A New Blind Beamforming Technique for the Alignment and Enhancement of Seismic Signals

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***Abstract*—Blind beamforming constitutes a unified framework for the solution of two very important problems in seismological applications involving ensembles of similar signals, namely the signal alignment and signal enhancement problems. The former problem translates into the estimation of the time delays that exist between the signals, while the second problem deals with the optimal weighting of the signals, so that the SNR of their weighted average is maximized. A global optimization technique for the solution of the alignment problem with a sample-level accuracy, is proposed in this manuscript. The sample-level alignment problem is formulated as a combinatorial optimization problem and an approximate solution is proposed by using the technique of SDP relaxation. Finally, the signal enhancement problem is formulated as a quadratic maximization problem which in the vast majority of cases has an analytical solution, while in more challenging conditions can be approximately solved via SDP relaxation. The superior performance of the technique compared to other similar approaches is demonstrated through a number experiments involving numerical simulations with several signal and noise models.**

I. Introduction

The increasing use of sensor arrays in several fields including wireless communication, radar, sonar, seismic prospecting, remote sensing and others, has turned beamforming into a topic of intensive and ongoing research [1]. The enhancement effect of beamforming can improve the detectability of weak signals and facilitate further signal analysis, while the time-delays between the signals can be translated into information regarding the location of the source (e.g., the signal's direction of arrival). The specific solution to the problem of beamforming depends on the assumptions that can be made regarding the array's response, the characteristics of the recorded signals, as well as the statistical properties of the noise.

There is an extensive literature treating the problem of beamforming under the scenario of narrowband signals, plane-waves and precisely calibrated array responses [2]–[4]. On the other hand, robust beamforming deals primarily with the effects of imperfect knowledge regarding the array's response [5]–[7]. In case the information regarding sensor placement and response is (totally or partially) missing, the beamforming problem is referred to as blind beamforming [8]. The majority of techniques that have been proposed for the solution of the blind beamforming problem assume narrowband signals of known characteristics. The constant modulus algorithms (CMA) [9] and the higher order statistics (HOS) methods [10]–[12] belong to this category of blind beamforming techniques.

In this manuscript, we treat blind beamforming in its most general form, meaning that no assumption is made regarding sensor placement, specific signal features, or specific noise models. We only assume an ensemble of similar (but not necessarily identical) signals, recorded with unknown delays, and corrupted by noise. The goal of the proposed technique is to obtain the time-delays that yield the optimal joint alignment of the signals, as well as the non-negative weights that must be applied to the aligned signals, so that the SNR of their weighted sum is maximized.

Although it constitutes a general alignment and enhancement framework, the proposed technique is especially suitable in applications dealing with signals of natural and not man-made origin, namely, where exact knowledge and control over the input is missing, as it is the case, for example, with seismological applications. Waveform similarity in seismological datasets occurs mainly under two scenarios. Either when the signal from a single source is recorded by several similar and closely spaced sensors, or when signals from several similar and closely spaced sources are recorded by a single sensor. Records from seismic arrays and datasets from seismic multiplets (that is, clusters of similar events), are prime representatives of the the first and second scenarios, respectively.

Typically, in seismic array processing, beamforming is performed on the basis of predicted time delays that are obtained by using simplified models of reality (e.g., homogeneous geology, plane waves) [13] and grid searching techniques. In this case, the proposed technique can be used for the design of data-driven instead of model-driven array methodologies. It should be noted that the advantages of this signal-based approach in seismic array processing, have already been demonstrated [14]. In seismic prospecting the proposed technique can be considered as a generalization of what is referred to as optimal stacking [15], [16], where the signals are assumed already aligned, with the optimization being targeted only towards the weights. Finally, in the study of seismic multiplets, the delay estimates, as well as the enhanced average signal returned by the proposed technique, can be used in re-location procedures that have been shown to improve the

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accuracy of the event location estimates by several orders of magnitude [17]–[19].

The remaining of this manuscript is organized as follows. In Section II we present a general formulation of the problem at hand and subsequently, in Sections III and IV the proposed solutions to the ensuing problems of signal alignment and weight assignment, respectively, are presented in detail. In Section V we present an outline of the proposed algorithm and hold a brief discussion regarding the estimation of the required parameters. Section VI contains our experimental results and finally, Section VII contains our conclusions.

II. Problem formulation

Let $x_i(n),\ i = 1, \dots, M$, denote an ensemble of M time series, defined as follows:

$$x_i(n) = s_i(n) + w_i(n),\ n = 0, 1, \dots, N-1, \qquad (1)$$

where $s_i(n)$, $w_i(n)$ denote the i-th signal and i-th noise process, respectively. The involved noise processes are assumed zero-mean, wide-sense stationary and jointly uncorrelated. Let also $\boldsymbol{\tau} = [\tau_1, \dots, \tau_M]^t$, $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_M]^T$ denote arrays of time-delays and of non-negative weights, respectively, and let the beam (or enhanced signal, depending on the application) $\bar{y}(n; \boldsymbol{\gamma}, \boldsymbol{\tau})$, be defined as the weighted average of the delayed version of the available ensemble, namely:

$$\bar{y}(n; \boldsymbol{\gamma}, \boldsymbol{\tau}) \triangleq \sum_{i=1}^{M} \gamma_i x_i(n - \tau_i). \qquad (2)$$

If we denote as $\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma}, \boldsymbol{\tau})$ the SNR of $\bar{y}(n; \boldsymbol{\gamma}, \boldsymbol{\tau})$, the problem at hand is finding the optimal $\boldsymbol{\gamma}$,$\boldsymbol{\tau}$, so that $\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma}, \boldsymbol{\tau})$ is maximized, i.e.:

$$\max_{\boldsymbol{\gamma}, \boldsymbol{\tau}} \mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma}, \boldsymbol{\tau}), \quad \text{s.t. } \boldsymbol{\gamma} \geq 0. \qquad (3)$$

By taking into account the assumed stationarity of the involved processes, the SNR of $\bar{y}(n; \boldsymbol{\gamma}, \boldsymbol{\tau})$, can be expressed compactly as follows:

$$\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma}, \boldsymbol{\tau}) = \frac{\boldsymbol{\gamma}^T R(\boldsymbol{\tau}) \boldsymbol{\gamma}}{\boldsymbol{\gamma}^T \Sigma \boldsymbol{\gamma}}, \qquad (4)$$

where $R(\boldsymbol{\tau})$, Σ, are the signal cross-correlation and noise cross-covariance matrices, respectively. The noise cross-covariance matrix Σ, is a diagonal matrix having σ_i^2 in its i-th diagonal position, where σ_i^2 denotes the variance of the $i-$th process.

As it becomes obvious from Eq. (4), the problem defined in (3) is non-convex, meaning that a closed form solution is not possible in the general case. However, satisfactory approximate solutions can be achieved in a numerical fashion, via an iterative scheme of alternating optimizations over the two sets of parameters. This approach yields the following two maximization problems:

- P_1: *Signal alignment*. For a fixed $\boldsymbol{\gamma}$, find the time-delays that maximize the numerator of Eq. (4).
- P_2: *Signal enhancement*. For a given $\boldsymbol{\tau}$ (that is, for a given alignment of the signals), find the weights that maximize the SNR of the average signals, namely $\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma})$.

III. Optimal sample-level alignment

A. A filtering-based reformulation

Let $\boldsymbol{h}_i = [h_{i,0}, \dots, h_{i,(L-1)}]^T,\ i = 1, \dots, M$ denote M ideal delay operators of length L each, defined as follows:

$$h_{i,n} = \gamma_i \delta(n - \tau_i),\ n = 0, \dots, L-1, \qquad (5)$$

where $\delta(n)$ denotes the Kronecker delta, $\gamma_i > 0$ and $0 \leq \tau_i \leq L-1$, and L is assumed greater than the greatest pairwise time-difference of the signals. Let also $\boldsymbol{s}_i(n) = [s_i(n), s_i(n-1), \dots, s_i(n-L+1)]^T$, $i = 1, \dots, M$ denote a signal array of length L. Then, we can write $s_i(n) * h_{i,n} = \boldsymbol{h}_i^T \boldsymbol{s}_i(n) = \gamma_i s(n - \tau_i)$, meaning that, after some mathematical manipulations, the alignment problem P_1 can be expressed as follows:

$$\max_{\mathbf{h}} \quad \mathbf{h}^T \mathbf{R} \mathbf{h} \qquad (6)$$

$$\text{s.t.} \quad \boldsymbol{h}_i \text{ satisfies (5)},\ 1 \leq i \leq M, \qquad (7)$$

where,

$$\mathbf{R} = \begin{bmatrix} 0 & R_{12} & \cdots & R_{1M} \\ R_{21} & 0 & \cdots & R_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ R_{M1} & R_{M2} & \cdots & 0 \end{bmatrix},\ \mathbf{h} = \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \vdots \\ \boldsymbol{h}_M \end{bmatrix}, \qquad (8)$$

where R_{ij} is an $L \times L$ Toeplitz matrix whose (p, q)-th element equals $r_{ij}(p - q)$, and $r_{ij}(\kappa)$ denotes the cross-correlation of the (i, j)-th signal pair at lag κ. Thus, for a given set of γ_i's, the signal alignment problem as formulated in (6)-(7), seeks the optimal placement of the non-zero coefficient in each $\boldsymbol{h}_i$, so that the quadratic form in Eq. (6) is maximized. This is of course a problem of combinatorial complexity.

A very useful property stated without proof is the following:

Proposition 1: Condition (5) is equivalent to the combination of the following three constraints:

$$\|\boldsymbol{h}_i\|_2 = \gamma_i, \quad \|\boldsymbol{h}_i\|_1 = \gamma_i, \quad \boldsymbol{h}_i \geq 0. \qquad (9)$$

By combining Proposition 1 with the property $\mathrm{trace}(ABC) = \mathrm{trace}(BCA)$ (for any compatible matrices A, B, C), the original version of the signal alignment problem can be equivalently formulated as the following trace maximization problem:

$$\phi_o^* = \max_{\mathbf{H}} \quad \mathrm{trace}(\mathbf{RH}), \qquad (10)$$

$$\text{s.t.} \quad \mathrm{trace}(H_{ii}) = \gamma_i^2,\ 1 \leq i \leq M, \qquad (11)$$

$$\mathbf{1}^T H_{ii} \mathbf{1} = \gamma_i^2,\ 1 \leq i \leq M, \qquad (12)$$

$$\mathbf{H} \geq 0, \qquad (13)$$

$$\mathrm{rank}(\mathbf{H}) = 1, \qquad (14)$$

where $\mathbf{H} = \mathbf{h}\mathbf{h}^T$ is a rank-one, $LM \times LM$ matrix consisting of M^2 $L \times L$ blocks, arranged as follows:

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1M} \\ H_{12}^T & H_{22} & \cdots & H_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ H_{1M}^T & H_{2M}^T & \cdots & H_{MM} \end{bmatrix}, \qquad (15)$$

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and $H_{ij} = \boldsymbol{h}_i \boldsymbol{h}_j^T$. The only non-convex constraint in (10)-(14) is the rank-one constraint (14), since all other constraints and the objective function are linear in $\mathbf{H}$. By applying the SDP relaxation technique [20] we drop the non-convex constraint (14) and replace it with $\mathbf{H} \succeq 0$, which is convex. Thus, the relaxed version of (10)-(14) takes the following form:

$$
\begin{aligned}
\phi_r^* = \max_{\mathbf{H}} \quad & \mathrm{trace}(\mathbf{R}\mathbf{H}), && (16)\\
\text{s.t.} \quad & \mathrm{trace}(H_{ii}) = \gamma_i^2,\ 1 \le i \le M, && (17)\\
& \mathbf{1}^T H_{ii} \mathbf{1} = \gamma_i^2,\ 1 \le i \le M, && (18)\\
& \mathbf{H} \ge 0, && (19)\\
& \mathbf{H} \succeq 0, && (20)
\end{aligned}
$$

which constitutes a semi-definite program with variable $\mathbf{H} \in \Re^{LM \times LM}$ that can be solved by using software packages like CVX [21].

B. Estimating $\boldsymbol{\tau}$ from the solution of the relaxed problem

Let $\mathbf{H}$ be the solution of the relaxed problem and $\hat{\mathbf{h}} = \boldsymbol{\xi}_{\max}(\mathbf{H})$, with $\boldsymbol{\xi}_{\max}(\mathbf{H})$ denoting the eigenvector that corresponds to the largest eigenvalue of $\mathbf{H}$.

It can be shown that, if $\mathbf{H}$ is a rank-one matrix, then $\hat{\mathbf{h}}\hat{\mathbf{h}}^T$ solves the original problem (10)-(14), meaning that in the equipartition $\hat{\mathbf{h}} = [\hat{\boldsymbol{h}}_1^T \cdots \hat{\boldsymbol{h}}_M^T]^T$ of $\hat{\mathbf{h}}$, every $\hat{\boldsymbol{h}}_i$ is a $L \times 1$ vector with a single non-zero (positive) element. In this case, the loci of the positive elements produce the set of time-delays that solve the signal alignment problem. In the general case, $\hat{\boldsymbol{h}}_i$ is a sparse vector with a dominant positive element. Thus, an estimate $\hat{\boldsymbol{\tau}}$ of the time-delays that lead to the optimal joint alignment of the signals, can be obtained as follows:

$$
\hat{\tau}_i = \underset{0 \le n \le L-1}{\mathrm{argmax}}\ \hat{h}_{i,n}, \tag{21}
$$

where $\hat{\tau}_i$, $\hat{h}_{i,n}$ denote the elements of $\hat{\boldsymbol{\tau}}$, $\hat{\boldsymbol{h}}_i$, respectively.

C. Computational complexity and over-relaxation

Although it is polynomially solvable, problem (16)-(20) is still computationally intensive. The main factor contributing to its complexity lies in Eq. (19) which introduces $LM(LM-1)/2$ inequality constraints to the problem. As a result, (16)-(20) can become impractical for large filter lengths, namely, for large values of L (e.g. for $L > 20$). This depends of course on the number of signals as well (namely, M), which, however, is not a parameter of the problem.

Since L needs to be greater than the greatest pairwise time-difference of the signals, in cases where the signals can be separated by large time intervals, we propose solving the following version of the problem:

$$
\begin{aligned}
\phi_{rr}^* = \max_{\mathbf{H}} \quad & \mathrm{trace}(\mathbf{R}_{+}\mathbf{H}), && (22)\\
\text{s.t.} \quad & \mathrm{trace}(H_{ii}) = \gamma_i^2,\ 1 \le i \le M, && (23)\\
& \mathbf{1}^T H_{ii} \mathbf{1} = \gamma_i^2,\ 1 \le i \le M, && (24)\\
& H_{ii} \ge 0,\ 1 \le i \le M, && (25)\\
& \mathbf{H} \succeq 0, && (26)
\end{aligned}
$$

where, $\mathbf{R}_{+}$ retains only the non-negative values of $\mathbf{R}$ (the rest are set to 0). The delay estimates $\hat{\boldsymbol{\tau}}$ are again obtaing by using Eq. (21).

IV. The optimal weights

As it is obvious from Eq. (4), for a given $\boldsymbol{\tau}$, $\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma})$ constitutes a generalized Rayleigh quotient, meaning that the following holds:

$$
\mathrm{snr}_{\bar{y}}(\boldsymbol{\gamma}) \le \lambda_{\max}(Q), \tag{27}
$$

with equality being reached for

$$
\boldsymbol{\gamma}_{\max} = \Sigma^{-1/2} \boldsymbol{\xi}_{\max}(Q), \tag{28}
$$

where $Q = \Sigma^{-1/2} R(\boldsymbol{\tau})\, \Sigma^{-1/2}$ and $\lambda_{\max}(Q)$, $\boldsymbol{\xi}_{\max}(Q)$ denote the largest eigenvalue of Q and the corresponding eigenvector, respectively. Thus, if $\boldsymbol{\gamma}_{\max}$ has non-negative elements, then $\boldsymbol{\gamma}_{\max}$ solves the signal enhancement problem. It should be stressed that, since the estimation of $\boldsymbol{\gamma}$ succeeds the alignment of the signals, it is reasonable to expect that in the vast majority of cases, the correlation matrix $R(\boldsymbol{\tau})$ will be non-negative, which is a sufficient condition for $\boldsymbol{\gamma}_{\max} \ge 0$ to hold [22]. If this is not the case, then, as we did with the alignment problem, we can relax the enhancement problem into a semi-definite program with variable $\Gamma \in \Re^{M \times M}$:

$$
\begin{aligned}
\max_{\Gamma} \quad & \mathrm{trace}(Q\Gamma), && (29)\\
\text{s.t.} \quad & \mathrm{trace}(\Gamma) = 1, && (30)\\
& \Gamma \ge 0, && (31)\\
& \Gamma \succeq 0. && (32)
\end{aligned}
$$

The optimal weights are then estimated as

$$
\hat{\boldsymbol{\gamma}} = \Sigma^{-1/2} \boldsymbol{\xi}_{\max}(\Gamma). \tag{33}
$$

The outline of the proposed technique is summarized in Algorithm 1 presented in the next section.

V. Algorithm

Algorithm 1 SDP-based blind beamforming

```
procedure SDP-BB
    Estimate correlation sequences, noise variances and L.
    Set γ = 1.
    Solve (22)-(26) and estimate initial τ.
    repeat
        Update L using max_ij max_κ r̂_ij(κ).
        Solve (16)-(20) and estimate τ using Eq. (21).
        Update γ using (28) or (29)-(33).
    until There is no change in τ.
    Return τ, γ.
end procedure
```

As it can be seen in Algorithm 1, the signals are initially aligned having been assigned equal weights. If an optimal (joint) alignment is reached, then the algorithm stops after the first iteration (in this case problem (3) is separable). Otherwise,

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it re-iterates by giving greater importance to signals with lager weights. Typically, no more than 2-3 iterations are needed. Parameter estimation is discussed in the next paragraph.

A. Parameter Estimation

Three data-related quantities are required by the proposed technique, namely, the signal correlation sequences $r_{ij}(\kappa)$, the filter length L and the noise variances σ_i^2.

Firstly, we assume that the noise variances can be estimated from "quiet" intervals of the data. Since the noise processes are assumed jointly uncorrelated, the signal correlation sequences $r_{ij}(\kappa)$ can be estimated from the pairwise correlation sequences of the available noisy data, i.e.:

$$r_{ij}^x(\kappa) = \frac{1}{N}\sum_{n=0}^{N-1} x_i(n)x_j(n+\kappa) \approx r_{ij}(\kappa) + E[w_i(n)w_j(n)],$$

as follows: $\hat{r}_{ij}(\kappa) = r_{ij}^x(\kappa)$, $i \neq j$, $\hat{r}_{ii}(0) = r_{ii}^x(0) - \sigma_i^2$, $1 \leq i,j \leq M$. Finally, since the filter length must be larger that the largest time separation of the signals, an indication for the value of L can be obtained by the maximum of the lags that maximize the pairwise correlation sequences $\hat{r}_{ij}(\kappa)$, namely the quantity $\max_{ij} \max_{\kappa} \hat{r}_{ij}(\kappa)$.

VI. Experimental Results

This section contains the most representative cases of simulation experiments that were conducted, with various signal/noise model combinations. The included results are focused on the signal alignment aspect of the problem, which constitutes the main contribution of the manuscript. The solution to the weighting problem is obtained in a closed form and it is in total agreement with almost all other beamforming techniques.

The synthetic signals (namely $s_i(n)$ in Eq. (1)) were modelled as low-pass filtered white Gaussian noise. The corner frequency of the filter was set at 10Hz, and the sampling frequency at 100Hz. In order to control the degree of similarity among the signals of the dataset (in their pure noiseless form), the M signals were produced by M different filters. Each filter occurred by adding a random perturbation of a controlled magnitude to a common low-pass filter prototype. On the other hand, the noise processes were modelled as first-order AR processes with a pole of magnitude 0.8, in order to test the performance of the proposed method in cases of correlated noise. Finally, in order to construct the synthetic dataset, every signal was delayed by an arbitrary number of samples (within specified limits) and multiplied by a constant gain in order to achieve the desired SNR.

The performance of the proposed method is compared against two other techniques that share the same assumptions with the proposed one (see also Section I). More specifically, the first one (referred to as L_1 in Fig. 1) is the technique proposed in [17] for jointly aligning waveforms of closely spaced seismic events with the purpose of improving phase arrival estimates. The author of [17] estimates the pairwise lags that maximize the respective cross-correlation sequences and seeks for a set of time-delays $\tau_1^*, \ldots, \tau_M^*$ that minimizes the error between the theoretical optimal lag, namely $\tau_i^* - \tau_j^*$ and the observed one, for all possible signal pairs. This leads to an overdetermined system of $M(M-1)/2$ equations with M unknowns. An approximate solution is obtained by minimizing the L_1-norm of the residual vector. The second technique included in our comparisons (referred to as MaxEig in Fig. 1) is the blind beamforming technique proposed in [8]. In this case the desired time-delays are obtained via the maximal eigenvector of a correlation matrix which is identical to the matrix $\mathbf{R}$ defined in (8), with the exception that the diagonal blocks of the matrix used in [8] contain the autocorrelation matrices of the M time series, rather than being empty.

Experiment I

The goal of the first experiment is to asses the sensitivity of the proposed sample-level alignment technique with respect to the quality (i.e., the SNR) of the available dataset. To this end, we used signals with a very high degree of similarity in their pure form (having pairwise correlation coefficients in the neighbourhood of 0.9), by limiting the amount of perturbations introduced to the signal-producing filters. We then tested the performance of the technique against the techniques of [17], [8], under the AR-modelled noise scenario, for several SNR values. For every SNR value 100 synthetic datasets were constructed, each containing 15 signals with arbitrary delays of up to 10 samples from a reference point (pairwise delays of up to 20 samples) and corrupted by noise.

The superior performance of the proposed technique becomes readily apparent from the (empirical) cumulative distribution functions (CDFs) of the delay-estimation error, shown in Fig. 1.(a), The errors are in samples and a point (x, y) on the curve signifies that the probability of having an estimation error of at most x samples, is equal to y. The filter length was set to $L = 25$ for the first step of the proposed technique (i.e. for the solution of the proposed over-relaxed problem), as well as for the technique of [8]. The relaxed problem was solved with a filter length of $L = 6$ (for a correction of up to ± 5 samples), having the solution of the over-relaxed problem as a starting point.

Experiment II

In the second experiment we evaluated the performance of the proposed technique in the presence of outliers (that is, irrelevant signals) and under the scenario of low-similarity datasets, respectively. In the first case, out of the 15 signals contained in every dataset, 7 were outliers, while in the second, we increased the perturbations introduced to the signal-producing filters, leading to signals with pairwise correlation coefficient in the neighbourhood of 0.6.

The parameters used were identical to the first experiment and the results obtained for the AR noise case with SNR= -6 dB are shown in Fig. 1.(b) & (c). Although there is an apparent drop in the performance of all used techniques (see also Fig. 1.(a) in comparison), especially in the case of low-similarity

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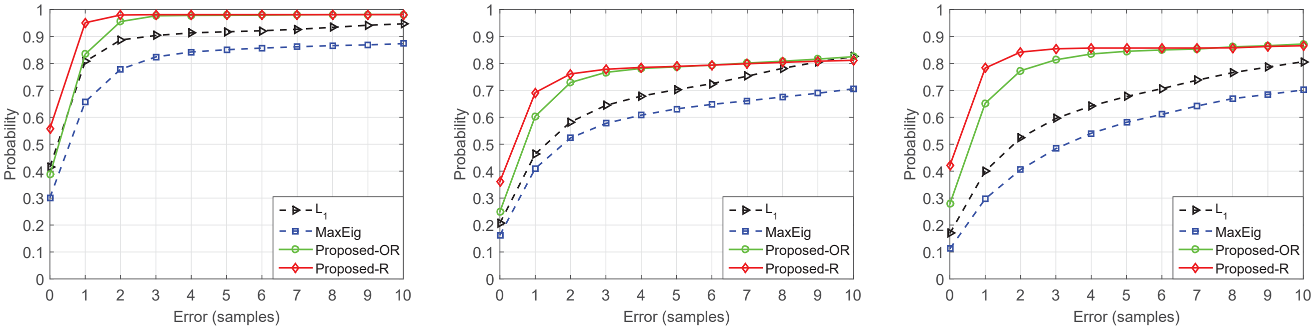

(a) High-similarity signals

(b) Low-similarity signals

(c) Outliers

Fig. 1: Empirical CDFs of the pairwise time-delay estimation errors obtained for the scenarios of (a) high-similarity datasets (b) low-similarity datasets, and (c) presence of outliers. In all shown cases the signals were corrupted by AR-modelled noise with an SNR of -6 dB.

signals, the proposed technique manages to outperform its rivals by a safe margin.

VII. Conclusions

Blind beamforming is formulated as a combination of the signal alignment and signal enhancement problems. The sample-level signal alignment problem is a problem of combinatorial complexity which is relaxed and approximately solved by using the SDP relaxation technique. An efficient over-relaxed version of the problem is also proposed. On the other hand, the signal enhancement problem leads to a quadratic maximization problem which in the vast majority of cases has an exact solution, while in more challenging conditions is solved in an approximate fashion. The main contribution of the manuscript is the use of SDP for the blind solution of the signal alignment problem, which results in very robust and accurate estimations of the pairwise time delays between the signals. This makes it especially suitable for seismological applications that rely in blind time-delay estimation for the solution of enhancement or localization problems. The superiority of the proposed technique was demonstrated through a number of numerical simulations.

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