Binary Sequences Set with Small ISL for MIMO Radar Systems

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***Abstract*—In this paper, we aim at designing a set of binary sequences with good aperiodic auto- and cross-correlation properties for Multiple-Input-Multiple-Output (MIMO) radar systems. We show such a set of sequences can be obtained by minimizing the Integrated Side Lobe (ISL) with the binary requirement imposed as a design constraint. By using the block coordinate descent (BCD) framework, we propose an efficient monotonic algorithm based on Fast Fourier Transform (FFT), to minimize the objective function which is non-convex and NP-hard in general. Simulation results illustrate that the ISL of designed binary set of sequences is the neighborhood of the Welch bound, indicating its superior performance.**

***Index Terms*—Binary Sequences Set, Block Coordinate Descent (BCD), Integrated Sidelobe Level (ISL), Multiple-Input-Multiple-Output (MIMO), Radar Waveform Design.**

I. Introduction

Orthogonal waveforms are the key to many advantages in colocated/widely-separated MIMO radar systems. In colocated MIMO radar systems, orthogonal waveforms create a filled virtual aperture, which enhances spatial resolution of the receive array, improves detection performance and refines parameter identifiability [1]–[5]. In widely separated MIMO radar systems, orthogonal waveforms are used to ensure the capability of separating and processing the waveforms individually at the receive side [1].

In order to achieve waveform orthogonality in MIMO radar systems, several approaches, including frequency-division-multiplexing (FDM) [6], [7], Doppler-division multiplexing (DDM) [8], [9], time-division-multiplexing (TDM) [10] and code-division-multiplexing (CDM) [2], [11], [12] have been developed. Among them, FDM, DDM, and TDM can provide almost perfect orthogonality. However, comparing with CDM, they suffer from strong azimuth-Doppler coupling, lower amount of maximum Doppler frequency and shorter target detection range, respectively [13]. Therefore, designing sets of phase coded waveforms with small auto-correlation sidelobes[1] and low cross-correlation[2] for CDM-MIMO radar systems is the point of common interest (see [2], [14]–[19] and references therein as some examples).

However, most of the recent works have not considered practical constraints such as discrete-phase or binary set of sequences. In active sensing and radar systems, due to the both simplicity of the implementation and Doppler tolerance, set of sequences whose entries are $+1$ or -1, are typically preferred to the continuous phase[3] sequences [20], [21]. Such a set of sequences, provided that have small aperiodic auto-correlation sidelobes and low cross-correlations values, are intrinsically suited for both separation of signals from noise and discrimination of the waveforms at output of the matched filter. Unfortunately, neither the well-known Minimum Peak Sidelobe (MPS) sequences [22], nor Gold, Kasami or M-sequences, which are prevalent in single-input-single-output (SISO) radar systems [23], have good properties in terms of auto- and cross-correlation functions.

In this paper, we propose a mathematical approach for designing binary sequences set by minimizing the important measure of ISL[4] quantifying the goodness in correlation functions. The proposed method can be used in MIMO radar waveform design, as well as other signal processing applications, including spread spectrum communications, channel estimation, fast start-up equalization and sonar systems. To tackle the NP-hard binary sequence design problem, the paper proposes an iterative block coordinate descent algorithm. The proposed method uses an efficient algorithm based on the FFT to obtain sets of binary sequences which almost meet the Welch lower bound [12], [19], [24].

II. Problem Formulation

Let us consider a MIMO radar system with N_T transmit antennas. The m-th antenna transmits a code vector

[1] Small auto-correlation sidelobes, indicates that any sequence in the set is approximately uncorrelated with its own time shifted versions and therefore it avoids masking weak targets within the range sidelobes of a strong target.

[2] Low cross-correlation means that any member of the sequences in the set is roughly uncorrelated with any other members at any shift.

[3] Sequences which have arbitrary phases in $[0, 2\pi)$.

[4] A mathematical definition for the ISL in MIMO radar systems is provided in the next section.

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composed of N sub-pulses that can be expressed as,

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \dots, x_m(N)]^T \in \mathbb{C}^N, \quad m \in [1, N_T], \tag{1}$$

where $x_m(n)$ is the n-th sub-pulse of the transmit code vector $\boldsymbol{x}_m$. Let $\{\boldsymbol{x}_m\}_{m=1}^{N_T}$ be columns of the code matrix $\boldsymbol{X}$, viz.,

$$\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_{N_T}] \in \mathbb{C}^{N \times N_T}. \tag{2}$$

The aperiodic cross-correlation [25] of $\{x_m(n)\}_{n=1}^{N}$ and $\{x_l(n)\}_{n=1}^{N}$ at lag k is defined as,

$$\begin{aligned} r_{ml}(k) &= \sum_{n=1}^{N-k} x_m(n) x_l^*(n+k) = r_{lm}^*(-k), \\ & m, l = 1, \dots, N_T, \quad -N+1 \le k \le N-1, \end{aligned} \tag{3}$$

when $m = l$, (3) becomes the aperiodic auto-correlation of $\{x_m(n)\}_{n=1}^{N}$. Notice that, the in-phase lag of auto-correlation function (i.e., $k = 0$), represents the energy component of the sequence whereas the out-of-phase lag (i.e., $k \neq 0$) represent the sidelobes. The commonly used metric for the *goodness* of correlation function for the code matrix $\boldsymbol{X}$ is the ISL which is defined as [11], [12],

$$\text{ISL} = \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1 \\ m \neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2. \tag{4}$$

Since the design problem is constrained to the family of binary sequences, the n-th sub-pulse at m-th transmit antenna can be written as,

$$x_m(n) \in \{-1, 1\}, \quad m = 1, \dots, N_T \quad \text{and} \quad n = 1, \dots, N \tag{5}$$

Therefore, the optimization problem can be cast as,

$$\begin{cases} \min\limits_{\boldsymbol{X}} & \sum\limits_{m=1}^{N_T} \sum\limits_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum\limits_{\substack{m,l=1 \\ m \neq l}}^{N_T} \sum\limits_{k=-N+1}^{N-1} |r_{ml}(k)|^2 \\ \text{s.t.} & x_m(n) \in \{-1, +1\}, \ {\scriptstyle m=1,\dots,N_T \atop n=1,\dots,N} \end{cases} \tag{6}$$

which is a non-convex NP-hard problem [26].

III. The Proposed Method

This section introduces an iterative derivative-free optimization algorithm, based on the BCD minimization procedure, by updating just one or a few blocks of variables at a time, rather than updating all the blocks together (the batch update) [27]. Indeed, to handle the minimization problem of this paper, we need to loop over all the coordinates and resort to the following sub-problems:

- **Outer loop;** Pick a coordinate t from $1, \dots, N_T$ and design a code vector $\boldsymbol{x}_t$ keeping the other code vectors fixed.
- **Inner loop;** Pick a coordinate $d = 1, \dots, N$ in the selected coordinate t to optimize each scalar variable $x_t(d)$ of $\boldsymbol{x}_t$, keeping fixed the other entries of the code vector $\boldsymbol{x}_t$.

Therefore, by solving a sequence of simpler optimization problems, each subproblem will have a lower dimension in the minimization procedure, and thus can typically be solved easier than the original problem. To tackle Problem (6), we define

$$\widetilde{f}_1(\boldsymbol{X}) = \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} |r_{mm}(k)|^2,$$

$$\widetilde{f}_2(\boldsymbol{X}) = \sum_{\substack{m,l=1 \\ m \neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2,$$

where $\widetilde{f}_1(\boldsymbol{X})$ and $\widetilde{f}_2(\boldsymbol{X})$ stand for the summation of the auto-correlation sidelobes of all different N_T sequences, and summation of the cross-correlation between all different N_T sequences, respectively. Consequently, the ISL metric defined in (4), can be written as $\widetilde{f}(\boldsymbol{X}) = \widetilde{f}_1(\boldsymbol{X}) + \widetilde{f}_2(\boldsymbol{X})$. We aim to design the good set of sequences $\boldsymbol{X}^\star$ by solving the following optimization problem,

$$P_{\boldsymbol{X}} = \begin{cases} \min\limits_{\boldsymbol{X}} & \widetilde{f}(\boldsymbol{X}) \\ \text{s.t.} & x_m(n) \in \{-1, +1\}, \ {\scriptstyle m=1,\dots,N_T \atop n=1,\dots,N} \end{cases} \tag{7}$$

according to the BCD framework. The idea to tackle $P_{\boldsymbol{X}}$ is summarized below:

- Pick coordinate t from $1, 2, \dots, N_T$.
- Set $\boldsymbol{x}_t^{(i+1)} = \arg\min\limits_{\boldsymbol{x}_t} \widetilde{f}(\boldsymbol{x}_t, \boldsymbol{X}_{-t}^{(i)})$.

where $\boldsymbol{X}_{-t}^{(i)}$ represent all other coordinates which kept fixed during the iteration $(i+1)$ of the outer loop, i.e.,

$$\boldsymbol{X}_{-t}^{(i)} = \left[\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(i)}, \dots, \boldsymbol{x}_{t-1}^{(i)}, \boldsymbol{x}_{t+1}^{(i)}, \dots, \boldsymbol{x}_{N_T}^{(i)}\right] \in \mathbb{C}^{N \times N_T - 1}.$$

Further, to obtain the optimal code entry $x_t(d)$, we undertake the following steps:

- Pick coordinate d from $1, 2, \dots, N$.
- Set $x_t^{(h+1)}(d) = \arg\min\limits_{x_t(d)} \widehat{g}(x_t(d), \boldsymbol{x}_{t,-d}^{(h)})$.

where $\boldsymbol{x}_{t,-d}^{(h)}$ represent all other coordinates of the code vector $\boldsymbol{x}_t$ which are keeping fixed at iteration $(h+1)$ of the inner loop. Accordingly, in the outer loop, the optimization Problem $P_{\boldsymbol{X}}$ at iteration $(i+1)$ boils down to

$$P_{t,\boldsymbol{X}^{(i)}} = \begin{cases} \min\limits_{\boldsymbol{x}_t} & \widetilde{f}(\boldsymbol{x}_t, \boldsymbol{X}_{-t}^{(i)}) \\ \text{s.t.} & x_t(n) \in \{-1, +1\}, \quad n = 1, \dots, N \end{cases} \tag{8}$$

where

$$\widetilde{f}(\boldsymbol{x}_t, \boldsymbol{X}_{-t}^{(i)}) = \widetilde{f}\left(\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(i)}, \dots, \boldsymbol{x}_{t-1}^{(i)}, \boldsymbol{x}_t, \boldsymbol{x}_{t+1}^{(i)}, \dots, \boldsymbol{x}_{N_T}^{(i)}\right).$$

Thus, denoting by $\boldsymbol{X}_t^{\star(i+1)}$ the optimal solution to $P_{t,\boldsymbol{X}^{(i)}}$, the optimized code matrix at iteration $(i+1)$ becomes,

$$\boldsymbol{X}_t^{\star(i+1)} = \left[\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(i)}, \dots, \boldsymbol{x}_{t-1}^{(i)}, \boldsymbol{x}_t^\star, \boldsymbol{x}_{t+1}^{(i)}, \dots, \boldsymbol{x}_{N_T}^{(i)}\right].$$

In the inner loop, we go through t-th selected block and choose the scalar $x_t(d)$ as the variable to be optimized,

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put the remaining code entries at iteration $(h+1)$ in the vector $\boldsymbol{x}_{t,-d}^{(h)} \in \mathbb{C}^{N-1}$ defined as,

$$\boldsymbol{x}_{t,-d}^{(h)} = [x_t^{(h)}(1), \ldots, x_t^{(h)}(d-1), x_t^{(h)}(d+1), \ldots, x_t^{(h)}(N)]^T,$$

and form the optimization problem,

$$P_{d,\boldsymbol{x}_t^{(h)}} \begin{cases} \min\limits_{x_t(d)} & \widehat{g}\left(x_t(d); \boldsymbol{x}_{t,-d}^{(h)}\right) \\ \text{s.t.} & x_t(d) \in \{-1, +1\} \end{cases} \tag{9}$$

at iteration $(h+1)$ of the inner loop, where,

$$\widehat{g}(x_t(d); \boldsymbol{x}_{t,-d}^{(h)}) = \widehat{g}_1(x_t(d); \boldsymbol{x}_{t,-d}^{(h)}) + \widehat{g}_2(x_t(d); \boldsymbol{x}_{t,-d}^{(h)})$$
$$\equiv \widehat{g}(x_t^{(h)}(1), \ldots, x_t^{(h)}(d-1), x_t(d), x_t^{(h)}(d+1), \ldots, x_t^{(h)}(N)),$$

with $\widehat{g}_1(x_t(d); \boldsymbol{x}_{t,-d}^{(h)}) = \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{tt}(k)|^2$,

$$\widehat{g}_2(x_t(d); \boldsymbol{x}_{t,-d}^{(h)}) = \sum_{\substack{l=1 \\ l\neq t}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{tl}(k)|^2.$$

Thus, the optimized code vector at the t-th transmit antenna is,

$$\boldsymbol{x}_t^{(h+1)} = [x_t^{(h)}(1), x_t^{(h)}(2), \ldots, x_t^{\star}(d), \ldots, x_t^{(h)}(N)]^T.$$

where $x_t^{\star}(d)$ is the solution to (9). As the result, starting from an initial code matrix $\boldsymbol{X}^{(0)}$, the code matrices $\boldsymbol{X}^{(1)}$, $\boldsymbol{X}^{(2)}$, $\boldsymbol{X}^{(3)}$, ... are obtained iteratively[5].

A. Code Entry Simplification

The reliance of $\widehat{g}_1(x_t(d); \boldsymbol{x}_{t,-d}^{(h)})$ and $\widehat{g}_2(x_t(d); \boldsymbol{x}_{t,-d}^{(h)})$ on the only variable $x_t(d)$ can be explicitly expressed as,

$$r_{tt}(k) = a_{dkt} x_t(d) + c_{dkt}, \; k = -N+1, \ldots, N-1 \tag{10}$$

where[6] $a_{dkt} \triangleq x_t^{(h)}(d+k)\mathbf{I}_A(d+k) + x_t^{(h)}(d-k)\mathbf{I}_A(d-k)$,

$$c_{dkt} \triangleq \sum_{n=1, n\neq\{d,d-k\}}^{N-k} x_t^{(h)}(n) x_t^{(h)}(n+k)\mathbf{I}_A(k+1) + \sum_{n=-k+1, n\neq\{d,d-k\}}^{N} x_t^{(h)}(n) x_t^{(h)}(n+k)\mathbf{I}_B(k),$$

with $\mathbf{I}_A(k)$ and $\mathbf{I}_B(k)$ being the indicator functions of sets $A = \{1, 2, \ldots, N\}$ and $B = \{-1, -2, \ldots, -N+1\}$ respectively, i.e., $\mathbf{I}_A(v) = 1$ if $v \in A$, otherwise $\mathbf{I}_A(v) = 0$. Similarly, the cross-correlation function $r_{tl}(k)$ with explicit dependence on $x_t(d)$ becomes,

$$r_{tl}(k) = a_{dkl} x_t(d) + c_{dkl}, \; k = -N+1, \ldots, N-1, \tag{11}$$

with $a_{dkl} \triangleq x_l^{(h)}(d+k)\mathbf{I}_A(d+k), \; k = -N+1, \ldots, N-1$,

$$c_{dkl} \triangleq \sum_{n=1, n\neq d}^{N-k} x_t^{(h)}(n) x_l^{(h)}(n+k)\mathbf{I}_A(k+1) + \sum_{n=-k+1, n\neq d}^{N} x_t^{(h)}(n) x_l^{(h)}(n+k)\mathbf{I}_B(k).$$

[5]The super scripts (i) and $(i+1)$ for $\boldsymbol{x}_t$ and $\boldsymbol{x}_t^{\star}$ is implicit and omitted for simplicity.

[6]For notational simplicity, the dependency of auto- and cross-correlation to the iteration index h is implicitly assumed and hence omitted.

B. Binary Code Design

Discrete Fourier Transform (DFT) is the motivation to tackle Problem $P_{d,\boldsymbol{x}_t^{(h)}}$. Notice that the phase variable $\phi_t(d) = \arg(x_t(d)) \in \{0, \pi\}$, the problem can be recast as,

$$\begin{cases} \min\limits_{\phi_t(d)} & \sum\limits_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{tt,\phi_t(d)}(k)|^2 + \sum\limits_{\substack{l=1 \\ l\neq t}}^{N_T} \sum\limits_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{tl,\phi_t(d)}(k)|^2 \\ \text{s.t.} & \phi_t(d) \in \{0, \pi\} \end{cases} \tag{12}$$

with $r_{tz,\phi_t(d)}(k) = a_{dkz} e^{\jmath\phi_t(d)} + c_{dkz}$ where z stands for either t or l. The following lemma provides a key result to tackle above optimization problem.

Lemma I

Let $\boldsymbol{\nu}_{dkz} = \left[|r_{tz,\phi_t(1)}(k)|^2, |r_{tz,\phi_t(2)}(k)|^2\right]^T \in \mathcal{R}^2$ *with* $\phi_t(q) = \pi(q-1)$, $q = 1, 2$ *and* $\boldsymbol{\zeta}_{dkz} = [a_{dkz}, c_{dkz}]^T \in \mathcal{R}^2$, *then,*

$$\boldsymbol{\nu}_{dkz} = |DFT(\boldsymbol{\zeta}_{dkz})|^2, \tag{13}$$

with $DFT(\boldsymbol{\zeta}_{dkz})$ *is the two-points DFT of the vector* $\boldsymbol{\zeta}_{dkz}$ *and can be efficiently calculated through FFT. Also, the square modulus is element wise.* ■

Proof. The 2-point DFT of $\boldsymbol{\zeta}_{dkz}$ is,

$$\mathcal{F}_L(\boldsymbol{\zeta}_{dkz}) = \begin{bmatrix} a_{dkz} + c_{dkz} \\ a_{dkz} - c_{dkz} \end{bmatrix}$$

Next, observe that,

$$r_{tz,k}(\phi_t(d)) e^{-\jmath\phi_t(q)} = a_{dkz} + c_{dkz} e^{-\jmath\phi_t(q)}, \quad q = 1, 2.$$

and

$$\left| r_{tz,k}(\phi_t(d)) e^{-\jmath\phi_t(q)} \right| = |r_{tz,k}(\phi_t(d))|. \tag{14}$$

Therefore,

$$|\mathcal{F}_L(\boldsymbol{\zeta}_{dkz})| = \left[|r_{tz,k}(\phi_t(1))|, |r_{tz,k}(\phi_t(2))| \right]^T, \tag{15}$$

which proofs $\boldsymbol{\nu}_{dkz} = |\mathrm{DFT}(\boldsymbol{\zeta}_{dkz})|^2$. □

Inspired from **Lemma I**, we define the matrix $\boldsymbol{U}_z \in \mathcal{R}^{(2N-1)\times 2}$ whose[7] k-th row is $\boldsymbol{\nu}_{dkz}^T$. Let $\boldsymbol{u}_z^s \in \mathcal{R}^2$ be the vectors containing the summation of each columns of the matrix $\boldsymbol{U}_z$. We can write,

$$\boldsymbol{\omega}_t(d) = \boldsymbol{u}_t^s + \sum_{\substack{l=1 \\ l\neq t}}^{N_T} \boldsymbol{u}_l^s, \tag{16}$$

where $\boldsymbol{\omega}_t(d) \in \mathcal{R}^2$. Then, the solution to Problem (12), is given by

$$\phi_t^{\star}(d) = \pi(q^{\star} - 1), \tag{17}$$

where $q^{\star} = \arg\min\limits_{q=1,2} \left\{\boldsymbol{\omega}_t(d)\right\}$, and the optimal phase code entry can be computed as $x_t^{\star}(d) = e^{\jmath\phi_t^{\star}(d)}$.

[7]Notice that, the matrix $\boldsymbol{U}_t$ contains all possible auto-correlation sidelobes (all the values of $\boldsymbol{\nu}_{dkt}$ are zero when $k = 0$) of different lags (i. e., k), whereas all the cross-correlation values for different possible lags are written in $\boldsymbol{U}_l$.

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IV. Performance Analysis

This section is devoted to the performance analysis of the proposed algorithm for designing binary sequences set. In order to evaluate performance of the proposed method with a normalized measure, we use the definition

$$\text{ISLR (dB)} = 10\log_{10}\frac{\text{ISL}}{N^2},$$

which is the ratio of integrated energy of the sidelobes to the peak energy of the mainlobe. The "Multi-CAN" algorithm [2], is adopted as the benchmark since it meets the Welch lower bound on ISL when designing set of sequences with constant modulus but arbitrary phases [12], [24]. Therefore, we consider Multi-CAN(continuous phase) as the Welch lower bound on ISL and compare the performance of the proposed algorithm with this method. Notice that, even-though sets of Gold, Kasami and M-sequences have ideal periodic auto-correlation and perfect cross-correlation functions, they don't have optimal aperiodic auto- and cross-correlation functions.
Fig. 1 illustrates the convergence behavior of the proposed algorithm initialized by a set of random sequences and a set of Gold sequences. The code length $N = 63$ is adopted since the set of Gold sequences are only available when $N = 2^n - 1, n = 1, 2, \ldots$. The number of iterations depicted in the figure is based on the outer loop as described in section III. As expected, the ISLR values decreases monotonically and converge to a stationary point, when the iteration increases.
Fig. 2 and Fig. 3 provide a comparison between the Multi-CAN and proposed algorithm when both algorithms are initialized with sets of random sequences at lengths $N = \{8, 16, 24, 32, 40, 48, 56, 64\}$, with $N_T = 3, 4$. Indeed, Multi-CAN provides set of continuous phase sequences, by minimizing an almost equivalent metric of the ISL (see [2] for more details). As Fig. 2 illustrates, the proposed algorithm has provided a set of sequences with the ISL values very close to the set obtained via Multi-CAN (or the Welch lower bound), but interestingly with a binary alphabet. The average result over 10 independent trials are reported for both Multi-CAN and the proposed algorithm in this figure. Meanwhile, the averaged ISLR values (over 10 independent trials in each sequence length) of the initialization random set of sequences, are depicted in the figure. To observe the effectiveness of the proposed algorithm, we also have plotted the best set of binary sequences, obtained via quantization of the Multi-CAN sequences. Of course the quantization cannot provide a good set of sequences, particularly when the alphabet size is small. This fact can be proven by the comparison in Fig. 2, between the ISLR values obtained via the proposed algorithm and Multi-CAN(Binary) , for binary set of sequences.
Eventually, Fig. 3 illustrates a comparison between computational time[8] of the proposed and Multi-CAN algorithms, initialized with random set of sequences and averaged over 10 independent trials. It is known that, optimization algorithms which sequentially minimize the objective function (e. g. BCD), are slower than the methods that update all the variables simultaneously. However, considering small block sizes (e. g. $N_T = 3$, $N < 64$), the computational time of the proposed algorithm is not very higher than Multi-CAN.

[8]The computational time is reported using a laptop with a 2.90GHz Intel(R) Core(TM) i7-7600U CPU and 8 GB RAM.

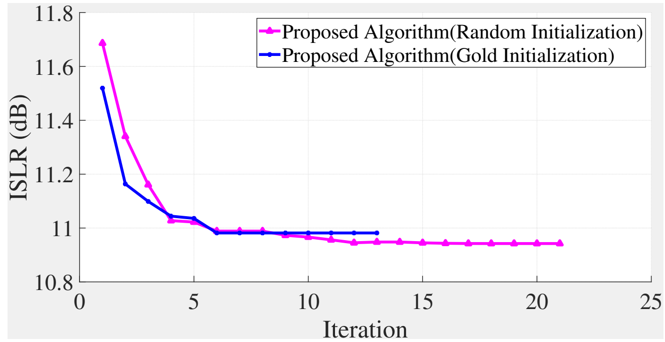
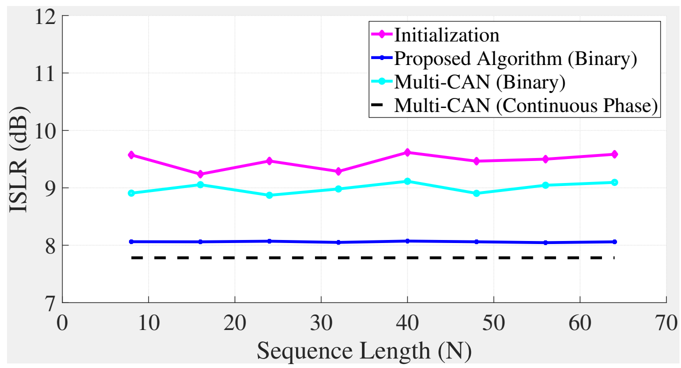
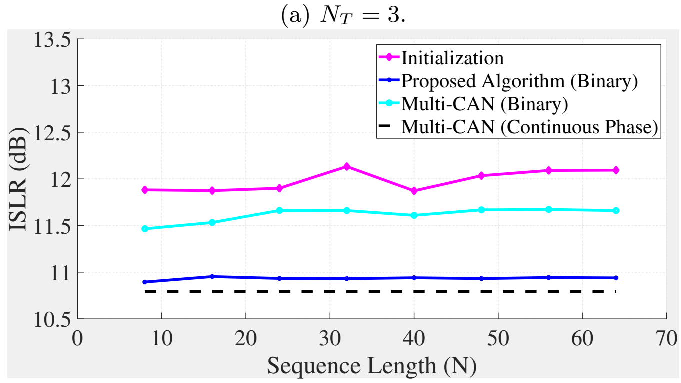


Fig. 1: Convergence behavior of the proposed algorithm ($N = 63$, $N_T = 4$).



(a) $N_T = 3$.



(b) $N_T = 4$.

Fig. 2: Comparison between the ISLR values of obtained sequence sets through proposed and Multi-CAN algorithms.

V. Conclusion

The performance of MIMO radar systems is strongly dependent on the waveform characteristics. When employing distinct binary sequences as waveforms transmit-

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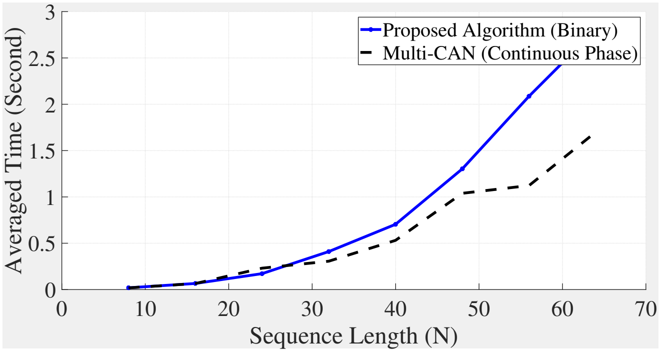

Fig. 3: Computational time of obtained sequence sets through proposed and Multi-CAN algorithms ($N_T = 3$).

ted from different antennas simultaneously in a MIMO radar, it is desirable they exhibit impulse-like auto-correlation functions with small cross-correlation values. In this paper, we have developed a promising approach for designing such a set of binary sequences with good aperiodic auto- and cross-correlation functions. Specifically, we devised a novel algorithm based on BCD and FFT, to tackle the non-convex NP-hard optimization problem. Simulation results have illustrated the effectiveness of the new algorithm in obtaining binary sequences with ISL near the Welch lower bound.

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