Tracking and Sensor Fusion in Direction of Arrival Estimation Using Optimal Mass Transport

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***Abstract*—In this work, we propose new methods for information fusion and tracking in direction of arrival (DOA) estimation by utilizing an optimal mass transport framework. Sensor array measurements in DOA estimation may not be consistent due to misalignments and calibration errors. By using optimal mass transport as a notion of distance for combining the information obtained from all the sensor arrays, we obtain an approach that can prevent aliasing and is robust to array misalignments. For the case of sequential tracking, the proposed method updates the DOA estimate using the new measurements and an optimal mass transport prior. In the case of sensor fusion, information from several, individual, sensor arrays is combined using a barycenter formulation of optimal mass transport.**

***Index Terms*—Optimal mass transport, Spectral estimation, Direction of arrival, Sensor fusion, Target tracking**

I. Introduction

The problem of direction of arrival (DOA) estimation is ubiquitous in the field of signal processing, with applications in, e.g., communications, radar, and sonar [1]–[3]. Often formulated as a (spatial) spectral estimation problem, the spatial directions of wave-emitting sources are determined by studying the spatial distribution of spectral power impinging on an array of sensors. Given its significance, a multitude of approaches have been developed to address different aspects and formulations of the DOA estimation problem, including data-adaptive beamforming approaches [4], [5] and subspace-based methods, such as MUSIC [6] and ESPRIT [7]. More recently, different solutions that allow for calibration errors have been examined (see, e.g., [8]–[11]). Another focus has been on methods that seek a suitable spectrum consistent with the estimated covariance matrix, often with some regularization, such as maximum entropy [3], [12], [13], or more recently approaches based on L_1-regularization, exploiting sparsity in the spectrum [14].

Inherent in DOA estimation is the phenomenon of spatial ambiguity, i.e., several DOAs may be consistent with the observed covariance matrix. This is, for example, the case in scenarios where the array is a uniform linear array (ULA), with sensor spacing larger than half of the impinging wave's wavelength [3]. Similarly, for a ULA, DOA estimation is limited to directions in one of the half-planes defined by the array. Apart from alleviating this by considering other array geometries, there have been methods proposed that consider measurements obtained from several individual arrays; by matching peaks in spectra estimated from the individual arrays, the correct DOAs may under some conditions be identified and erroneous spectral peaks discarded [15]. However, in order to perform the matching, the individual spectra may not be too poorly resolved, preventing application of such methods to scenarios such as, e.g., estimation of end-fire DOAs for settings with ULAs.

In this work, we consider the problem of combining DOA measurements, obtained from several individual arrays or from the same array at different time points, in order to alleviate spatial ambiguity. To this end, we consider the sets of spectra consistent with the observed covariance matrices, and then select the spectrum for which the sum of the distances to those sets is minimized. As a notion of distance, we use the optimal mass transport framework (see, e.g., [16]). The optimal transport problem has earlier been used as a distance measure between spectra [17]–[19], and has been shown to be robust against misalignments [20]. Herein, we demonstrate that the proposed method allows for a flexible formulation for combining covariance matrices from several different arrays in order to form estimates of the spatial spectrum, alleviating the problem of spectral ambiguity also for cases when the spectrum estimates corresponding to the individual arrays suffer in resolution.

II. Background

A. Direction of arrival estimation

In order to introduce notation, we begin by briefly reviewing the DOA problem, which strives to estimate directions and magnitudes of a set of planar waves that simultaneously arrive at an area containing the sensors. Here, the sensors measure the superposition of the waves and the spatial characteristics of the sensor array dictate ambiguity and sensitivity to certain directions. To model this, let the array geometry be specified by the sensor positions, $x_k \in \mathbb{R}^d$, for $k = 1, \ldots, n$. The incoming planar wave arrive from the directions $\mathcal{S} = \{u \in \mathbb{R}^d \,|\, \|u\|_2 = 1\}$, e.g., represented in spherical coordinates as

$$u(\theta) = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}, \quad u(\theta) = \begin{pmatrix} \cos(\theta_1)\sin(\theta_2) \\ \sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_2) \end{pmatrix}$$

for $d = 2$ and $d = 3$, respectively. The spacial distribution of the incoming waves can thus be represented by a non-negative

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function or measure Φ on $\mathcal{S}$, denoted the power spectrum. Letting λ denote the wavelength of the impinging waves, the covariance matrix of the measurements is (see, e.g., [21])

$$R = \int_{\mathcal{S}} a(u)\Phi(u)a(u)^H ds(u), \tag{1}$$

where $a(u)$ is the array manifold vector [2, Chapter 2.2], i.e.,

$$a(u) = \left(e^{2\pi i u^T x_1/\lambda} \quad \dots \quad e^{2\pi i u^T x_n/\lambda} \right)^T, \tag{2}$$

with $(\cdot)^T$ denoting the transpose, and $ds(u)$ the normalized surface element.[1] It is worth noting that although any covariance matrix R is positive semidefinite, there may, for a given positive semidefinite matrix R, not exist a representing spectrum Φ. For example, a matrix that is not Toeplitz structured cannot be a covariance matrix corresponding to a ULA. Let the linear mapping from non-negative spectra on $\mathcal{S}$ to covariance matrices be $\Gamma : \mathcal{M}_+(\mathcal{S}) \to \mathbb{M}^n$, being defined as

$$\Gamma(\Phi) \triangleq \int_{\mathcal{S}} a(u)\Phi(u)a(u)^H ds(u). \tag{3}$$

Then, a positive semidefinite matrix R is a valid covariance for a given spatial array if and only if $R = \Gamma(\Phi)$ for some non-negative spectrum $\Phi \in \mathcal{M}_+(S)$. The set of such covariances can also be shown to be the closed convex cone

$$\left\{ R : R = \sum_{i=1}^{N} a(u_i)a(u_i)^H m_i, \quad u_i \in \mathcal{S}, m_i \geq 0 \right\}. \tag{4}$$

Specific classes of solutions Φ to (1), such as the exponential family or rational family, may be parameterized and solved explicitly [21], [22]. In this work, we will use a non-parametric approach of representing all solutions to (1) in order solve optimization problem that contain, e.g., regularization terms and optimal mass transport costs.

B. Optimal mass transport

The optimal mass transport problem is the problem of finding a transport plan between two given mass distributions with minimal total cost [16]. Consider two non-negative distributions Φ_0 and Φ_1 on the underlying space $\mathcal{S}$ with equal total mass. A transport plan is a non-negative measure $M \in \mathcal{M}_+(\mathcal{S}^2)$, where $M(u_0, u_1)$ represents the amount of mass transported from location u_0 to location u_1, i.e., any feasible transport plan satisfies

$$\Phi_0(u_0) = \int_{\mathcal{S}} M(u_0, u_1) ds(u_1), \tag{5}$$

$$\Phi_1(u_1) = \int_{\mathcal{S}} M(u_0, u_1) ds(u_0). \tag{6}$$

Let the cost of moving a unit mass from location u_0 to location u_1 be given by $c(u_0, u_1)$. Then, the cost of the transport plan M is given by

$$\Psi(M) \triangleq \int_{\mathcal{S}^2} c(u_0, u_1) M(u_0, u_1) ds(u_0) ds(u_1). \tag{7}$$

[1]That is, $ds(u)$ is the normalized Haar measure on $\mathcal{S}$.

The optimal mass transport problem is thus to find the feasible transport plan from Φ_0 to Φ_1 with minimal transportation cost, i.e.,

$$\begin{aligned} T(\Phi_0, \Phi_1) \triangleq & \min_{M \in \mathcal{M}_+(\mathcal{S}^2)} \Psi(M) \\ & \text{subject to } \Phi_0(u_0) = \int_{\mathcal{S}} M(u_0, u_1) ds(u_1) \\ & \qquad\qquad \Phi_1(u_1) = \int_{\mathcal{S}} M(u_0, u_1) ds(u_0). \end{aligned} \tag{8}$$

Here, the minimal cost $T(\Phi_0, \Phi_1)$ will be used as a measure of similarity between the two mass distributions Φ_0 and Φ_1. The idea of utilizing the optimal mass transport cost as a distance measure has been used, e.g., for defining metrics on the space of power spectra [17], clustering in fundamental frequency estimation [23], and the corresponding transport plan has been used for tracking and morphing signals with smoothly varying spectral content [18], [24]. It may be noted that, due to the marginal constraints, the distance measure T is only defined for spectra of the same total mass. However, T may be generalized in order to allow for unbalanced masses [17], [25]. In the numerical section of this work, we will consider the cost fucntion $c(u_0, u_1) = \|u_0 - u_1\|_2^2$.

III. Tracking and Sensor Fusion via Optimal Mass Transport

A. DOA estimation and tracking

Consider the problem of tracking several slowly moving targets, i.e., of forming estimates of time-varying DOAs in a sequential fashion. Specifically, assume that approximate information of the spatial spectrum at an earlier time instance is available in the form of a spectrum Φ_{prior} and that new data, in the form of the sensor array covariance matrix, R, is received. Then, an estimate of the spatial spectrum may be formed as the spectrum in the set of spectra consistent with R that minimizes the distance to the prior Φ_{prior} in the optimal mass transport sense (cf. [26]). That is, the estimated spatial spectrum Φ solves

$$\underset{\Phi \in \mathcal{M}_+(\mathcal{S})}{\text{minimize}} \quad T(\Phi_{\text{prior}}, \Phi) \quad \text{subject to} \quad \Gamma(\Phi) = R. \tag{9}$$

Here, the optimal mass transport cost T enforces slow variations in the spectral distribution of power, corresponding to smoothly moving targets. As noted above, a given covariance matrix R may not have a spectral representation, a scenario that may arise in the presence of calibration errors in the array manifold $a(u)$. In order to allow for such situations, i.e., to increase robustness, one may introduce small perturbations to the consistency constraint. This may be formulated as

$$\begin{aligned} & \underset{\Phi \in \mathcal{M}_+(\mathcal{S}),\, \Delta}{\text{minimize}} && T(\Phi_{\text{prior}}, \Phi) + \gamma \|\Delta\|_F^2 \\ & \text{subject to} && \Gamma(\Phi) = R + \Delta, \end{aligned} \tag{10}$$

where $\gamma > 0$ is a user-defined regularization parameter, penalizing the magnitude of the introduced perturbation matrix Δ, as measured by the Frobenius norm.

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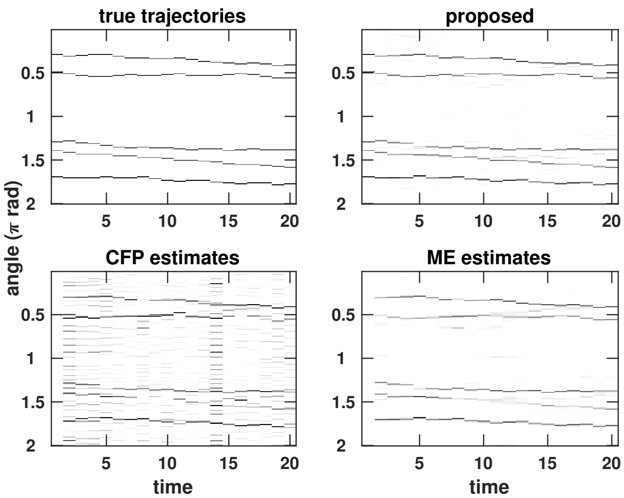

Fig. 1. Trajectories obtained from sequential DOA estimation.

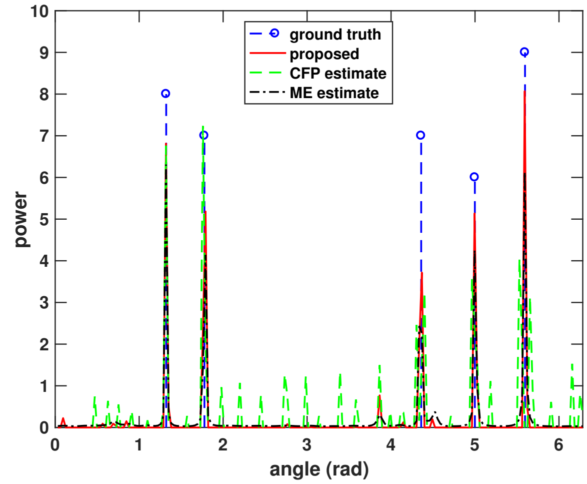

Fig. 2. Final DOA estimates after 20 time steps.

B. Sensor fusion

Consider the case of DOA estimation in a scenario where the wave-emitting sources impinge on a set of J arrays, with array manifold vectors $a_j(u)$, for $u \in \mathcal{S}$, giving rise to covariance matrices R_j, $j = 1, 2, \ldots, J$. Assuming knowledge of the individual array geometries, i.e., a_j, but not of the common geometry, we then aim to form an estimate of the underlying, spatial spectrum, given estimates of the J covariance matrices. Note that the ideal approach would be to concatenate the different arrays to a large array and then form the spectral estimates. However, in some scenarios, the global geometry may be unavailable, prohibiting calibration of the large array, prompting the need of fusing information from the individual arrays. Similarly, when information from several arrays is combined at a central processing node, one will strive to minimize the information transmitted to the central node. Here, we propose to utilize all the available data, in the form of covariance matrix estimates, by forming the estimates of the DOAs using the (not necessarily unique) spectrum that is closest to all the observed covariance matrices in optimal mass transport sense. Then, the sought spectrum, Φ, corresponding to the desired DOAs, may be estimated as the solution to the convex barycenter problem

$$\begin{aligned} \underset{\Phi \in \mathcal{M}_+(\mathcal{S}), \Phi_j \in \mathcal{M}_+(\mathcal{S})}{\text{minimize}} \quad & \sum_{j=1}^{J} T(\Phi, \Phi_j) \\ \text{subject to} \quad & \Gamma_j(\Phi_j) = R_j, \; j = 1, 2, \ldots, J, \end{aligned} \tag{11}$$

where $T(\Phi, \Phi_j)$ is the transport distance defined in (8) and where Γ_j is the function mapping the spectrum to the covariance matrix of array j, i.e.,

$$\Gamma_j(\Phi) = \int_{\mathcal{S}} a_j(u) \Phi(u) a_j(u)^H ds(u). \tag{12}$$

As in (10), one may extend the formulation to allow for inexact covariance estimates, due to, e.g., sensor noise or calibration errors, by allowing perturbations in the covariances according to

$$\begin{aligned} \underset{\substack{\Phi \in \mathcal{M}_+(\mathcal{S}), \Phi_j \in \mathcal{M}_+(\mathcal{S}) \\ \Delta_j}}{\text{minimize}} \quad & \sum_{j=1}^{J} T(\Phi, \Phi_j) + \gamma \sum_{j=1}^{J} \|\Delta_j\|_F^2 \\ \text{subject to} \quad & \Gamma_j(\Phi_j) = R_j + \Delta_j, \; j = 1, 2, \ldots, J. \end{aligned} \tag{13}$$

Note that the obtained optimization problem is still convex.

IV. Numerical results

A. DOA estimation and tracking

We initially illustrate the proposed method's ability to track smoothly moving targets. Consider an L-shaped array consisting of nine sensors with equidistant spacing, equal to the signal wavelength λ, along the array. Using this setup, we consider the problem of tracking five smoothly moving targets during 20 time instances using (10). Specifically, we provide the proposed method with an accurate prior spectrum Φ_{prior} that is used in the estimation of the spectrum in the first time instance. In each step, the previously estimated spectrum serves as the prior spectrum for the next time instance. The sensor measurements are corrupted by spatially and temporally white Gaussian noise of variance 0.1 and each covariance is then estimated using the sample covariance estimate from 200 snapshots (cf. [27]). The realized target trajectories are shown in Figure 1, along with estimates obtained using the proposed method. Also included are estimates obtained using a Carathéodory-Fejér-Pisarenko (CFP) [28]–[30] and a Maximum Entropy (ME) method [12], [27], neither of which employ any prior information.

As may be seen from Figure 1, the proposed estimator is able to accurately track the targets, whereas the CFP and ME estimators suffer from the spectral ambiguity inherent in estimating a spectrum from a finite covariance matrix estimate, resulting in both erroneous peaks in the obtained spectra as well as spectral leakage. Figure 2 displays the target locations at the last time instance, together with the spectral estimates

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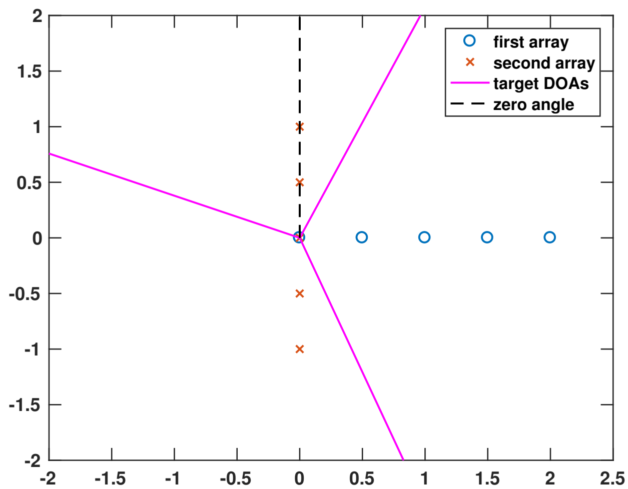

Fig. 3. Array and DOA setting for the sensor fusion scenario.

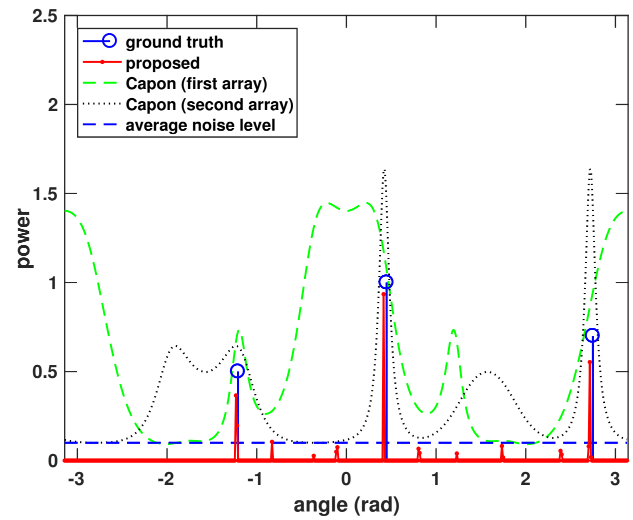

Fig. 4. Obtained DOA spectra from one instance of the sensor fusion scenario.

obtained from the proposed method as well as the CFP and ME estimates. As can be seen, the CFP estimate contains several spurious peaks. It may be noted that the proposed method is superior in estimating both the location and the power of the signal sources.

B. Sensor fusion

Proceeding, we consider a sensor fusion scenario with two separate ULAs, both consisting of 5 sensors, and three sources impinging on the arrays, as illustrated in Figure 3, where the unit of the axes is the signal wavelength. The three sources are modelled as uncorrelated Gaussian processes with variances 0.5, 0.7, and 1, respectively. Adding spatially and temporally white Gaussian sensor noise of variance 0.1, we evaluate the performance of the proposed sensor fusion formulation in equation (13) using 100 Monte Carlo simulations. In each simulation, we perturb the location of each sensor slightly by adding zero-mean, Gaussian distributed numbers with standard deviation corresponding to 1% of the nominal sensor spacing in both the x- and y-directions, and generate 100 snapshots of the sources impinging on the arrays, from which estimates of the covariance matrices for the two arrays are formed using the sample covariance estimate. Note that the corresponding operators Γ_j, $j = 1, 2$, are defined using the assumed ULA structure of the arrays as seen in Figure 3, i.e., there will be calibration errors due to the perturbation of the sensor locations.

Figure 4 displays the estimates obtained from one of the realizations. As can be seen, the proposed estimator is able to identify the three different DOAs, without having any prior information of the number of sources. Note also that the estimate also contains some small peaks, corresponding to the sensor noise. As comparison, we have also included estimates obtained using the Capon estimator for each individual array. Here, it should be noted that the individual Capon estimates, as expected, display erroneous peaks of power, due to the spatial ambiguity caused by the geometry of the arrays. Interestingly,

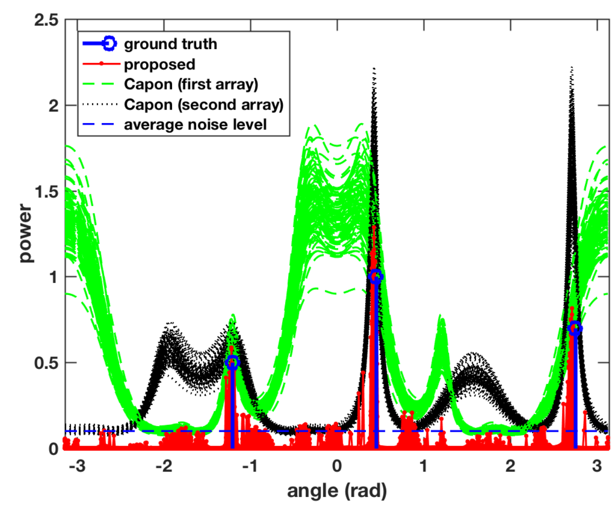

Fig. 5. Obtained DOA spectra from 100 instances of the sensor fusion scenario.

the peaks of the two Capon estimates only coincide well for one of the DOAs, indicating that a peak matching approach such as [15] would not succeed in identifying the correct number of sources, let alone their locations. The results from 100 Monte Carlo simulations, superimposed in the same plot, are shown in Figure 5. It may be noted that high-power estimates are consistently concentrated to the DOAs of the three sources.

V. Conclusions

In this work, we have presented a framework for performing tracking and information fusion in direction of arrival estimation scenarios. Utilizing the concept of optimal mass transport as a notion of distance between power spectra, we have proposed convex optimization criteria allowing for the tracking of slowly varying spatial spectra, as well as combining measurements from individual sensor arrays. The proposed methods have been shown to alleviate spatial ambiguity caused by the array geometry, and also allow for perturbations to the nominal sensor positions.

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