of $Q = 0$ leads to a gain of almost 2dB at a BER of 2×10^{-2}. Then, the improvement is up to 4 dB at a BER of 10^{-2}.

The influence of the number of iteration between the CPM Detector and the Channel Decoder and of the parameter Q is shown in Fig.5 for several values of Q. We can see that the choice of those two parameters (number of iteration and Q) has a critical impact on the BER of the overall receiver. However, there is a trade-off to find between performance and computational complexity because increasing Q will increase

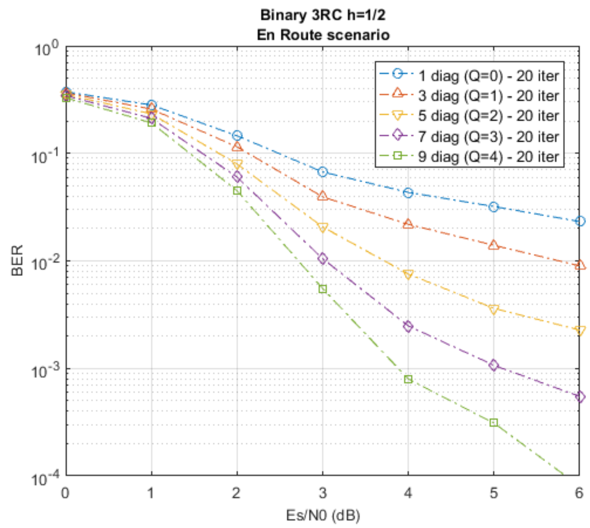

Fig. 4. Influence of the parameter Q

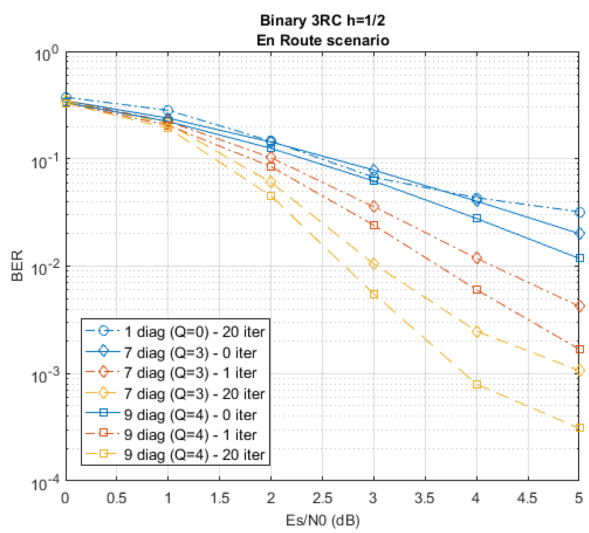

Fig. 5. Influence of the parameter Q and of the number of iteration

the complexity of the equalizer, whereas increasing the number of iterations will increase the overall complexity and also the latency of our receiver.

Finally, Fig.6 shows the performance of our receiver in the context of [10] and compare it to the LTV-MMSE receiver. We consider an uncoded binary GMSK with $h = 1/2$ and a

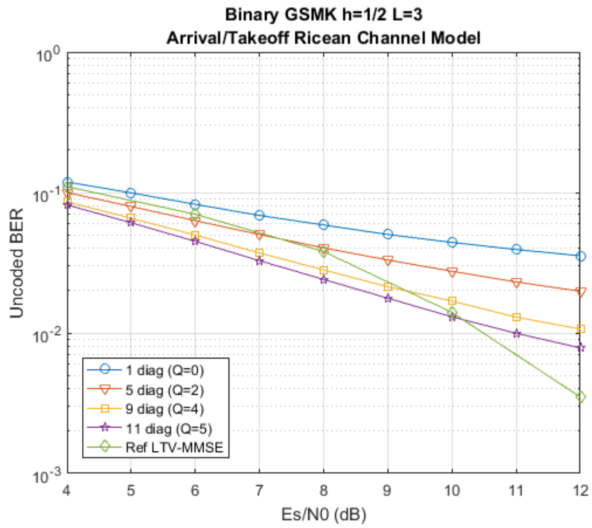

Fig. 6. Comparison with the LTV-MMSE [10]

memory of $L_{\text{CPM}} = 3$. We can see that by choosing $Q \geq 3$, our low complexity equalizer outperforms the MMSE-LTV equalizer in the low SNR region. However, at high SNR, the MMSE-LTV is performing better. It can be explained by the fact that our equalizer does not take into account all the Doppler spread, unlike [10], producing a residual interference between symbols. This drawback should be nevertheless balanced by the fact that, when considering iterative detection and decoding, low SNR behaviour of the detector mainly conditioned the performance of the iterative receiver in the waterfall region. Thus having enhanced performance of the proposed detector in the low SNR regime is a good feature for the coded case when performing iterative detection and decoding.

VI. Conclusion

In this paper, we have derived a new low-complexity Frequency Domain MMSE Equalizer for CPM in case of frequency-selective time-varying channels. We have shown that its computational complexity can be reduced by enforcing the band structure of the channel and that this solution enables good performance for a slight increase of complexity compared to the time-invariant case. It is also shown that the proposed solution enables to have a lower complexity compared to state of the art time domain equalizers for CPM over time-varying channels at the price of a reasonable loss of performance at high SNRs in the uncoded case, while performing better at low SNRs, which is a good feature for iterative detection and decoding in the coded case.

VII. Future Work

Future works will deal with channel estimation for time-varying channel and also possible extension to Widely-Linear techniques as in [10].

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