Efficient Semi-Blind Subspace Channel Estimation For MIMO-OFDM System

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***Abstract*—This paper deals with channel estimation for Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) wireless communications systems. Herein, we propose a semi-blind (SB) subspace channel estimation technique for which an identifiability result is first established for the subspace based criterion. Our algorithm adopts the MIMO-OFDM system model without cyclic prefix and takes advantage of the circulant property of the channel matrix to achieve lower computational complexity and to accelerate the algorithm's convergence by generating a group of sub vectors from each received OFDM symbol. Then, through simulations, we show that the proposed method leads to a significant performance gain as compared to the existing SB subspace methods as well as to the classical last-squares channel estimator.**

1. Introduction

Channel estimation is of paramount importance to equalization and symbol detection problems in most wireless communications systems. Many approaches have been developed and can be classified into two main categories.

The first one concerns blind channel estimation methods which have been extensively studied and are based on the statistical or structural properties of the transmitted symbols (e.g. [1]).

The second one, adopted in most communications standards [2], relies on the insertion of pilots in the physical packet according to a given arrangement type (block, comb or lattice) [3], [4].

Each channel estimation class has its own benefits and drawbacks. Generally, the second class, i.e. pilot-based channel estimator provides an easier and more robust channel estimation than the blind estimation class. However, it decreases the spectral efficiency and the throughput as compared to the blind methods. Therefore, it would be advantageous to retain the benefits of the two techniques through the use of semi-blind estimation methods [5], [6], [7] which exploit both data and pilots to achieve the desired channel identification.

Research work on semi-blind methods can be divided into two categories. The first category groups works that aim to improve the performance of the channel estimation through the joint use of pilots and data symbols. This is the case, for example, of [8] where the authors used a subspace approach or [5] which proposes a decomposition of the channel matrix into a whitening matrix and another unitary. The second category includes works that focus on reducing the size of the transmitted pilot signals in order to improve the throughput gain (see for example [9]). In [10], the authors exploit the semi-blind approach to reduce the transmitted power ("green communications").

This article proposes a semi-blind channel estimation method based on the subspace decomposition (in signal subspace and noise subspace) of the covariance matrix of the received signal. The derivation of subspace methods depends on the matrix system model. In our case, we use an appropriate windowing that increases the convergence rate together with the circular Toeplitz block structure of the system matrix associated with an OFDM symbol. First, we establish a subspace identifiability result linked to this structure before using it for semi-blind channel estimation. Note that in the literature there exist already several versions of the subspace method, for example [8], [11] differ from the one proposed in this paper by incorporating the cyclic prefix (CP) and virtual carriers (VC) into the system model which changes the size and structure of the system channel matrix. The latter methods are efficient only for large sample sizes and hence a fast alternative approach has been introduced in [12]. Compared to this last method, our solution does not rely on the presence of VC and has a lower computational complexity. Finally, we present simulation results with comparative study that assess the performance gain achieved by the proposed solution.

2. MIMO-OFDM system model

This section presents the MIMO-OFDM wireless system model illustrated in Fig. 1. It is composed of N_t transmit antennas and N_r receive antennas. The transmitted signal is assumed to be an OFDM one, composed of K samples (sub-carriers) and L Cyclic Prefix samples. The CP length is assumed to be greater or equal to the maximum multipath channel delay denoted N (i.e. $N \leq L$).

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The received signal $\mathbf{y}$ at the N_r receivers of the MIMO-OFDM system is given by [13] (after CP removal):

$$\mathbf{y} = \boldsymbol{\mathcal{H}}\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{y} = \left[\mathbf{y}_1^T \cdots \mathbf{y}_{N_r}^T\right]^T$ and $\mathbf{x} = \left[\mathbf{x}_1^T \cdots \mathbf{x}_{N_t}^T\right]^T$. The noise $\mathbf{v} = \left[\mathbf{v}_1^T \cdots \mathbf{v}_{N_r}^T\right]^T$ is assumed to be additive independent white Circular Complex Gaussian (CCG) satisfying $E\left[\mathbf{v}(k)\mathbf{v}(i)^H\right] = \sigma_{\mathbf{v}}^2 \mathbf{I}_K \delta_{ki}$; $(.)^H$ being the Hermitian operator; $\sigma_{\mathbf{v}}^2$ the noise variance; $\mathbf{I}_K$ the identity matrix of size $K \times K$ and δ_{ki} the Dirac operator. The channel matrix $\boldsymbol{\mathcal{H}}$ is given by:

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,N_t} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N_r,1} & \cdots & \mathbf{H}_{N_r,N_t} \end{bmatrix}. \tag{2}$$

Each sub-block $\mathbf{H}_{i,j}$ (with $i = 1, \cdots, N_r$ and $j = 1, \cdots, N_t$) of the matrix $\boldsymbol{\mathcal{H}}$ is a circulant $K \times K$ Toeplitz matrix. The first row of the (i,j)-th block contains the propagation channel coefficients between the i-th transmitter and the j-th receiver $\mathbf{h}_{i,j}$ $\left(\mathbf{h}_{i,j} = [\mathrm{h}_{i,j}(0) \ \cdots \ \mathrm{h}_{i,j}(N-1)]^T\right)$, given by: $\left[\mathrm{h}_{i,j}(0) \quad \mathbf{0}_{1\times(K-N)} \quad \mathrm{h}_{i,j}(N-1) \ \cdots \ \mathrm{h}_{i,j}(1)\right]$. The signal $\mathbf{x}_i$, sent by the i-th transmitter is an OFDM signal, modulating the data signal $\mathbf{d}_i$, using the inverse Fourier transform IFFT, as follows

$$\mathbf{x}_i = \frac{\mathbf{W}^H}{\sqrt{K}}\mathbf{d}_i, \tag{3}$$

where $\mathbf{W}$ represents the K-point Fourier matrix. Equation (1), can be rewritten as:

$$\mathbf{y} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{W}}\mathbf{d} + \mathbf{v} = \boldsymbol{\mathcal{A}}\mathbf{d} + \mathbf{v}, \tag{4}$$

where $\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{W}}$ and $\boldsymbol{\mathcal{W}} = \mathbf{I}_{N_t} \otimes \mathbf{W}$, $\otimes$ refers to the Kronecker product. The transmitted data are regrouped in $\mathbf{d} = \left[\mathbf{d}_1^T \cdots \mathbf{d}_{N_t}^T\right]^T$.

In the sequel the received OFDM symbols are assumed to be i.i.d and the N_p pilots are arranged according to the block-type scheme followed by N_d data OFDM symbols. To take into account the time index (ignored in equations (1) and (4)), we will refer to the t-th OFDM symbol by $\mathbf{y}(t)$ instead of $\mathbf{y}$.

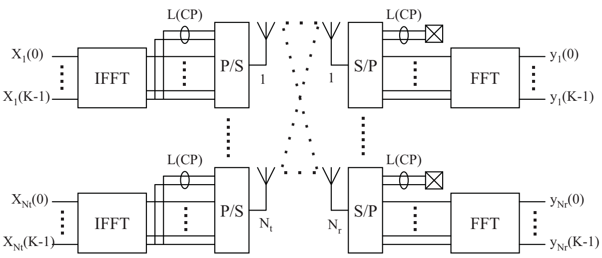

Figure 1. MIMO-OFDM system model

3. MIMO channel estimation

This section first reminds the well known Least Squares estimator, denoted LS, based on the pilot symbols known at the receiver side. Our subspace blind estimator is then introduced to ultimately derive the proposed semi-blind estimation solution. This is formulated by the minimization of a cost function that incorporates both the pilot and the blind (data) part.

3.1. Pilot-based channel estimation

In order to derive LS estimator, based on the training sequences, equation (1) is rewritten as:

$$\mathbf{y} = \tilde{\mathbf{X}}\mathbf{h} + \mathbf{v}, \tag{5}$$

where $\mathbf{h} = \left[\mathbf{h}_1^T \cdots \mathbf{h}_{N_r}^T\right]^T$ is a vector of size $N_r N_t N \times 1$ representing the MIMO channel taps (where $\mathbf{h}_r = \left[\mathbf{h}_{1,r}^T \cdots \mathbf{h}_{N_t,r}^T\right]^T$). $\tilde{\mathbf{X}} = \mathbf{I}_{N_r} \otimes \mathbf{X}$, with $\mathbf{X} = [\mathbf{X}_1 \cdots \mathbf{X}_{N_t}]$ where $\mathbf{X}_i$ is a circulant $K \times N$ Toeplitz matrix containing the elements of $\mathbf{x}_i$. Each column is obtained by a simple down cyclic shift of the previous one with the first column being the vector $\mathbf{x}_i$.

The LS channel estimator $\hat{\mathbf{h}}_{LS}$, using N_p pilot OFDM symbols, $\tilde{\mathbf{X}}_p = \left[\tilde{\mathbf{X}}(1)^T \cdots \tilde{\mathbf{X}}(N_p)^T\right]^T$, is obtained by the minimization of the following cost function:

$$C(\mathbf{h}) = \left\|\tilde{\mathbf{y}}_p - \tilde{\mathbf{X}}_p\mathbf{h}\right\|^2, \tag{6}$$

with $\tilde{\mathbf{y}}_p = \left[\mathbf{y}(1)^T \cdots \mathbf{y}(N_p)^T\right]^T$.

Then the LS estimator is given by [14]:

$$\hat{\mathbf{h}}_{LS} = \left(\tilde{\mathbf{X}}_p^H \tilde{\mathbf{X}}_p\right)^{-1} \tilde{\mathbf{X}}_p^H \tilde{\mathbf{y}}_p. \tag{7}$$

3.2. Subspace based SB channel estimation

In this section, we consider the subspace approach for the data model given in equation (1). Based on the data model assumptions, the data covariance matrix is equal to:

$$\mathbf{C}_y = E(\mathbf{y}\mathbf{y}^H) = \sigma_{\mathbf{x}}^2 \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H + \sigma_{\mathbf{v}}^2 \mathbf{I}_{KN_r} \tag{8}$$

Hence, the signal subspace (principal subspace of $\mathbf{C}_y$) coincides with the range space of $\boldsymbol{\mathcal{H}}$ while the noise subspace is its orthogonal complement. These subspaces can be estimated from the eigenvalue decomposition (EVD) of $\mathbf{C}_y$ according to:

$$\mathbf{C}_y = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H = [\mathbf{U}_s \,|\mathbf{U}_n] \begin{bmatrix} \boldsymbol{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}, \tag{9}$$

where $\mathbf{C}_y$ is estimated using N_d data OFDM symbols as follows:

$$\hat{\mathbf{C}}_y = \frac{1}{N_d} \sum_{t=1}^{N_d} \mathbf{y}(t)\mathbf{y}(t)^H \tag{10}$$

$\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues in descending order, the matrix $\mathbf{U}_s$ of size $KN_r \times KN_t$ contains

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the eigenvectors associated with the largest eigenvalues representing the signal subspace. The noise subspace $\mathbf{U}_n$ is associated with the $K(N_r - N_t)$ smallest eigenvalues, i.e.

$$[\mathbf{U}_s \,|\mathbf{U}_n] = [\mathbf{u}_1 \,\cdots\, \mathbf{u}_{KN_t} \,|\mathbf{u}_{KN_t+1} \,\cdots\, \mathbf{u}_{KN_r}]. \quad (11)$$

Now, the subspace identification applies only when the range space of matrix $\boldsymbol{\mathcal{H}}$ ($range(\boldsymbol{\mathcal{H}})$) characterizes uniquely the channel vector $\mathbf{h}$ (up to certain inherent indeterminacies [15]). For this purpose, we have proved the following identifiability result:

Lemma: Let $\mathbf{H}(z)$ be the $N_r \times N_t$ polynomial filtering matrix which (i,j)-th entry is given by $\mathbf{h}_{i,j}(z) = \sum_{k=0}^{N} h_{i,j}(k) z^{-k}$. Under the assumption that $\mathbf{H}(z)$ is irreducible (i.e. $rank(\mathbf{H}(z)) = N_t$ for all z), the range space of matrix $\boldsymbol{\mathcal{H}}$ characterizes the channel as follows: For any polynomial matrix $\mathbf{H}'(z)$ of degree N, we have $range(\boldsymbol{\mathcal{H}}') = range(\boldsymbol{\mathcal{H}})$ if and only if $\mathbf{H}'(z) = \mathbf{H}(z)\mathbf{Q}$, where $\mathbf{Q}$ is a constant $N_t \times N_t$ matrix representing the inherent indeterminacy of the blind approach [15].

Using the previous lemma, we can blindly identify the channel vector through the orthogonality relation between the noise and signal subspaces according to:

$$\mathbf{u}_i^H \boldsymbol{\mathcal{A}} = 0 \quad i = KN_t + 1, \cdots, KN_r, \quad (12)$$

where $\boldsymbol{\mathcal{A}}$ is the channel matrix given in equation (4).

Solving this orthogonality relation in the least squares sense leads to:

$$C(\boldsymbol{\mathcal{H}}) = \sum_{i=KN_t+1}^{KN_r} \left\| \mathbf{u}_i^H \boldsymbol{\mathcal{A}} \right\|^2 = \sum_{i=KN_t+1}^{KN_r} \left\| \mathbf{u}_i^H \boldsymbol{\mathcal{H}} \boldsymbol{\mathcal{W}} \right\|^2. \quad (13)$$

By partitioning vector $\mathbf{u}_i$ of dimension $KN_r \times 1$ into N_r vectors $\mathbf{v}_r^i \, (r = 1, \cdots, N_r)$ of size K as follows:

$$\mathbf{u}_i = \left[\begin{array}{ccc} \mathbf{v}_1^{i\,T} & \cdots & \mathbf{v}_{N_r}^{i\;T} \end{array} \right]^T, \quad (14)$$

one can generate the $NN_r \times K$ matrix $\mathbf{V}_i$ as:

$$\mathbf{V}_i = \left[\begin{array}{c} \mathbf{V}_1^i \cdots \mathbf{V}_{N_r}^i \end{array} \right]^T, \quad (15)$$

where each matrix $\mathbf{V}_r^i$ is circulant of size $N \times K$ constructed from the vector $\mathbf{v}_r^i$. Each line is obtained by a simple left cyclic shift of the previous one with the first line being the vector $\mathbf{v}_r^{i\,T}$. The cost function given by equation (13), can then be rewritten in the following form:

$$C(\underline{\mathbf{H}}) = \sum_{i=KN_t+1}^{KN_r} \left\| \underline{\mathbf{H}}^T \mathbf{V}_i{}^* \boldsymbol{\mathcal{W}} \right\|^2 = \sum_{i=KN_t+1}^{KN_r} \left\| \underline{\mathbf{H}}^T \mathbf{V}_i{}^* \right\|^2, \quad (16)$$

where

$$\begin{aligned} \underline{\mathbf{H}} &= \left[\underline{\mathbf{h}}_1 \,\cdots\, \underline{\mathbf{h}}_{N_t} \right] \\ \underline{\mathbf{h}} &= \left[\underline{\mathbf{h}}_1^T \,\cdots\, \underline{\mathbf{h}}_{N_t}^T \right]^T \\ \underline{\mathbf{h}}_i &= [\mathrm{h}_{1,i}(0) \,\cdots\, \mathrm{h}_{1,i}(N-1) \,\cdots \\ &\qquad \mathrm{h}_{N_r,i}(0) \,\cdots\, \mathrm{h}_{N_r,i}(N-1)]^T. \end{aligned} \quad (17)$$

This criterion reduces finally to:

$$\begin{aligned} C(\underline{\mathbf{h}}) = \sum_{i=1}^{N_t} \underline{\mathbf{h}}_i^T \boldsymbol{\Phi} \underline{\mathbf{h}}_i^* &= \underline{\mathbf{h}}^T (\mathbf{I}_{N_t} \otimes \boldsymbol{\Phi}) \, \underline{\mathbf{h}}^* \\ &= \underline{\mathbf{h}}^H (\mathbf{I}_{N_t} \otimes \boldsymbol{\Phi}^*) \, \underline{\mathbf{h}}, \end{aligned} \quad (18)$$

where

$$\boldsymbol{\Phi} = \sum_{i=KN_t+1}^{KN_r} \mathbf{V}_i{}^* \mathbf{V}_i{}^T, \quad (19)$$

The cost function in the semi-blind subspace case is composed of two cost functions: the least squares based on the pilots and the one related to the subspace blind estimation:

$$C(\underline{\mathbf{h}}) = \left\| \tilde{\mathbf{y}}_p - \tilde{\mathbf{X}}_p \mathbf{P} \underline{\mathbf{h}} \right\|^2 + \alpha \underline{\mathbf{h}}^H (\mathbf{I}_{N_t} \otimes \boldsymbol{\Phi}^*) \, \underline{\mathbf{h}}, \quad (20)$$

where α is a weighting factor[1] for the subspace method and $\mathbf{P}$ is a permutation matrix such that $\mathbf{h} = \mathbf{P}\underline{\mathbf{h}}$. The minimization of the latest cost function, leads to the semi-blind channel estimation as:

$$\hat{\underline{\mathbf{h}}} = \left(\mathbf{P}^H \tilde{\mathbf{X}}_p^H \tilde{\mathbf{X}}_p \mathbf{P} + \alpha \left(\mathbf{I}_{N_t} \otimes \boldsymbol{\Phi}^* \right) \right)^{-1} \mathbf{P}^H \tilde{\mathbf{X}}^H \tilde{\mathbf{y}}_p. \quad (21)$$

The channel estimation performance is strongly related to the estimation quality of covariance matrix, which is relatively poor when the number of data OFDM symbols is small. To alleviate to this and also to reduce the computational cost (via a reduced size EVD), we introduce next a windowing technique that helps obtaining 'closed to optimal' performance with small number of OFDM symbols.

3.3. Fast semi-blind channel estimation

In this part, we propose to subdivide each OFDM symbol into N_g OFDM subvectors, according to a specific shift which will be detailed hereafter. Using one received OFDM symbol $\mathbf{y}$ given in equation (1), one can define a set of sub-vectors $\mathbf{y}_{(g)}$ of size $N_r G \times 1$ ($G < K$ being a chosen window size) as follows[2]

$$\mathbf{y}_{(g)} = \left[\mathbf{y}_1(g : g + G - 1)^T \cdots \mathbf{y}_{N_r}(g : g + G - 1)^T \right]^T \quad (22)$$

where $g = 1, \cdots, K - G + 1$. Then, we group the N_g ($N_g = K - G + 1$) vectors into one matrix $\mathbf{Y}_G = \left[\mathbf{y}_{(1)} \cdots \mathbf{y}_{(N_G)} \right]$ that is given by:

$$\mathbf{Y}_G = \boldsymbol{\mathcal{H}}_G \mathbf{X}_G + \mathbf{V}_G, \quad (23)$$

where the new channel matrix $\mathbf{H}_G$ ($N_r G \times N_t K$) is extracted from the matrix $\mathbf{H}$ given in (2) as:

$$\boldsymbol{\mathcal{H}}_G = \left[\begin{array}{ccc} \mathbf{H}_{1,1}(1:G,:) & \cdots & \mathbf{H}_{1,N_t}(1:G,:) \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N_r,1}(1:G,:) & \cdots & \mathbf{H}_{N_r,N_t}(1:G,:) \end{array} \right]. \quad (24)$$

1. The optimal weighting can be derived as in [16] using a two step approach.

2. For simplicity, we adopt here some MATLAB notations.

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and the input data matrix is given by $\mathbf{X}_G = \left[\mathbf{x}_{(0)} \cdots \mathbf{x}_{(N_G-1)}\right]$, where $\mathbf{x}_{(g)}$ is obtained from vector $\mathbf{x}$ by applying g up-cyclic shifts. Using equation (3), one can establish the relation between the i-th transmitted signal $\mathbf{x}^i_{(g)}$ and the data $\mathbf{d}_i$ as:

$$\mathbf{x}^i_{(g)} = \frac{\mathbf{W}^H}{\sqrt{K}}\mathbf{D}^g\mathbf{d}_i = \frac{\mathbf{W}^H}{\sqrt{K}}\mathbf{d}^i_{(g)}, \tag{25}$$

where $\mathbf{D}^g$ is $(K \times K)$ diagonal phase matrix given by:

$$\mathbf{D}^g = \frac{1}{\sqrt{(K)}} diag\{e^{j2\pi(g)(0)} \cdots e^{j2\pi(g)(K-1)}\} \tag{26}$$

Then, $\mathbf{x}_{(g)} = \boldsymbol{\mathcal{W}}\mathbf{d}_{(g)}$, where $\mathbf{d}_{(g)} = \left[(\mathbf{d}^1_{(g)})^T \cdots (\mathbf{d}^{N_t}_{(g)})^T\right]^T$. Finally, by concatenating all the data vectors in one $N_tK \times N_g$ matrix $\mathbf{D}_G = \left[\mathbf{d}_{(0)} \cdots \mathbf{d}_{(N_G-1)}\right]$, equation (23) becomes:

$$\mathbf{Y}_G = \boldsymbol{\mathcal{H}}_G\boldsymbol{\mathcal{W}}\mathbf{D}_G + \mathbf{V}_G \tag{27}$$

The estimation of the correlation matrix is done using the N_dN_g vectors (instead of using only N_d vectors), which leads to fast convergence speed:

$$\hat{\mathbf{C}}_G = \frac{1}{N_dN_G}\sum_{t=1}^{N_d}\mathbf{Y}_G(t)\mathbf{Y}_G(t)^H \tag{28}$$

As in the previous section, under the condition that matrix $\boldsymbol{\mathcal{H}}_G$ is full column rank (and hence $GN_r > KN_t$), one can use the subspace orthogonality relation as in (12) to estimate the channel vector using the EVD of $\hat{\mathbf{C}}_G$.

4. Performance analysis and discussions

In this section, we analyze the performance of the subspace semi-blind channel estimators in terms of the normalized Root Mean Square Error (NRMSE) given by equation (29) for the two subspace methods presented in this paper i.e. when considering one symbol OFDM and the case when we split this OFDM symbol into several subvectors.

$$NRMSE = \sqrt{\frac{1}{NN_tN_rN_{mc}}\sum_{i=1}^{N_{mc}}\frac{\left\|\hat{\mathbf{h}}^{(i)} - \mathbf{h}\right\|^2}{\|\mathbf{h}\|^2}}, \tag{29}$$

where $N_{mc} = 500$ represents the number of Monte Carlo realizations. The considered MIMO-OFDM wireless system is related to the IEEE 802.11n standard [2] composed of two transmitters ($N_t = 2$) and three receivers ($N_r = 3$). The pilot sequences (or training sequences) correspond to those specified in the IEEE 802.11n standard, where each pilot is represented by one OFDM symbol ($K = 64$ samples) of power P_{x_p} = 23 dBm completed by a CP ($L = 16$ samples) at its front. The data signal power is P_{x_d} = 20 dBm. The channel model is of type B with path delay [0 10 20 30] μs and an average path gains of [0 -4 -8 -12] dB.

The Signal to Noise Ratio associated with pilots at the reception is defined as $SNR = \frac{\|\boldsymbol{\mathcal{H}}\mathbf{x}_p\|^2}{N_rN_pK\sigma^2_{\mathbf{v}}}$.

Fig. 2 presents a comparison between the proposed SB method, the SB method in [12] ($\mathbf{h}^{G=45}_{SB}$[12]), the LS method ($\mathbf{h}_{LS}$) and the SB Cramèr Rao bound CRB_{SB}, detailed in [13], for $N_p = 4$ and $N_d = 150$. For the subspace method, we considered the full-OFDM symbol case[3] with $G = K = 64$ ($\mathbf{h}^{G=64}_{SB}$) and the windowed case with $G = 45$ ($\mathbf{h}^{G=45}_{SB}$). The curves represent the NMSE versus the SNR for all considered methods. Several observations can be made out of this experiment: First, both SB methods (the proposed one and the SB method in [12]) have the same estimation performance but our algorithm has a reduced computational cost due to the reduced size of matrix $\mathbf{Y}_G$ as compared to the one used in [12] and to the circulant matrix structure which helps reducing the cost of the calculation of matrix $\mathbf{\Phi}$ in equation (19). Second, by comparing the cases $G = K = 64$ and $G = 45$, one can see that the windowing is of high importance to achieve the SB gain for small sample sizes. Finally, comparing the obtained results with the CRB, we observe a gap of few dBs with the optimal estimation.

Fig. 3 presents the performance of the SB method with $G = K = 64$ and $G = 45$ versus the number of data OFDM symbols (N_d). Also, as a benchmark, we compare the results with the case where the covariance matrix for $G = K = 64$ is perfectly estimated ($\mathbf{h}^E_{SB}$) and given by equation (10). One can see that without windowing a large number of OFDM symbols (more that 300) is needed to achieve the gain of the SB approach, while the proposed windowing allows us to converge with about 20 OFDM symbols only. Another observation is that increasing the window size G improves the estimation accuracy when a large number of OFDM symbols is available.

For a given $SNR = 10dB$, Fig. 4 illustrates the impact of the size of the partitioned OFDM symbol[4] (G) on the estimation performance for the cases $N_d = 40$ (small sample size), $N_d = 150$ (moderate sample size) and $N_d = 300$ (large sample size). We notice that the window size choice has a strong impact on the estimation performance and for small and moderate sample sizes, an optimal value of G exists and depends on N_d. For large sample sizes, the optimal window size is $G = K$ which confirms the observation made previously in Fig. 3.

5. Conclusion

A new version of the semi-blind subspace method for channel estimation is proposed in the context of MIMO-OFDM systems. For that, we have introduced a new blind subspace estimation method for which an identifiability result has been established. This SB method exploits the circulant matrix structure to reduce the computational complexity and an appropriate windowing technique to improve the estimation accuracy for small or moderate sample sizes.

3. For this case, the method in [12] does not work without the use of the VC and hence its corresponding plot is not provided.

4. Note that for $\boldsymbol{\mathcal{H}}_G$ to be tall and full column rank, G belongs to the range $[43, 64]$.

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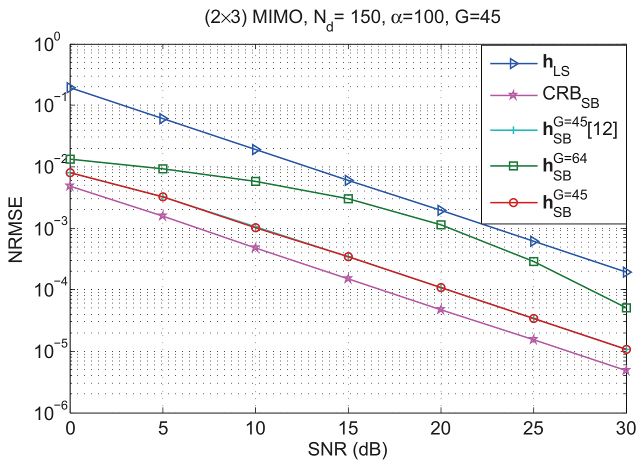

Figure 2. NRMSE versus SNR.

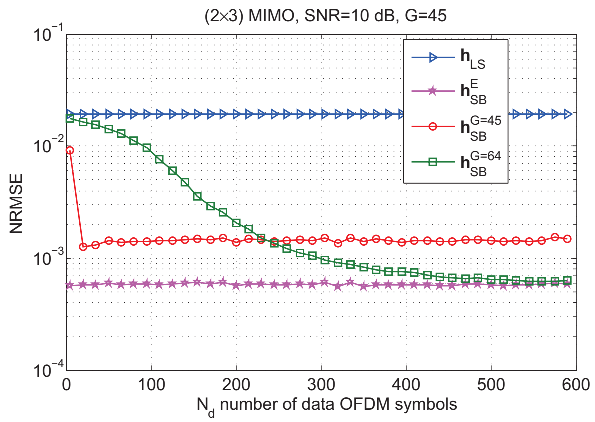

Figure 3. NRMSE versus the number of data OFDM symbols N_d ($SNR = 10$ dB).

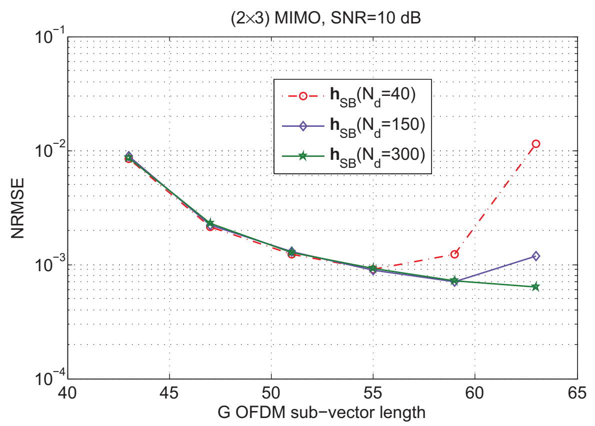

Figure 4. NRMSE versus the Size of the partitioned symbol G.

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