

# Steerable Circular Differential Microphone Arrays

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**Abstract**—An efficient continuous beam steering method, applicable to differential microphones of any order, has been recently developed. Given two identical reference beams, pointing in two different directions, the method allows to derive a beam of nearly constant shape continuously steerable between those two directions. In this paper, the steering method is applied to robust Differential Microphone Arrays (DMAs) characterized by uniform circular array geometries. In particular, a generalized filter performing the steering operation is defined. The definition of such a filter enables the derivation of closed-form formulas for computing the white noise gain and the directivity factor of the designed steerable differential beamformers for any frequency of interest. A study on the shape invariance of the steered beams is conducted. Applications of the steering approach to first-, second- and third-order robust circular DMAs are presented.

## I. INTRODUCTION

The interest in Differential Microphone Arrays (DMAs) [1], [2] is continuously increasing in the audio signal processing community due to their promise to build nearly frequency-invariant responses. DMA theory has been developed to design directional beams starting from arrays of physical omnidirectional microphones. DMAs with different array geometries, e.g. Uniform Linear Arrays (ULAs) [2], [3], Non-Uniform Linear Arrays [4] or Uniform Circular Arrays (UCAs) [5], appeared in the literature. While in linear DMAs the endfire direction is the preferred steering direction, circular DMAs are characterized by the same behavior in all azimuthal directions identified by a physical sensor [5]. Moreover, the theory on Robust Circular DMAs (RCDMAs) formalized in [5] to minimize the white noise amplification, decouples the number of physical sensors from the order of the designed beams, unlike what happens with traditional DMAs [2], [6], [7]. In the latest years, procedures to achieve continuous beam steering using DMAs have been developed. One of these is the method based on eigenbeams originally developed by Elko in [8], that, employing square array geometries, allows to design arbitrarily-shaped steerable first-order beams. A second approach, proposed by Wu *et al.* in [9], allows to derive steerable second-order beams. However, that approach is constrained to an ad-hoc nonuniform 2D array geometry of 7 sensors. A novel class of beamformers, derived by Huang *et al.* in [10] and called Frequency-Invariant Beampatterns with Least-Squares Error (FIB-LSE), allows to build perfectly steerable frequency-invariant beams of any order. However, the design of the coefficients used in FIB-LSE requires the evaluation of Bessel functions which can cause significant degradation of the beamformer when their output approaches

zero [10]. Two solutions to this problem have been proposed by Huang *et al.* [10]: one is based on 3D arrays and the other on concentric circular arrays [11]. Both solutions still require the evaluation of Bessel functions. This last task could result in computationally heavy implementations for systems with small processing capabilities, e.g., embedded systems.

An alternative efficient steering method applicable to differential microphones with arbitrary beampatterns (BPs) has been introduced in [12], [13]. The method is simple to implement, since it is based on a weighted sum of identical beams positioned in the same point of the 2D plane and pointing towards two different directions. Assuming that the angular displacement between the two mainlobe directions of the two reference beams is sufficiently small, the steering method allows to derive a beam of nearly constant shape continuously steerable between those two directions. In [12] the method was formalized for general DMAs, independently from the actual geometry, assuming frequency-independent beampatterns; only a brief example of application to ULAs was presented.

In this paper, a frequency domain analysis of the steering method presented in [12], applied to RCDMAs with UCA geometry, is conducted. A new expression for the frequency domain filter that performs the steering between two directions identified by two adjacent sensors of the UCA is derived. The definition of such a filter enables the derivation of closed-form formulas for computing the White Noise Gain (WNG) and the Directivity Factor (DF) of the designed steerable differential beamformers for any frequency of interest. Section II provides a background on RCDMAs. Section III resumes the continuous beam steering method presented in [12]. Section IV introduces steerable RCDMAs; a derivation of the frequency domain filter for the steering operation is presented, along with a definition of WNG and DF of steerable RCDMAs. It is shown that the WNG and the DF of steerable RCDMAs are similar but not identical to traditional RCDMAs. In particular, steered RCDMAs exhibit slightly better WNG and slightly worse DF. Moreover, an index for measuring the similarity between steered beams and ideal ones is defined. Section V discusses how the proposed method can be used in conjunction with the method presented in [10] and concludes this paper.

## II. BACKGROUND ON ROBUST CIRCULAR DMAs

Let us consider the UCA geometry with radius  $r$  and  $M$  omnidirectional microphones shown in Fig. 1. The UCA is placed in an anechoic acoustic environment also containing an

acoustic source in far-field. The acoustic signal from the source sensed by the UCA is modeled as a plane wave propagating at the speed of sound, i.e.,  $c = 340$  m/s. The reference point of the array is the center of the UCA and it coincides with the the origin of the Cartesian coordinate system. The time delay between the  $m$ th microphone and the center of the array is computed as

$$\tau_m = \frac{r}{c} \cos(\theta - \psi_m), \quad m = 1, \dots, M \quad (1)$$

where  $\theta$  is the azimuth angle, measured anti-clockwise from the axis containing the first microphone characterized by index 1, and  $\psi_m = 2\pi(m-1)/M$  is the angular position of the  $m$ th sensor. It follows that the steering vector of length  $M$  characterizing the UCA is

$$\mathbf{d}(\omega, \theta) = [e^{j\omega\tau_1} \dots e^{j\omega\tau_M}]^T, \quad (2)$$

where the superscript  $T$  denotes transposition,  $\omega = 2\pi f$  is the radian frequency and  $f > 0$  is the temporal frequency.

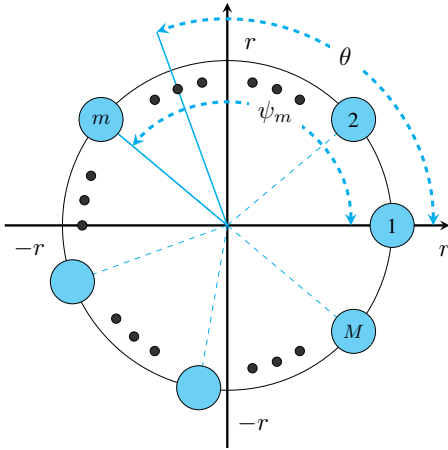


Fig. 1. Illustration of an UCA in the Cartesian coordinate system.

Arbitrary order RCDMAs have been theorized in [5] and they are characterized by the UCA geometry in Fig. 1.  $N$ th order RCDMAs can be designed with an arbitrary number of microphones  $M$ , provided that the condition  $N \leq \lfloor M/2 \rfloor$  is satisfied. It follows that first-, second- and third-order RCDMAs can be designed disposing of at least  $M = 2$ ,  $M = 4$  and  $M = 6$  microphones, respectively. The frequency-dependent beampattern of a RCDMA is given by

$$\mathcal{B}(\omega, \theta) = \mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta), \quad (3)$$

where the superscript  $H$  indicates the Hermitian operator and the minimum-norm vector filter  $\mathbf{h}(\omega) = [H_1(\omega) \dots H_M(\omega)]^T$  is computed as

$$\mathbf{h}(\omega) = \mathbf{A}^H(\omega, \theta) [\mathbf{A}(\omega, \theta) \mathbf{A}^H(\omega, \theta)]^{-1} \mathbf{b}, \quad (4)$$

where the constraint matrix  $\mathbf{A}(\omega, \theta)$  and the constraint vector  $\mathbf{b}$  are built as described by Benesty *et al.* in chapters 3 and 7 of [5]. An important property of RCDMAs is that the frequency-dependent beampattern in eq. (3) is almost

frequency-invariant over large frequency ranges [5] and it is very close to a reference  $N$ th order frequency-independent beampattern, defined as

$$\mathcal{B}(\theta) = 1 - \sum_{n=1}^N a_n + \sum_{n=1}^N a_n \cos^n(\theta), \quad (5)$$

where  $a_1, \dots, a_N$  are real coefficients. Given the desired order  $N$ , the number of sensors  $M$  and the beampattern coefficients  $a_1, \dots, a_N$ ,  $\mathbf{A}(\omega, \theta)$  and  $\mathbf{b}$  are uniquely determined [5].

The WNG  $\mathcal{G}_{\text{wn}}[\mathbf{h}(\omega)]$  of a RCDMA is given by

$$\mathcal{G}_{\text{wn}}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega) \mathbf{d}(\omega, 0)|^2}{\mathbf{h}^H(\omega) \mathbf{h}(\omega)}, \quad (6)$$

while the DF  $\mathcal{G}_{\text{dn}}[\mathbf{h}(\omega)]$  is given by

$$\mathcal{G}_{\text{dn}}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega) \mathbf{d}(\omega, 0)|^2}{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\text{dn}}(\omega) \mathbf{h}(\omega)}, \quad (7)$$

where the entry at row  $i$  and column  $j$  of matrix  $\mathbf{\Gamma}_{\text{dn}}(\omega)$  is defined as  $[\mathbf{\Gamma}_{\text{dn}}(\omega)]_{ij} = \text{sinc}(\omega \delta_{ij}/c)$ , being  $\delta_{ij} = 2r |\sin[\pi(i-j)/M]|$  the distance between sensors  $i$  and  $j$ .

### III. CONTINUOUS BEAM STEERING METHOD

In this Section, the continuous beam steering method presented in [12], [13] is revised. Since DMAs are known to be nearly frequency-invariant for large frequency ranges [2], frequency-independent beampatterns are considered in this Section; it follows that, as a first approximation, the formulas are valid independently from the actual array geometry and the used DMA design approach. In Section IV the method will be specifically applied to RCDMAs and the more accurate frequency-dependent beampattern model in eq. (3) will be considered.

Let us consider a frequency-independent beampattern  $\mathcal{B}(\theta)$ , defined as in eq. (5).  $\mathcal{B}(\theta)$  is assumed to have a mainlobe pointing at  $\theta_0$  and a shape symmetric w.r.t. the mainlobe axis. Let us then define  $\bar{\mathcal{B}}(\theta) = \mathcal{B}(\theta - \rho)$  as the rotated version of  $\mathcal{B}(\theta)$  in the same point on the 2D plane and with the mainlobe pointing in  $\bar{\theta}$ , where  $\rho = \bar{\theta} - \theta_0$  is the angular displacement and  $0 < \rho \leq \pi/2$ . Without loss of generality, let us assume that  $\theta_0 = 0$  and  $\bar{\theta} = \rho$ . Mathematically, we define a weighted sum between  $\mathcal{B}(\theta)$  and  $\bar{\mathcal{B}}(\theta)$  as

$$\mathcal{B}_{\text{ws}}(\theta) = \alpha \mathcal{B}(\theta) + \bar{\alpha} \bar{\mathcal{B}}(\theta) = \bar{\alpha} [\beta \mathcal{B}(\theta) + \mathcal{B}(\theta - \rho)] \quad (8)$$

where  $\alpha > 0$ ,  $\bar{\alpha} > 0$  and  $\beta = \alpha/\bar{\alpha}$  are real weights. In order to steer the mainlobe of  $\mathcal{B}_{\text{ws}}(\theta)$  in a desired direction  $\theta_d$ , with  $0 < \theta_d < \rho$ , we impose the constraint

$$\mathcal{D}_{\text{ws}}(\theta_d) = \left. \frac{\partial \mathcal{B}_{\text{ws}}(\theta)}{\partial \theta} \right|_{\theta=\theta_d} = \bar{\alpha} [\beta \mathcal{D}_{\mathcal{B}}(\theta_d) + \mathcal{D}_{\mathcal{B}}(\theta_d - \rho)] = 0, \quad (9)$$

where

$$\mathcal{D}_{\mathcal{B}}(\theta) = \frac{\partial \mathcal{B}(\theta)}{\partial \theta} = - \sum_{n=1}^N n a_n \sin(\theta) \cos^{n-1}(\theta). \quad (10)$$

From constraint (9), we deduce the following expression for the parameter  $\beta$

$$\beta = \frac{D_{\mathcal{B}}(\theta_d - \rho)}{-D_{\mathcal{B}}(\theta_d)}. \quad (11)$$

In order to normalize the beampattern  $\mathcal{B}_{\text{ws}}(\theta)$ , we impose the second constraint  $\mathcal{B}_{\text{ws}}(\theta_d) = 1$ , such that we deduce the following expression for the weight  $\bar{\alpha}$

$$\bar{\alpha} = \frac{1}{\beta \mathcal{B}(\theta_d) + \mathcal{B}(\theta_d - \rho)}. \quad (12)$$

The other weight  $\alpha$  can be easily computed as  $\alpha = \beta \bar{\alpha}$ . Assuming that  $\rho$  is sufficiently small and computing  $\beta$  and  $\bar{\alpha}$  using (11) and (12), respectively,  $\mathcal{B}_{\text{ws}}(\theta)$  is steered in  $\theta_d$  and it has a shape similar to  $\mathcal{B}(\theta)$ , according to the definition of similarity given in [12].

#### IV. STEERABLE ROBUST CIRCULAR DMAS

In this Section, firstly, the continuous beam steering method revised in Section III is applied to non-ideal frequency-dependent patterns. The reference beams are now designed according to RCDMA theory and their pointing directions are identified by pairs of adjacent microphones, i.e.,  $\theta_0 = \psi_m$  and  $\bar{\theta} = \psi_{m+1}$ . The proposed approach allows to build a beam of nearly constant shape steerable in the circular sector  $\rho = \psi_{m+1} - \psi_m$ . An expression for the filter describing the steerable beamformer is derived. A comparison between a beamformer based on frequency-dependent weights and another based on frequency-independent weights is performed. Finally, closed-form formulas for the WNG and DF are provided.

##### A. Filter Derivation

The frequency-dependent beampattern of a RCDMA, defined in eq. (3), can also be written as

$$\mathcal{B}(\omega, \theta) = \sum_{m=1}^M H_m^*(\omega) e^{(j\omega r/c) \cos(\theta - \psi_m)}. \quad (13)$$

It follows that

$$\begin{aligned} \mathcal{D}_{\mathcal{B}}(\omega, \theta) &= \frac{\partial \mathcal{B}(\omega, \theta)}{\partial \theta} = \\ &= -j\omega r/c \sum_{m=1}^M \sin(\theta - \psi_m) H_m^*(\omega) e^{(j\omega r/c) \cos(\theta - \psi_m)}. \end{aligned} \quad (14)$$

Employing an UCA composed of  $M$  sensors results in an angular displacement  $\rho = 2\pi/M$  between two microphones. The weighted sum between the two RCDMA beams  $\mathcal{B}(\omega, \theta)$  and  $\mathcal{B}(\omega, \theta - \rho)$  is

$$\begin{aligned} \mathcal{B}_{\text{ws}}(\omega, \theta) &= \alpha^*(\omega) \mathcal{B}(\omega, \theta) + \bar{\alpha}^*(\omega) \mathcal{B}(\omega, \theta - \rho) \\ &= \bar{\alpha}^*(\omega) [\beta^*(\omega) \mathcal{B}(\omega, \theta) + \mathcal{B}(\omega, \theta - \rho)], \end{aligned} \quad (15)$$

where

$$\beta^*(\omega) = \frac{\mathcal{D}_{\mathcal{B}}(\omega, \theta_d - \rho)}{-\mathcal{D}_{\mathcal{B}}(\omega, \theta_d)} \quad (17)$$

and

$$\bar{\alpha}^*(\omega) = \frac{1}{\beta^*(\omega) \mathcal{B}(\omega, \theta_d) + \mathcal{B}(\omega, \theta_d - \rho)} \quad (18)$$

are the complex conjugates of the frequency-dependent weights to be applied to the reference beams.

Defined  $\mathbf{I}$  as the  $M \times M$  identity matrix, we can write  $\mathbf{I} = [\mathbf{i}_1 \cdots \mathbf{i}_M]$ , where  $\mathbf{i}_m$ , with  $1 \leq m \leq M$ , is a column vector of all zeros except for a 1 as  $m$ th element. Let us now define a permutation matrix  $\mathbf{P}$  such that  $\mathbf{P} = [\mathbf{i}_2 \mathbf{i}_3 \cdots \mathbf{i}_M \mathbf{i}_1]$ , we can build  $\mathcal{B}(\omega, \theta_d - \rho)$  as

$$\mathcal{B}(\omega, \theta_d - \rho) = \mathbf{h}^H(\omega) \mathbf{P}^T \mathbf{d}(\omega, \theta). \quad (19)$$

Substituting (19) in (15) we obtain a generalized formula for the steerable RCDMA beam

$$\begin{aligned} \mathcal{B}_{\text{ws}}(\omega, \theta) &= \alpha^*(\omega) \mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta) + \bar{\alpha}^*(\omega) \mathbf{h}^H(\omega) \mathbf{P}^T \mathbf{d}(\omega, \theta) \\ &= \mathbf{h}_{\text{ws}}^H(\omega, \theta) \mathbf{d}(\omega, \theta), \end{aligned} \quad (20)$$

where

$$\mathbf{h}_{\text{ws}}(\omega, \theta) = [\alpha(\omega) \mathbf{I} + \bar{\alpha}(\omega) \mathbf{P}] \mathbf{h}(\omega) \quad (21)$$

or, equivalently,

$$\mathbf{h}_{\text{ws}}(\omega, \theta) = \bar{\alpha}(\omega) [\beta(\omega) \mathbf{I} + \mathbf{P}] \mathbf{h}(\omega) \quad (22)$$

is the vector filter of the steerable beamformer.

A slightly less accurate, though more efficient, implementation of the vector filter employs frequency-independent weights. The expression of the vector filter becomes

$$\hat{\mathbf{h}}_{\text{ws}}(\omega, \theta) = \bar{\alpha} [\beta \mathbf{I} + \mathbf{P}] \mathbf{h}(\omega). \quad (23)$$

where  $\alpha$ ,  $\bar{\alpha}$  and  $\beta$  are the frequency-independent weights defined in Section III. It is worth noticing that the implementation of (23) requires the calculation of two scalar weights applied to all the frequency bins; therefore, once  $\mathbf{h}(\omega)$  is computed, continuous steering can be performed extremely efficiently.

##### B. White Noise Gain and Directivity Factor Definition

Disposing of a definition of the filter, i.e. eq. (21), closed-form formulas for the WNG and the DF of the proposed steerable beamformer are easily derived. The WNG  $\mathcal{G}_{\text{wn}}[\mathbf{h}_{\text{ws}}(\omega)]$  of a steerable RCDMA is given by

$$\mathcal{G}_{\text{wn}}[\mathbf{h}_{\text{ws}}(\omega)] = \frac{|\mathbf{h}_{\text{ws}}^H(\omega) \mathbf{d}(\omega, \theta_d)|^2}{\mathbf{h}_{\text{ws}}^H(\omega) \mathbf{h}_{\text{ws}}(\omega)}, \quad (24)$$

while the DF  $\mathcal{G}_{\text{dn}}[\mathbf{h}_{\text{ws}}(\omega)]$  is given by

$$\mathcal{G}_{\text{dn}}[\mathbf{h}_{\text{ws}}(\omega)] = \frac{|\mathbf{h}_{\text{ws}}^H(\omega) \mathbf{d}(\omega, \theta_d)|^2}{\mathbf{h}_{\text{ws}}^H(\omega) \mathbf{\Gamma}_{\text{dn}}(\omega) \mathbf{h}_{\text{ws}}(\omega)}. \quad (25)$$

In Fig. 2, we show the WNG of hypercardioids of the first-, second- and third-order. From these plots, it is clear that, fixed  $\rho$  and  $a_1, \dots, a_N$ , the WNG of a steered RCDMA is better than the WNG of a traditional RCDMA. In particular, we see that the highest WNG is achieved at  $\theta_d = \rho/2$ . It is also clear that the WNG improves significantly with third-order beams, while, in the first- and second-order case the WNG of the steered RCDMA is very close to the WNG of the standard RCDMA. In Fig. 3, we show the DF of hypercardioids of the first-, second- and third-order. It is clear that the steered RCDMA, fixed  $\rho$  and  $a_1, \dots, a_N$ , has worse DF w.r.t. the standard RCDMA. It is also noticeable that, for higher orders beams, the DF of the steered RCDMA worsen.

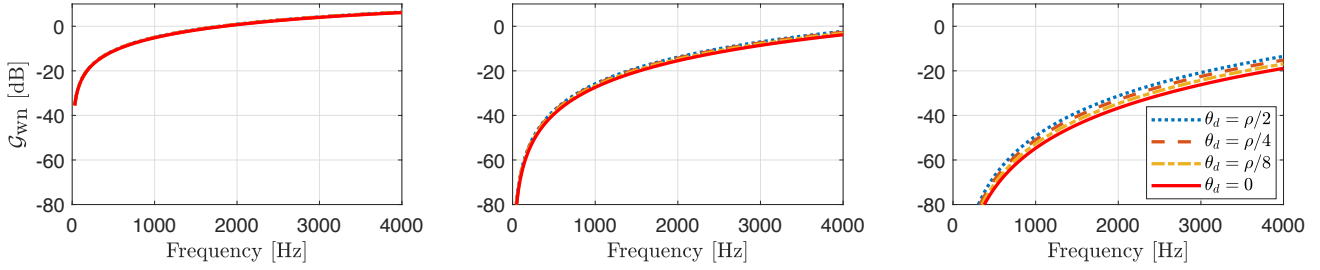


Fig. 2. WNG of first-order (left), second-order (center) and third-order (right) hypercardioids. Conditions of simulation:  $M = 8$ ,  $r = 1$  cm,  $\theta_d = \rho/2$  and  $f = 1$  kHz.

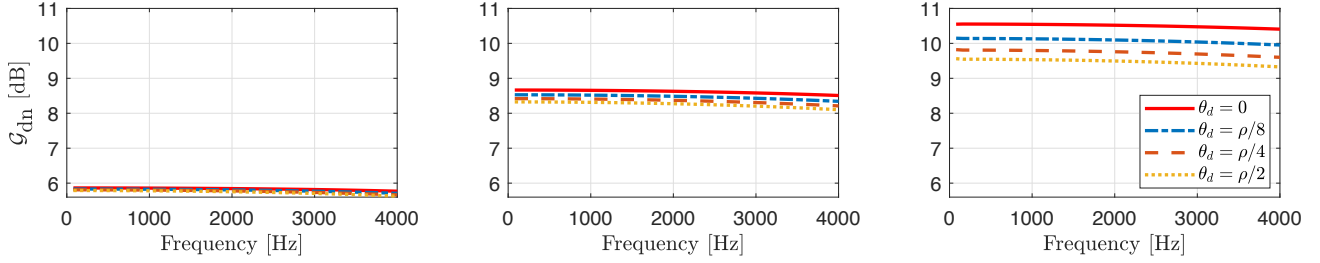


Fig. 3. DF of first-order (left), second-order (center) and third-order (right) hypercardioids. Conditions of simulation:  $M = 8$ ,  $r = 1$  cm,  $\theta_d = \rho/2$  and  $f = 1$  kHz.

### C. Measuring Steered Beam Shape Invariance

As already mentioned in [12],  $\mathcal{B}_{ws}(\theta)$  is generally similar to the reference directivity pattern  $\mathcal{B}(\theta - \theta_d)$ , but not exactly identical. This is also true for the steered RCDMA pattern  $\mathcal{B}_{ws}(\omega, \theta)$ . To compute a degree of similarity between the two patterns, the *percentage similarity error*  $J_E$  is defined in [12]. This measure is based on the shape error

$$\mathcal{E}(\omega, \theta) = \mathcal{B}_{ws}(\omega, \theta) - \mathcal{B}(\theta - \theta_d). \quad (26)$$

The percentage similarity error  $J_E$ , relative to the ideal frequency-independent beam  $\mathcal{B}(\theta - \theta_d)$ , is here redefined as

$$J_E(\omega) = 100 \times \frac{\int_0^{2\pi} |\mathcal{E}(\omega, \theta)|^2 d\theta}{\int_0^{2\pi} |\mathcal{B}(\theta - \theta_d)|^2 d\theta}. \quad (27)$$

In Fig. 4 we show how the  $J_E$  varies w.r.t. the steering angle  $\theta_d$ ,  $0 \leq \theta_d \leq \rho$ ; first-, second- and third-order steered cardioids, designed both with filter  $\mathbf{h}_{ws}(\omega, \theta)$  and with filter  $\hat{\mathbf{h}}_{ws}(\omega, \theta)$ , are considered. From the plots it is clear that, when filter  $\mathbf{h}_{ws}(\omega, \theta)$  is used, the largest  $J_E$  always corresponds to  $\theta_d = \rho/2$ ; while, the  $J_E$  profile changes when filter  $\hat{\mathbf{h}}_{ws}(\omega, \theta)$  is employed. We also deduce that using  $\hat{\mathbf{h}}_{ws}(\omega, \theta)$  is always worse in terms of  $J_E$  than using  $\mathbf{h}_{ws}(\omega, \theta)$ , except for the case  $\theta_d = \rho/2$  in which the two approaches lead to the same percentage similarity error. In Fig. 4  $\omega = 2\pi f$  with  $f = 1$  kHz. However, similar considerations hold when  $\omega$  is changed. Fig. 5 shows how the  $J_E$  improves when the number of sensors  $M$  is increased.

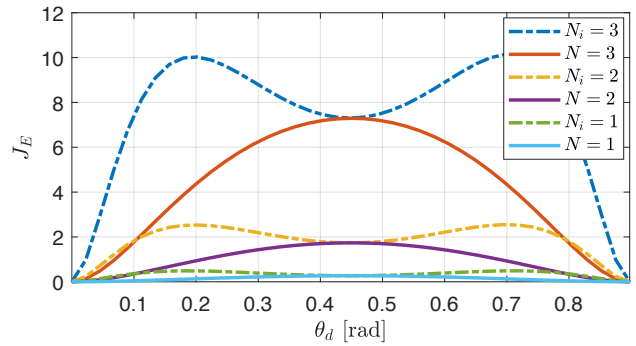


Fig. 4. Percentage similarity error  $J_E$  of first-, second- and third-order steered cardioids, designed with filter  $\mathbf{h}_{ws}(\omega, \theta)$  (continuous line and order  $N$ ) and filter  $\hat{\mathbf{h}}_{ws}(\omega, \theta)$  (dashed line and order  $N_i$ ), as a function of the steering direction  $\theta_d$ ,  $0 \leq \theta_d \leq \rho$ . Conditions of simulation:  $M = 7$ ,  $r = 1$  cm and  $f = 1$  kHz.

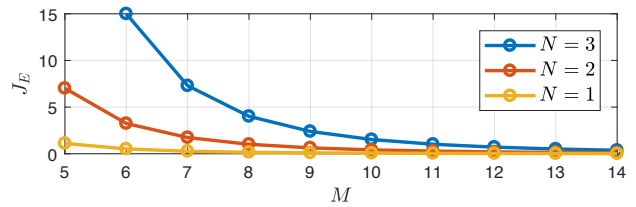


Fig. 5.  $J_E$  of first-, second- and third-order steered cardioids as a function of  $M$ . Conditions of simulations:  $\rho = 2\pi/M$ ,  $\theta_d = \rho/2$ ,  $r = 1$  cm and  $f = 1$  kHz.

In Fig. 6 we show some comparisons between steered beams of frequency-dependent RCDMA and ideal frequency-independent steered patterns, when  $\theta_d = \rho/2$ . From these plots, it is evident that the weighted sum approach performs better, in terms of beam shape conservation, when pattern of lower orders are designed.

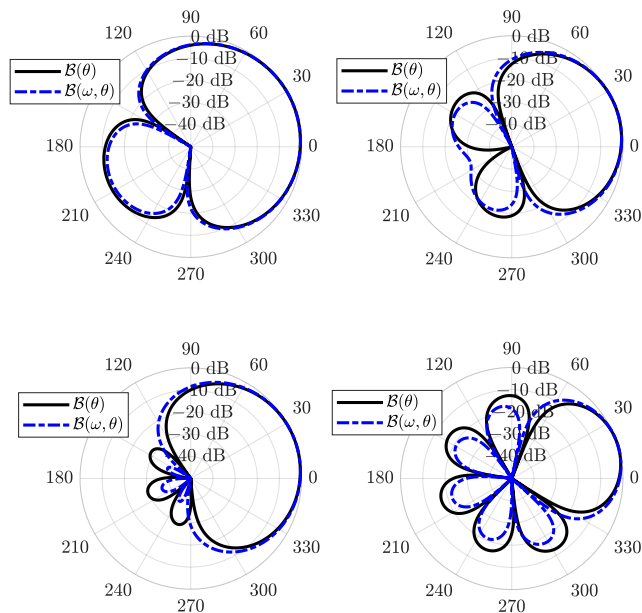


Fig. 6. Directivity patterns of steered RCDMAs with  $\theta_d = \rho/2$ : (upper-left) hypercardioid,  $N = 1$ ,  $M = 7$ , (upper-right) cardioid,  $N = 2$ ,  $M = 9$ , (bottom-left) supercardioid,  $N = 2$ ,  $M = 9$  and (bottom-right) hypercardioid,  $N = 3$ ,  $M = 10$ . Conditions of simulation:  $r = 1$  cm and  $f = 1$  kHz.

## V. DISCUSSION AND CONCLUSION

As the proposed steering method is very efficient, in the following, we will discuss how and when it can be used as an alternative to, or in conjunction with, the state-of-the-art steering method proposed in [10].

### A. Discussion on FIB-LSE and steerable RCDMA

The FIB-LSE approach [10] allows to perform continuous beam steering preserving the shape of the beampattern. However, the filter design procedure of the FIB-LSE method requires the evaluation of Bessel functions of the first kind, which can cause significant degradation of the beamformer when their output approaches zero. Moreover, the evaluation of Bessel functions could be computationally heavy for systems with small processing capabilities, and, eventually, it would require tabulation methods.

For these reasons, the steering method proposed in this paper, though less accurate, could be used as an alternative to, or even in conjunction with, the method in [10] in order to reduce the computational cost. In fact, since the proposed steering approach works independently from the method used to construct the pair of reference beams, it would be also possible to derive the reference beams using the FIB-LSE

methodology. This would allow to deal with angular displacements  $\rho$  that are different from  $2\pi/M$ . For instance, such an approach could be useful in scenarios in which the Direction of Arrival (DOA) of the acoustic source needs to be estimated, performing searching procedures based on continuous steering in an angular sector and on maximization of the energy of the beamformer output. Since the design of the coefficients of FIB-LSE depends on the steering direction, performing continuous steering, during the DOA estimation procedure, could be computationally costly. Therefore, the efficient steering method based on a weighted sum could be used during the DOA estimation procedure and, once a DOA is estimated, the FIB-LSE approach could be finally employed.

### B. Conclusion

In this paper, we showed how the continuous beam steering approach introduced in [12] can be applied to RCDMAs. We defined a filter performing steering of RCDMAs, and we derived closed-form formulas to compute the WNG and the DF at each frequency of interest. The presented steerable RCDMA approach, although being less accurate w.r.t. the FIB-LSE method, could be used, eventually in conjunction with FIB-LSE, for designing steerable beamformers, which are more flexible in terms of computational cost.

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