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Complexity-Reduced Solution for TDOA Source Localization in Large Equal Radius Scenario with Sensor Position Errors

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***Abstract*—This paper presents a new algebraic solution for source localization using time difference of arrival (TDOA) measurements in the large equal radius (LER) scenario when the known sensor positions have random errors. The proposed method utilizes the LER condition to directly approximate the true TDOAs so that the originally nonlinear TDOA equations can be recast into ones linearly related to the source position. This enables the use of the closed-form weighted least squares (WLS) technique for source localization and makes the proposed method have lower complexity than the existing technique. The approximate efficiency of the new algorithm is established analytically under strong LER condition. The associated approximation bias is also derived and it is shown numerically to be greater than that of the benchmark technique, especially when LER condition is weak. However, through iterating the proposed method once with bias correction, the proposed method yields comparable localization accuracy with reduced complexity. The theoretical developments are validated by computer simulations.**

I. Introduction

Source localization refers to determining the position of an unknown source. It is an active research area with wide applications in navigation, wireless communications, mobile positioning, and search and rescue [1], [2]. To locate the source, positioning parameters including the angle of arrival (AOA) [3], time difference of arrival (TDOA) [4] and frequency difference of arrival (FDOA) [5] are typically utilized.

We shall consider in this work using TDOA measurements for source localization. The inherent nonlinearity of TDOAs with respect to the unknown source position often makes it non-trivial to obtain the optimal solution. Many research efforts have been devoted to addressing this difficulty. Among the available TDOA positioning techniques, a straightforward approach is to perform exhaustive grid search. But the associated computational cost is prohibitive for real-time applications. Besides the iterative methods such as the Taylor-series (TS) based methods [6], [7], there also exist algebraic closed-form solutions such as the well known two-stage weighted least squares (TS-WLS) algorithm [8]. The TS-WLS technique is computationally attractive as it does not require initial solution guesses. More importantly, it provides the Cramér-Rao lower bound (CRLB) accuracy under small noise condition.

In some localization applications, the sensors may be deployed in a way that they have almost identical ranges from the source to be localized. Such a localization situation is called the large equal radius (LER) scenario [9], where all the TDOA measurements would approach zeros. A typical LER scenario occurs in the geolocation of an object on Earth using a number of satellites. Another example is positioning an object near the center of a sensor network using sensors at the boundary.

The TS-WLS method is known to be unable to produce a correct localization solution in the LER scenario as the matrix to be inverted in the WLS solution can be ill-conditioned, leading to a poor estimate [10]. To address this deficiency, alternative methods such as the dual-root minimum variance least squares (DMVLS) [11] and separated constrained weighted least squares (SCWLS) [10] algorithms have been proposed. To ensure good performance, however, they either require a proper initial solution guess or need complicated root selection strategies. Recently, [1] developed a geometric interpretation based method for TDOA source localization in the LER scenario. It leads to a closed-form solution, eliminating the requirement for good initialization and cumbersome root-finding process. However, its computation cost can be large for real-time applications especially when the number of sensors increases. This motivates the development of a new method.

This paper proposes a new closed-form solution for TDOA positioning in the LER scenario when the known sensor positions have random errors. It is derived by approximating the true TDOAs directly via exploiting the LER condition. The resulting TDOA equations are linearly dependent on the unknown source position and have a closed-form WLS solution. It is shown analytically that the new method is able to achieve the CRLB accuracy under small observation noise and strong LER condition, as the benchmark technique from [1]. The localization bias from the approximation is also derived and shown numerically to be larger than that of the method in [1]. But complexity analysis demonstrates that the new method is conceptually simpler and computationally cheaper. Simulation results verify that with iterating the algorithm once and bias correction, the new method yields comparable localization performance close to the CRLB as the one developed in [1] with reduced computational complexity.

The remainder of this paper is organized as follows. Section II describes the considered TDOA source localization scenario. Section III presents the new localization solution with its

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performance and complexity analysis. Section IV gives the simulation results and conclusions are drawn in Section V.

II. Localization scenario

We shall consider the 3D localization scenario depicted in Fig. 1. The objective is to locate an unknown source at $\mathbf{u}^o = [u_x^o, u_y^o, u_z^o]^T$ using M sensors, whose true positions are $\mathbf{s}_i^o = [s_{x,i}^o, s_{y,i}^o, s_{z,i}^o]^T$, $i = 1, 2, \ldots, M$. Under the LER condition [9], the sensors are equidistant or almost equidistant from the origin $\mathbf{p}$. Besides, the source-origin distance $r^o = ||\mathbf{u}^o||$ is much smaller than the sensor-origin distances $R_i^o = ||\mathbf{s}_i^o||$, where $\| * \|$ is the Euclidean norm. Mathematically, we have

$$\frac{R_i^o}{R_j^o} \simeq 1, \ \forall i \neq j \quad \text{and} \quad \frac{r^o}{R_i^o} \simeq 0, \ \forall i. \tag{1}$$

When the LER condition is strong, (1) would essentially become equalities.

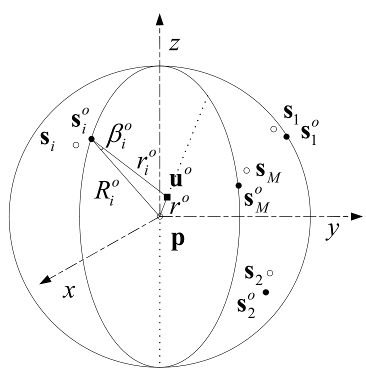

Fig. 1. 3D source localization in the LER scenario with noisy sensor positions.

The true TDOA between the sensor pair i and 1, after being multiplied with the signal propagation speed, is

$$r_{i1}^o = r_i^o - r_1^o = \|\mathbf{u}^o - \mathbf{s}_i^o\| - \|\mathbf{u}^o - \mathbf{s}_1^o\|. \tag{2}$$

The measured TDOAs can thus be collectively denoted as

$$\mathbf{r} = [r_{21}, r_{31}, ..., r_{M1}]^T = \mathbf{r}^o + \mathbf{n}_r \tag{3}$$

with $\mathbf{r}^o = [r_{21}^o, r_{31}^o, \ldots, r_{M1}^o]^T$. The measurement noise vector is $\mathbf{n}_r = [n_{r,21}, n_{r,31}, ..., n_{r,M1}]^T$ such that $r_{i1} = r_{i1}^o + n_{r,i1}$. $\mathbf{n}_r$ is assumed to follow the multivariate Gaussian distribution with zero mean and covariance matrix $\mathbf{Q}_r$.

It is clear from (2) that in the considered LER localization scenario, the true TDOAs will become close to zero. This renders many existing TDOA-positioning techniques such as those developed in [8] and [12] inapplicable, especially under strong LER condition. The underlying reason is that near zero-valued TDOAs can make some matrices to be inverted in these algorithms ill-conditioned and cause meaningless results.

The true positions of the sensors $\mathbf{s}_i^o$ are not available and the known sensor positions have errors $\mathbf{n}_{s,i}$ such that

$$\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, ..., \mathbf{s}_M^T]^T = \mathbf{s}^o + \mathbf{n}_s \tag{4}$$

where $\mathbf{s}^o = [\mathbf{s}_1^{oT}, \mathbf{s}_2^{oT}, \ldots, \mathbf{s}_M^{oT}]^T$. The sensor position error vector $\mathbf{n}_s = [\mathbf{n}_{s,1}^T, \mathbf{n}_{s,2}^T, ..., \mathbf{n}_{s,M}^T]^T$ is Gaussian distributed with zero-mean and covariance matrix $\mathbf{Q}_s$. We further assume that $\mathbf{n}_s$ and $\mathbf{n}_r$ are independent of each other.

III. Algorithm and Analysis

We shall present a new closed-form solution to the LER localization problem formulated in the previous section. As the existing method developed in [1], the new algorithm does not require an initial solution guess or intermediate variables, and it is shown analytically to be able to attain the CRLB accuracy under small Gaussian noise and strong LER condition. More importantly, the new method is conceptually and computationally simpler than the method in [1] and other root-finding based techniques such as those in [10], [11].

A. New Closed-Form Solution and Estimation Bias

1) Closed-Form Solution: Note from Fig. 1 that the sensor-origin distance R_i^o can be related to the source-sensor distance r_i^o and unknown source position $\mathbf{u}^o$ via

$$R_i^o = r_i^o \cos\beta_i^o + \boldsymbol{\rho}_i^{oT}\mathbf{u}^o \tag{5}$$

where $\boldsymbol{\rho}_i^o = \mathbf{s}_i^o / R_i^o$ is a unit vector from the origin $\mathbf{p}$ to the true position of sensor i, $\mathbf{s}_i^o$. From the LER condition (1), we have that β_i^o would approach zero if the source becomes close to the origin $\mathbf{p}$, which renders $\cos\beta_i^o \approx 1$. Using this approximation in (5) yields [1]

$$r_i^o \approx R_i^o - \boldsymbol{\rho}_i^{oT}\mathbf{u}^o. \tag{6}$$

Substituting (6) into (2), we arrive at

$$r_{i1}^o = r_i^o - r_1^o \approx R_i^o - R_1^o - (\boldsymbol{\rho}_i^o - \boldsymbol{\rho}_1^o)^T\mathbf{u}^o. \tag{7}$$

In other words, under the LER condition, the true TDOA is now approximately linear with respect to the unknown source position $\mathbf{u}^o$. This distinguishes the new solution from the existing one in [1] (see more discussions in Section III.C).

To estimate $\mathbf{u}^o$ as accurately as possible when the TDOA measurements and sensor positions are both subject to errors, we express the true values in (7) in terms of their noisy quantities and apply the following first-order approximations

$$R_i^o \approx R_i - \boldsymbol{\rho}_i^T\mathbf{n}_{\mathbf{s},i} \tag{8}$$

$$\boldsymbol{\rho}_i^o \approx \boldsymbol{\rho}_i - \frac{1}{R_i}\mathbf{P}_i^{\perp}\mathbf{n}_{\mathbf{s},i}\,, \quad \mathbf{P}_i^{\perp} = \mathbf{I} - \boldsymbol{\rho}_i\boldsymbol{\rho}_i^T. \tag{9}$$

R_i and $\boldsymbol{\rho}_i$ have the same functional form as R_i^o and $\boldsymbol{\rho}_i^o$ except that the true sensor position $\mathbf{s}_i^o$ is replaced with its known but imprecise version $\mathbf{s}_i$.

Putting (8) and (9) into (7), we obtain the solution equation for the considered LER localization problem, which is

$$\varepsilon_{i1} \approx r_{i1} - (R_i - R_1) + (\boldsymbol{\rho}_i - \boldsymbol{\rho}_1)^T\mathbf{u}^o. \tag{10}$$

The equation error ε_{i1} is

$$\varepsilon_{i1} = n_{r,i1} + \mathbf{g}_i^T\mathbf{n}_{\mathbf{s},i} - \mathbf{g}_1^T\mathbf{n}_{\mathbf{s},1}\,, \quad \mathbf{g}_i = \frac{1}{R_i}\mathbf{P}_i^{\perp}\mathbf{u}^o - \boldsymbol{\rho}_i. \tag{11}$$

Stacking (10) yields the solution equation in matrix form

$$\boldsymbol{\varepsilon} = \mathbf{h} - \mathbf{G}\mathbf{u}^o \tag{12}$$

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$$\mathbf{h} = \begin{bmatrix} r_{21} - (R_2 - R_1) \\ \vdots \\ r_{M1} - (R_M - R_1) \end{bmatrix}, \mathbf{G} = - \begin{bmatrix} \boldsymbol{\rho}_2^T - \boldsymbol{\rho}_1^T \\ \vdots \\ \boldsymbol{\rho}_M^T - \boldsymbol{\rho}_1^T \end{bmatrix}. \tag{13}$$

The equation error vector $\boldsymbol{\varepsilon}$ is equal to

$$\boldsymbol{\varepsilon} = \mathbf{n}_r + \mathbf{D}\mathbf{n}_s \tag{14}$$

where $\mathbf{D}$ is a $(M-1) \times 3M$ matrix and its $(i-1)$th row, $i = 2, 3, ..., M$, is defined as

$$\mathbf{D}(i-1, :) = [-\mathbf{g}_1^T, \mathbf{0}_{3(i-2)\times 1}^T, \mathbf{g}_i^T, \mathbf{0}_{3(M-i)\times 1}^T]. \tag{15}$$

The WLS solution to (12), also the newly proposed localization solution for the LER scenario, is

$$\mathbf{u} = (\mathbf{G}^T\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{W}\mathbf{h} \tag{16}$$

where the weighting matrix $\mathbf{W}$ is

$$\mathbf{W} = E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T)^{-1} = (\mathbf{Q}_r + \mathbf{D}\mathbf{Q}_s\mathbf{D}^T)^{-1}. \tag{17}$$

It is important to point out that evaluating (16) requires any three sensors are not colinear in order to make the matrix $\mathbf{G}$ having full column rank. This can be satisfied in the LER localization scenario by ensuring that no two sensors are very near to each other.

To obtain the weighting matrix $\mathbf{W}$ that depends on the unknown source position (see (15) and (11) for the definition of $\mathbf{D}$), we first find an initial source position estimate with $\mathbf{W} = \mathbf{Q}_r^{-1}$ and using the result to produce a better approximation of $\mathbf{W}$ for obtaining improved estimate $\mathbf{u}$.

2) Localization Bias: The localization error in $\mathbf{u}$ can be found by subtracting both sides of (16) the true source position $\mathbf{u}^o$. We have

$$\triangle\mathbf{u} = \mathbf{u} - \mathbf{u}^o = (\mathbf{G}^T\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{W}\mathbf{b} \tag{18}$$

where

$$\mathbf{b} = [b_{21}, b_{31}, ..., b_{M1}]^T = \mathbf{h} - \mathbf{G}\mathbf{u}^o. \tag{19}$$

Even in the absence of TDOA noise and sensor position errors, $\mathbf{b}$ would still be non-zero due to the use of the approximation (6). In such a noiseless case, from (13) and using (2) and (5), the $(i-1)$th element of $\mathbf{b}$ can be shown to be equal to

$$\begin{aligned} b_{i1} &= r_{i1}^o - (R_i^o - R_1^o) + (\boldsymbol{\rho}_i^o - \boldsymbol{\rho}_1^o)^T\mathbf{u}^o \\ &= r_i^o(1 - \cos\beta_i^o) - r_1^o(1 - \cos\beta_1^o). \end{aligned} \tag{20}$$

Putting (20) into (19) and substituting the result back into (18) yield the localization error in the *noise-free* scenario, which is indeed the estimation bias in the new localization solution (16). It is clear from (20) that the localization bias would be prominent as the LER condition becomes weak and β_i^o increases. On the contrary, under strong LER condition, β_i^o, β_1^o and hence b_{i1} in (20) would approach zero, which leads to an almost unbiased localization solution.

To reduce the estimation bias in $\mathbf{u}$ from (16), a straightforward approach is to subtract an estimate of the localization bias from $\mathbf{u}$ for producing the reduced-bias solution

$$\tilde{\mathbf{u}} = \mathbf{u} - \triangle\tilde{\mathbf{u}}. \tag{21}$$

$\triangle\tilde{\mathbf{u}}$ is obtained via replacing $\mathbf{u}^o$ with $\mathbf{u}$ and evaluating (18).

To further improve performance under weak LER condition, we can adopt the technique in [1] to iterate the proposed localization method once as follows. In particular, we can set the coordinate origin $\mathbf{p}$ to $\tilde{\mathbf{u}}$ and repeat the process that produces $\tilde{\mathbf{u}}$. The newly obtained localization result is then added to $\tilde{\mathbf{u}}$ to generate the final localization output.

B. *Performance Analysis*

The CRLB of the true source position $\mathbf{u}^o$ is [13]

$$\mathrm{CRLB}(\mathbf{u}^o) = (\mathbf{U}^T\mathbf{Q}_r^{-1}\mathbf{U} - \mathbf{U}^T\mathbf{Q}_r^{-1}\mathbf{V}\mathbf{Z}^{-1}\mathbf{V}^T\mathbf{Q}_r^{-1}\mathbf{U})^{-1}. \tag{22}$$

Putting the definition of $\mathbf{Z}$, $\mathbf{Z} = \mathbf{V}^T\mathbf{Q}_r^{-1}\mathbf{V} + \mathbf{Q}_s^{-1}$, and applying the matrix inversion Lemma [14], we have

$$\mathrm{CRLB}(\mathbf{u}^o) = (\mathbf{U}^T(\mathbf{Q}_r + \mathbf{V}\mathbf{Q}_s\mathbf{V}^T)^{-1}\mathbf{U})^{-1}. \tag{23}$$

$\mathbf{U} = \partial\mathbf{r}^o/\partial\mathbf{u}^o$ and $\mathbf{V} = \partial\mathbf{r}^o/\partial\mathbf{s}^o$ are the partial derivatives. They both have $(M-1)$ rows and their $(i-1)$th rows are

$$\mathbf{U}(i-1, :) = \frac{\partial r_{i1}^o}{\partial \mathbf{u}^{oT}} = \bar{\boldsymbol{\rho}}_i^{oT} - \bar{\boldsymbol{\rho}}_1^{oT} \tag{24}$$

$$\mathbf{V}(i-1, :) = \frac{\partial r_{i1}^o}{\partial \mathbf{s}^{oT}} = [\bar{\boldsymbol{\rho}}_1^{oT}, \mathbf{0}_{3(i-2)\times 1}^T, -\bar{\boldsymbol{\rho}}_i^{oT}, \mathbf{0}_{3(M-i)\times 1}^T] \tag{25}$$

where $i = 2, 3, ..., M$ and $\bar{\boldsymbol{\rho}}_i^o = (\mathbf{u}^o - \mathbf{s}_i^o)/r_i^o$.

Under strong LER condition, the proposed localization solution $\mathbf{u}$ is approximately unbiased (see the discussion under (20)). Moreover, when the noise is small, the covariance matrix of $\mathbf{u}$ can be found by multiplying $\Delta\mathbf{u}$ in (18) with its transpose, using $\boldsymbol{\varepsilon}$ given by (14) for $\mathbf{b}$ and taking expectation. We have, after putting the definition of the weighting matrix $\mathbf{W}$ in (17),

$$\mathrm{cov}(\mathbf{u}) \approx (\mathbf{G}^T(\mathbf{Q}_r + \mathbf{D}\mathbf{Q}_s\mathbf{D}^T)^{-1}\mathbf{G})^{-1}. \tag{26}$$

Comparing (26) with (23) reveals that they would be approximately equal to each other, if

$$\mathbf{U} \approx \mathbf{G},\ \mathbf{V} \approx -\mathbf{D}. \tag{27}$$

From (11), (13), (15), (24) and (25), we can establish (27). First, we have the approximation $\|\frac{1}{R_i^o}\mathbf{P}_i^{o\perp}\mathbf{u}^o\| \leq \frac{r^o}{R_i^o} \approx 0$, which is valid under strong LER condition. Second, by expanding $\bar{\boldsymbol{\rho}}_i^o$ with respect to the source position around the origin as

$$\begin{aligned} \bar{\boldsymbol{\rho}}_i^o &\approx -\boldsymbol{\rho}_i^o + \left.\frac{\partial\bar{\boldsymbol{\rho}}_i^o}{\partial\mathbf{u}^o}\right|_{\mathbf{u}^o=\mathbf{0}} \mathbf{u}^o \\ &= -\boldsymbol{\rho}_i^o + \frac{1}{R_i^o}\mathbf{P}_i^{o\perp}\mathbf{u}^o \approx -\boldsymbol{\rho}_i^o \approx -\boldsymbol{\rho}_i \end{aligned} \tag{28}$$

where $\mathbf{P}_i^{o\perp} = \mathbf{I} - \boldsymbol{\rho}_i^o\boldsymbol{\rho}_i^{oT}$. The last approximation comes from the small sensor position error assumption. Therefore, we can arrive at, under small noise and strong LER condition,

$$\mathrm{cov}(\mathbf{u}) \approx \mathrm{CRLB}(\mathbf{u^o}). \tag{29}$$

The newly proposed localization solution is approximately efficient.

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TABLE I
TRUE SOURCE AND SENSOR POSITIONS IN SPHERICAL COORDINATES

	Source	Sensor						
Azimuth (^{o})	114	116	79	169	11	-63	-142	40
Elevation (^{o})	-15	76	-39	47	46	-22	12	-77
Range (m)	r^o	$R^o = 100$						

C. Complexity Analysis

A closed-form WLS solution to the considered LER localization problem described in Section II was developed in [1], which is based on the TDOA model equation

$$r_{i1}^o \approx R_i^o - R_1^o - \left[\frac{2(\mathbf{s}_i^o - \mathbf{s}_1^o)^T}{R_i^o + R_1^o} - \frac{r_{i1}^o(\boldsymbol{\rho}_i^o - \boldsymbol{\rho}_1^o)^T}{R_i^o + R_1^o} \right] \mathbf{u}^o. \quad (30)$$

It is straightforward to verify that the proposed WLS localization solution and the one developed from (30) in [1] would be equivalent to each other under strong LER condition. But the TDOA equation adopted in this work (see (7)) is simpler. In particular, the derivation of the solution equation (10) in this work does not require ignoring second-order and higher order error terms. On the contrary, the establishment of the solution equation from (30) does need to neglect them due to the presence of the product $r_{i1}^o(\boldsymbol{\rho}_i^o - \boldsymbol{\rho}_1^o)$ and the division $\frac{1}{R_i^o + R_1^o}$. Another advantage of adopting the approximate TDOA equation (7) is the reduced computational complexity. Compared with the method in [1], it is no longer necessary to compute the division $\frac{1}{R_i^o + R_1^o}$ when evaluating the WLS localization solution using (16). Besides, finding the estimate of the localization bias with (18) is simpler, as the regression matrix $\mathbf{G}$ has a less complicated form. These lead to a reduction of the number of divisions by $M - 1$ and the number of multiplications by $11(M - 1)$. Simulation results presented in the following Section verify that the new algorithm takes significantly smaller amount of running time than the method in [1].

IV. SIMULATION RESULTS

We shall locate a source using $M = 7$ sensors, which are distributed on a sphere centered at the origin with a radius $R^o = 100$m. The spherical coordinates of the true source and sensor positions are listed in Table I. The radial distance of the source, r^o, will be varied to generate different LER conditions.

In each experiment, $L = 2000$ ensemble runs are performed. Noisy TDOA measurements are generated by adding to the true values independent zero-mean Gaussian noise with covariance matrix $\mathbf{Q}_r = \frac{1}{2}\sigma_r^2(\mathbf{I}_{6\times6} + \mathbf{1}_{6\times1}\mathbf{1}_{6\times1}^T)$ [8], where σ_r^2 is the TDOA noise variance. The erroneous sensor positions are produced in a similar way but with covariance matrix $\mathbf{Q}_s = \sigma_s^2$diag([1,2,10,4,25,30,40])$\otimes\mathbf{I}_{3\times3}$. Here, $\otimes$ denotes the Kronecker product and σ_s^2 is the variance of the sensor position errors. We shall investigate via simulations the performance of the proposed solution in terms of the localization mean square error (MSE) under different LER conditions, and various TDOA and sensor position noise variances.

In the first simulation experiment, we fix the TDOA noise power at $\sigma_r^2 = 10^{-7}$m^2 and sensor position error variance at $\sigma_s^2 = 0$m^2 (i.e, the sensor positions are known accurately). This setup facilitates the investigation of the localization bias of the proposed solution, as it is nearly a noise-free scenario. We plot in Fig. 2 the localization MSEs of the proposed solution in (16) and its reduced-bias version given in (21) as a function of the range ratio R^o/r^o. Clearly, the larger the range ratio is, the stronger the LER condition would be. For comparison, the associated source position CRLB and the localization performance of the solution in [1] are also shown.

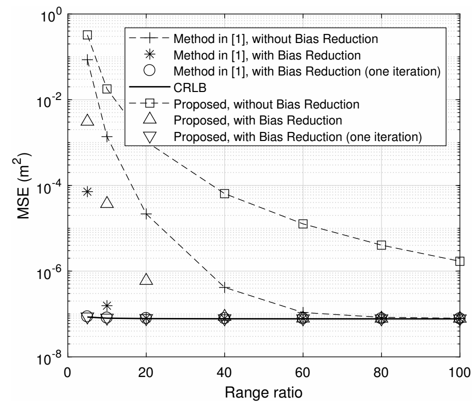

Fig. 2. Localization MSEs of the proposed method and the solution from [1] as a function of the range ratio R^o/r^o.

Fig. 2 shows that the localization MSE of the proposed solution in (16) gradually approaches the CRLB, as the range ratio R^o/r^o increases. In fact, it can reach the CRLB when R^o/r^o is larger than 300. This is consistent with the performance analysis result in Section III.B that the proposed solution is approximately efficient under strong LER condition. The bias reduction solution using (21) can significantly improve the localization performance, where its MSE is close to the CRLB when the range ratio is as small as 40 in this simulation.

Compared with the method from [1], however, the proposed localization solution requires larger range ratio to reach the CRLB performance, apparently due to having higher estimation bias. This comes from that the TDOA modeling error due to the use of the approximation (7) is higher than that obtained via using (30) (results not shown because of page limit).

We next repeat the experiment for Fig. 2 but this time, both the proposed solution and the method from [1] are repeated once after origin update, as suggested at the end of Section III.B. With just one iteration, the proposed and original methods have comparable performance and attain the CRLB accuracy even under very weak LER conditions. This is due to the improvement in the LER condition during iterations.

As pointed out in Section III.C, the proposed solution achieves such performance with less computation burden. To quantify the amount of computational saving, we run the MATLAB codes for the proposed method and the one developed in [1], both with bias reduction and one iteration, on a desktop with an Intel i5-8250U CPU clocked at 1.60GHz and 8GB RAM. The number of sensors used for localization is varied from 4 to 10 for a systematic evaluation. The

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TABLE II
COMPARISON OF EXECUTION TIMES IN SECONDS FOR $L = 50000$ RUNS AS THE NUMBER OF SENSORS VARIES

M	4	5	6	7	8	9	10
Proposed	5.85	5.58	5.68	5.91	6.10	6.16	6.35
Method in [1]	7.52	7.73	7.71	7.96	8.16	8.36	8.40
Percentage (%)	22.2	27.8	26.3	25.7	25.2	26.3	24.4

execution times for 50,000 runs are listed in Table II, where in each ensemble run, the azimuth and elevation angles of the sensors are randomly generated. The proposed method requires 22.2%∼27.8% less running time than the solution from [1].

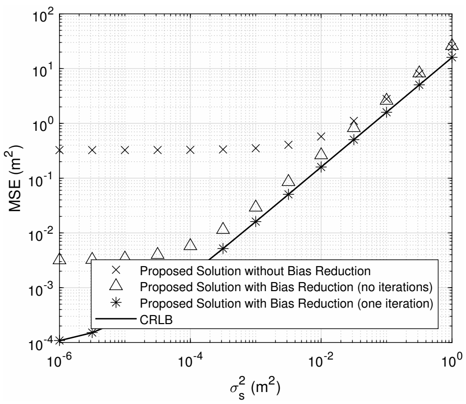

Fig. 3. Localization MSE of the proposed solution as a function of the sensor position error variance σ_s^2 with $\sigma_r^2 = 10^{-4}\mathrm{m}^2$.

We next fix $\sigma_r^2 = 10^{-4}\mathrm{m}^2$ and study the impact of the sensor position error on the localization accuracy of the proposed method. The simulation is only considered for the weak LER scenario with a small range ratio $R/r^o = 5$, as the proposed solution is confirmed in Fig. 2 to be efficient only under strong LER condition. In Fig. 3, the simulation MSE of the proposed solution is plotted against the CRLB when varying the sensor position noise power σ_s^2. Bias reduction provides some improvement of the performance in weak LER conditions. However, again by performing one iteration, the LER condition improves and the proposed solution achieves the CRLB accuracy. We then set $\sigma_s^2 = 10^{-5}\mathrm{m}^2$ and vary the TDOA measurement noise power, while keeping $R^o/r^o = 5$. The proposed method with bias reduction and one iteration also attains the CRLB accuracy, as shown in Fig. 4.

V. CONCLUSIONS

In this paper, a new closed-form solution with bias reduction for TDOA based LER localization was developed. Its performance was shown analytically and verified by simulations to be able to achieve the CRLB in the strong LER scenario. Though the method in [1] can reach the CRLB performance under weaker LER condition, the proposed estimator is less complex and computationally more efficient. Simulation results have shown that by applying bias reduction and repeating the processing once, they can both attain the CRLB accuracy under typical LER condition even if it is quite weak.

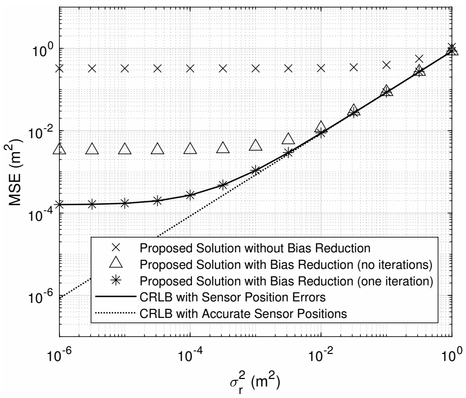

Fig. 4. Localization MSE of the proposed solution as a function of the TDOA measurement noise power σ_r^2 with $\sigma_s^2 = 10^{-5}\mathrm{m}^2$.

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