Stochastic Geometry Modeling of Cellular Networks: A New Definition of Coverage and its Application to Energy Efficiency Optimization

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Abstract— **In this paper, we analyze and optimize the energy efficiency of downlink cellular networks. With the aid of tools from stochastic geometry, we introduce a new closed-form analytical expression of the potential spectral efficiency. Unlike currently available analytical frameworks, the proposed analytical formulation explicitly depends on the transmit power and density of the base stations. This is obtained by generalizing the definition of coverage probability and by taking into account the sensitivity of the receiver not only during the detection of information data, but during the cell association phase as well. Based on the new analytical representation of the potential spectral efficiency, the energy efficiency is formulated in a tractable closed-form expression. The resulting optimization problem is studied and it is mathematically proved that the energy efficiency is a unimodal and strictly pseudo-concave function in the transmit power. Numerical results are illustrated and discussed.**

I. Introduction

The Energy Efficiency (EE) is considered to be a key performance metric towards the optimization of operational cellular networks and the network planning and deployment of emerging communication systems [1]. The EE is defined as a benefit-cost ratio where the benefit is given by the amount of information data per unit time and area that can be reliably transmitted in the network, i.e., the network spectral efficiency, and the cost is represented by the amount of power per unit area that is consumed to operate the network, i.e., the network power consumption. Analyzing and designing a communication network from the EE standpoint necessitate appropriate mathematical tools, which are usually different from those used for optimizing the network spectral efficiency and the network power consumption individually [2].

In the last few years, the system-level modeling and analysis of cellular networks have been facilitated by capitalizing on the mathematical tool of stochastic geometry and, more precisely, on the theory of spatial point processes [3]-[9]. It has been empirically validated that, from the system-level standpoint, the locations of the BSs can be abstracted as points of a homogeneous Poisson Point Process (PPP) whose intensity coincides with the average number of BSs per unit area in the geographical region of interest [10]. A comprehensive survey of recent results in this field of research is available in [11].

A relevant performance metric for the design of cellular networks is the Potential Spectral Efficiency (PSE), which is the network information rate per unit area that corresponds to the minimum signal quality for reliable transmission. Under the PPP modeling assumption, the PSE can be obtained in two steps: i) first by computing the PSE at an arbitrary location of the network and assuming a given spatial realization for the locations of the BSs and ii) then by averaging the conditional PSE with respect to all possible realizations for the locations of the BSs. In the interference-limited regime, this approach allows one to obtain closed-form expressions of the PSE under the (henceforth called) *standard modeling assumptions*, i.e., single-antenna transmission, singular path-loss model, Rayleigh fading, fully-loaded BSs, cell association based on the highest average received power [3].

Motivated by these results, the PPP modeling approach for the locations of the BSs has been widely used for analyzing the trade-off between the network spectral efficiency and the network power consumption, e.g., [12], as well as for minimizing the network power consumption given some constraints on the network spectral efficiency or for maximizing the network spectral efficiency given some constraints on the network power consumption [13]. The PPP modeling approach has been applied for optimizing the EE of cellular networks as well [14]. Currently available approaches for modeling and optimizing the system-level EE of cellular networks are, however, insufficient and/or unsuitable for mathematical analysis. This issue is further elaborated in the next section.

A. Fundamental Limitations of Current Approaches for System-Level EE Optimization

Under the standard modeling assumptions, the authors of [3] have proved that the PSE (expressed in bit/sec/m^2) can be formulated, in the interference-limited regime, as follows:

$$\mathrm{PSE} = \frac{\lambda_{\mathrm{BS}} \mathrm{B_W} \log_2 \left(1+\gamma_D\right)}{{}_2F_1 \left(1, -2/\beta, 1-2/\beta, -\gamma_D\right)} \tag{1}$$

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where λ_{BS} is the density of the BSs, $\mathrm{B_W}$ is the transmission bandwidth, γ_D is the threshold for reliable decoding, $\beta > 2$ is the path-loss exponent, and ${}_2F_1(\cdot,\cdot,\cdot,\cdot)$ is the Gauss hypergeometric function.

Under the standard modeling assumptions, the network power consumption (expressed in Watt/m^2) is equal to $\mathrm{P_{grid}} = \lambda_{\mathrm{BS}}(\mathrm{P_{tx}} + \mathrm{P_{circ}})$, where $\mathrm{P_{tx}}$ is the transmit power of the BSs and $\mathrm{P_{circ}}$ is the static power consumption of the BSs, which accounts for the power consumed in all hardware blocks other than $\mathrm{P_{tx}}$, e.g., analog-to-digital and digital-to-analog converters, analog filters, cooling components, and digital signal processing [1].

The system-level EE (bit/Joule) is defined as the ratio between (1) and the network power consumption, i.e., $\mathrm{EE} = \mathrm{PSE}/\mathrm{P_{grid}}$. Since the PSE in (1) is independent of the transmit power of the BSs, $\mathrm{P_{tx}}$, and the network power consumption, $\mathrm{P_{grid}}$, linearly increases with $\mathrm{P_{tx}}$, we conclude that any EE optimization problems formulated based on (1) would result in the trivial optimal solution consisting of turning all the BSs off (the optimal transmit power is zero).

A system-level EE optimization problem formulated based on (1) would result, in addition, in a physically meaningless utility function, which provides a non-zero benefit-cost ratio at a zero cost, i.e., a strictly positive EE while transmitting zero power, i.e., $\mathrm{EE}(\mathrm{P_{tx}} = 0) = \mathrm{PSE}/(\lambda_{\mathrm{BS}}\mathrm{P_{circ}}) > 0$.

We conclude that a new mathematical formulation of the PSE that explicitly depends on the transmit power and density of the BSs, and that is tractable enough for system-level EE optimization is needed. From an optimization point of view, it is desirable that the PSE is formulated in a closed-form expression. In this paper, we introduce a new definition of PSE that fulfill these requirements.

The remainder of the present paper is organized as follows. In Section II, the system model is presented. In Section III, the new mathematical expression of the PSE is introduced. In Section IV, the system-level EE optimization problem is formulated and studied. In Section V, numerical illustrations are shown in order to validate the mathematical findings and to analyze the impact of various system parameters on the EE. Finally, Section VI concludes this paper.

II. System Model

In this section, the network model is introduced. With the exception of the load model, we focus our attention on a system where the standard modeling assumptions hold. The proposed approach can be readily generalized to more advanced system models such as that recently adopted in [9].

A. Cellular Network Modeling

A downlink cellular network is considered. The BSs are modeled as points of a homogeneous PPP, denoted by Ψ_{BS}, of density λ_{BS}. The MTs are modeled as another homogeneous PPP, denoted by Ψ_{MT}, of density λ_{MT}. Ψ_{BS} and Ψ_{MT} are independent of each other. The BSs and MTs are equipped with a single omnidirectional antenna. Each BS transmits with a constant power denoted by $\mathrm{P_{tx}}$. The mathematical frameworks are developed for the typical MT, denoted by MT_0, that is located at the origin (Slivnyak theorem [15, Th. 1.4.5]). The BS serving MT_0 is denoted by BS_0. The cell association criterion is introduced in Section II-C. Throughout this paper, the subscripts 0, i and n identify the intended link, a generic interfering link, and a generic BS-to-MT link, respectively. The set of interfering BSs is denoted by $\Psi_{\mathrm{BS}}^{(\mathrm{I})}$. As far as the transmission of data is concerned, the cellular network operates in the interference-limited regime, i.e., the noise is negligible compared with the inter-cell interference.

B. Channel Modeling

For each BS-to-MT link, path-loss and fast-fading are considered. Shadowing is not explicitly taken into account because its net effect lies in modifying the density of the BSs [9]. All BS-to-MT links are assumed to be mutually independent and identically distributed.

a) Path-Loss: Consider a generic BS-to-MT link of length r_n. The path-loss is defined as $l(r_n) = \kappa r_n^{\beta}$, where κ and β are the path-loss constant and the path-loss slope.

b) Fast-Fading: Consider a generic BS-to-MT link. The power gain due to small-scale fading is assumed to follow an exponential distribution with mean Ω. Without loss of generality, $\Omega = 1$ is assumed. The power gain of a generic BS-to-MT link is denoted by g_n.

C. Cell Association Criterion

A cell association criterion based on the highest average received power is assumed. Let $\mathrm{BS}_n \in \Psi_{\mathrm{BS}}$ denote a generic BS. The serving BS, BS_0, is obtained as follows:

$$\begin{aligned}\mathrm{BS}_0 &= \arg\max\nolimits_{\mathrm{BS}_n \in \Psi_{\mathrm{BS}}} \{1/l(r_n)\} \\ &= \arg\max\nolimits_{\mathrm{BS}_n \in \Psi_{\mathrm{BS}}} \{1/L_n\}\end{aligned} \tag{2}$$

where the short-hand notation $L_n = l(r_n)$ is used. As for the intended link, $L_0 = \min_{r_n \in \Psi_{\mathrm{BS}}} \{L_n\}$ holds.

D. Load Modeling

Based on (2), several or no MTs can be associated to a generic BS. In the latter case, the BS transmits zero power, i.e., $\mathrm{P_{tx}} = 0$, and, thus, it does not generate inter-cell interference.

Let $\mathrm{N_{MT}}$ denote the number of MTs associated to a generic BS and $\mathrm{B_W}$ denote the transmission bandwidth available to each BS. If $\mathrm{N_{MT}} = 1$, the single MT associated to the BS is scheduled for transmission and the entire bandwidth, $\mathrm{B_W}$, and transmit power, $\mathrm{P_{tx}}$, are assigned to it. If $\mathrm{N_{MT}} > 1$, the BS selects, at each transmission instance, all the $\mathrm{N_{MT}}$ MTs associated to it. The BS equally splits the available transmission bandwidth, $\mathrm{B_W}$, and evenly spreads the available transmit power, $\mathrm{P_{tx}}$, among the $\mathrm{N_{MT}}$ MTs. Thus, the bandwidth and power are viewed as continuous resources by the BS scheduler: Each MT is assigned a bandwidth equal to $\mathrm{B_W}/\mathrm{N_{MT}}$ and the power spectral density at the detector input (i.e., the typical MT, MT_0) is equal to $\mathrm{P_{tx}}/\mathrm{B_W}$.

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$$\begin{aligned}\mathrm{PSE}\left(\gamma_{\mathrm{D}},\gamma_{\mathrm{A}}\right) &= \mathbb{E}_{\bar{\mathrm{N}}_{\mathrm{MT}}}\left\{\mathrm{PSE}\left(\gamma_{\mathrm{D}},\gamma_{\mathrm{A}}\middle|\,\bar{\mathrm{N}}_{\mathrm{MT}}\right)\right\}\\ &= \sum_{u=0}^{+\infty}\lambda_{\mathrm{MT}}\frac{\mathrm{B_W}}{u+1}\log_2\left(1+\gamma_{\mathrm{D}}\right)\Pr\left\{\mathrm{SIR}\geq\gamma_{\mathrm{D}},\overline{\mathrm{SNR}}\geq\gamma_{\mathrm{A}}\right\}\Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}}=u\right\}\\ &= \lambda_{\mathrm{MT}}\mathrm{B_W}\log_2\left(1+\gamma_{\mathrm{D}}\right)\Pr\left\{\mathrm{SIR}\geq\gamma_{\mathrm{D}},\overline{\mathrm{SNR}}\geq\gamma_{\mathrm{A}}\right\}\sum_{u=0}^{+\infty}\frac{\Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}}=u\right\}}{u+1}\end{aligned} \tag{7}$$

E. Power Consumption Modeling

In the considered system model, the BSs can be in two different operating modes: i) they are in idle mode if no MTs are associated to them and ii) they are in transmission mode if at least one MT is associated to them. The widespread linear power consumption model for the BSs is adopted [1], [16], which accounts for the power consumption due to the transmit power, $\mathrm{P_{tx}}$, the static (circuits) power, $\mathrm{P_{circ}}$, and the idle power, $\mathrm{P_{idle}}$. If the BS is in idle mode, its power consumption is equal to $\mathrm{P_{idle}}$. If the BS is in transmission mode, its power consumption is a function of $\mathrm{P_{tx}}$, $\mathrm{P_{circ}}$, and depends on the number of MTs associated to it. The inequalities $0 \leq \mathrm{P_{idle}} \leq \mathrm{P_{circ}}$ are assumed.

III. A New Analytical Formulation of the PSE

In this section, we introduce and motivate a new definition of coverage probability, $\mathrm{P_{cov}}$, which overcomes the limitations of currently available analytical frameworks and is suitable for system-level optimization.

Definition 1: Let γ_{D} and γ_{A} be the reliability thresholds for successfully decoding the information data and for successfully detecting the serving BS, BS_0, respectively. The coverage probability, $\mathrm{P_{cov}}$, of the typical MT, MT_0, is as follows:

$$\mathrm{P_{cov}}\left(\gamma_{\mathrm{D}},\gamma_{\mathrm{A}}\right) = \begin{cases}\Pr\left\{\mathrm{SIR}\geq\gamma_{\mathrm{D}},\overline{\mathrm{SNR}}\geq\gamma_{\mathrm{A}}\right\} & \text{if } \mathrm{MT}_0 \text{ is selected}\\ 0 & \text{if } \mathrm{MT}_0 \text{ is not selected}\end{cases} \tag{3}$$

where the Signal-to-Interference-Ratio (SIR) and the average Signal-to-Noise-Ratio ($\overline{\mathrm{SNR}}$) can be formulated, for the network model under analysis, as follows:

$$\mathrm{SIR} = \frac{\mathrm{P_{tx}}g_0/L_0}{\sum_{\mathrm{BS}_i\in\Psi_{\mathrm{BS}}^{(\mathrm{I})}}\mathrm{P_{tx}}g_i/L_i \quad (L_i > L_0)} \tag{4}$$

$$\overline{\mathrm{SNR}} = \frac{\mathrm{P_{tx}}/L_0}{\sigma_{\mathrm{N}}^2} \tag{5}$$

where N_0 is noise power spectral density, $\sigma_{\mathrm{N}}^2 = \mathrm{B_W}\mathrm{N}_0$ is noise variance, and $(\cdot)$ is the indicator function.

Remark 1: The definition of $\mathrm{P_{cov}}$ in (3) reduces to the conventional definition in [3] if $\gamma_{\mathrm{A}} = 0$. □

Remark 2: The new definition of $\mathrm{P_{cov}}$ in (3) is based on the actual value of L_0 because a necessary condition for the typical MT to be in coverage is that it can detect the pilot signal of at least one BS during the cell association. If the BS that provides the highest average received power cannot be detected, then any other BSs cannot be detected either. The new definition of $\mathrm{P_{cov}}$ is based on the average SNR, i.e., the SNR averaged with respect to the fast fading, because the cell association is performed based on long-term statistics, i.e., the path-loss, to avoid too frequent handovers. □

TABLE I

Main auxiliary functions. $\dot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\cdot)$; $\ddot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\cdot)$ are the first- and second-order derivatives with respect to $\mathrm{P_{tx}}$. $\mathcal{A} = \lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}$, $\Delta\mathrm{P} = \mathrm{P_{circ}} - \mathrm{P_{idle}} \geq 0$, $\eta = \kappa\sigma_{\mathrm{N}}^2\gamma_{\mathrm{A}}$.

Functions Definition
$\mathcal{L}(\mathcal{A}) = 1 - (1+(1/\alpha)\mathcal{A})^{-\alpha}\ ;\ \mathcal{M}(\mathcal{A}) = \mathcal{A} - \mathcal{L}(\mathcal{A})$
$\mathcal{Q}(\mathrm{P_{tx}}) = 1 - \exp\left(-\pi\lambda_{\mathrm{BS}}(\mathrm{P_{tx}}/\eta)^{2/\beta}(1+\Upsilon\mathcal{L}(\mathcal{A}))\right)$
$\dot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\mathrm{P_{tx}}) = \pi\lambda_{\mathrm{BS}}(1/\eta)^{2/\beta}(2/\beta)(1+\Upsilon\mathcal{L}(\mathcal{A}))\mathrm{P_{tx}}^{2/\beta-1} \times \exp\left(-\pi\lambda_{\mathrm{BS}}(\mathrm{P_{tx}}/\eta)^{2/\beta}(1+\Upsilon\mathcal{L}(\mathcal{A}))\right)$
$\ddot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\mathrm{P_{tx}}) = \pi\lambda_{\mathrm{BS}}(1/\eta)^{2/\beta}(2/\beta)(1+\Upsilon\mathcal{L}(\mathcal{A}))\mathrm{P_{tx}}^{2/\beta-1} \times\left[-\dot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\mathrm{P_{tx}})\right] + \pi\lambda_{\mathrm{BS}}(1/\eta)^{2/\beta}(2/\beta)(2/\beta-1) \times(1+\Upsilon\mathcal{L}(\mathcal{A}))\mathrm{P_{tx}}^{2/\beta-2}\exp\left(-\pi\lambda_{\mathrm{BS}}(\mathrm{P_{tx}}/\eta)^{2/\beta}(1+\Upsilon\mathcal{L}(\mathcal{A}))\right)$
$\mathcal{S}_{\mathcal{P}}(\mathrm{P_{tx}}) = \mathcal{L}(\mathcal{A})\left[\frac{\mathcal{Q}(\mathrm{P_{tx}})}{\dot{\mathcal{Q}}_{\mathrm{P_{tx}}}(\mathrm{P_{tx}})} - (\mathrm{P_{tx}}+\Delta\mathrm{P})\right] - \mathrm{P_{circ}}\mathcal{M}(\mathcal{A})$

In the sequel, due to space limitations, lemmas, propositions, and theorem are reported without proof. Further details can be found in the extended version of the present work [19].

A. Analytical Formulation of the PSE

Let $\bar{\mathrm{N}}_{\mathrm{MT}}$ be the number of MTs that lie in the cell of the typical MT, MT_0, with the exception of MT_0. $\bar{\mathrm{N}}_{\mathrm{MT}}$ is a discrete random variable whose probability mass function can be formulated, in closed-form, as follows [18, Eq. (3)]:

$$\begin{aligned}f_{\bar{\mathrm{N}}_{\mathrm{MT}}}(u) &= \Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}}=u\right\}\\ &= \frac{3.5^{4.5}\Gamma(u+4.5)\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)^u}{\Gamma(4.5)\,\Gamma(u+1)\left(3.5+\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)^{u+4.5}}\end{aligned} \tag{6}$$

Lemma 1: The PSE (bit/sec/m^2) can be formulated as shown in (7) at the top of this page.

Let N_{MT} be the number of MTs that lie in a cell. The probability that the BS is in idle mode, $\mathbb{P}_{\mathrm{BS}}^{(\mathrm{idle})}$, and in transmission mode, $\mathbb{P}_{\mathrm{BS}}^{(\mathrm{tx})}$, can be formulated as follows [18, Prop. 1]:

$$\begin{aligned}\mathbb{P}_{\mathrm{BS}}^{(\mathrm{idle})} &= \Pr\left\{\mathrm{N}_{\mathrm{MT}}=0\right\} = 1-\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\\ \mathbb{P}_{\mathrm{BS}}^{(\mathrm{tx})} &= \Pr\left\{\mathrm{N}_{\mathrm{MT}}\geq 1\right\} = 1-\mathbb{P}_{\mathrm{BS}}^{(\mathrm{idle})} = \mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\end{aligned} \tag{8}$$

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where $\mathcal{L}(\cdot)$ is defined in Table I.

Proposition 1: The PSE (bit/sec/m^2) is as follows:

$$\mathrm{PSE} = \mathrm{B_W}\log_2\left(1+\gamma_{\mathrm{D}}\right)\frac{\lambda_{\mathrm{BS}}\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)}{1+\Upsilon\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)} \times \mathcal{Q}\left(\lambda_{\mathrm{BS}},\mathrm{P_{tx}},\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right) \quad (9)$$

where $\mathcal{Q}(\cdot,\cdot,\cdot)$ is defined in Table I and $\Upsilon = {}_2F_1\left(1,-2/\beta,1-2/\beta,-\gamma_{\mathrm{D}}\right)-1$.

Proposition 2: $\mathrm{P_{grid}}$ (Watt/m^2) is as follows:

$$\mathrm{P_{grid}} = \lambda_{\mathrm{BS}}\mathrm{P_{tx}}\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right) + \lambda_{\mathrm{MT}}\mathrm{P_{circ}} + \lambda_{\mathrm{BS}}\mathrm{P_{idle}}\left(1-\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\right) \quad (10)$$

IV. System-Level EE Optimization

The EE (bit/Joule) can be formulated as follows:

$$\begin{aligned}\mathrm{EE}\left(\mathrm{P_{tx}}\right) &= \frac{\mathrm{PSE}}{\mathrm{P_{grid}}} = \mathrm{B_W}\log_2\left(1+\gamma_{\mathrm{D}}\right)\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right) \\ &\times \mathcal{Q}\left(\lambda_{\mathrm{BS}},\mathrm{P_{tx}},\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\left[1+\Upsilon\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\right]^{-1} \\ &\times \left[\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\left(\mathrm{P_{tx}}+\mathrm{P_{circ}}-\mathrm{P_{idle}}\right)+\mathrm{P_{idle}} \right. \\ &\left. +\mathcal{M}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\mathrm{P_{circ}}\right]^{-1} \quad (11)\end{aligned}$$

where all the auxiliary functions are defined in Table I.

In the following, the optimal transmit power that maximizes the EE is studied. For this reason, $\mathrm{P_{tx}}$ is explicitly highlighted in (11). All the other parameters are kept fixed. A similar study can be conducted for the density of the BSs, λ_{BS} [19]. Due to space limitations, this study is not reported in this paper.

In mathematical terms, the optimization problem as a function of $\mathrm{P_{tx}}$ can be formulated as follows:

$$\max_{\mathrm{P_{tx}}} \mathrm{EE}\left(\mathrm{P_{tx}}\right) \quad \text{subject to } \mathrm{P_{tx}} \in \left[\mathrm{P_{tx}^{(min)}},\mathrm{P_{tx}^{(max)}}\right] \quad (12)$$

where $\mathrm{P_{tx}^{(min)}} \geq 0$ and $\mathrm{P_{tx}^{(max)}} \geq 0$ are the minimum and maximum power budget of the BSs, respectively. Without loss of generality, $\mathrm{P_{tx}^{(min)}} \to 0$ and $\mathrm{P_{tx}^{(max)}} \to \infty$.

Theorem 1: Let $\mathcal{S_P}(\cdot)$ be the function defined in Table I. The EE in (11) is a unimodal and strictly pseudo-concave function in $\mathrm{P_{tx}}$. The optimization problem in (12) has a unique solution given by $\mathrm{P_{tx}^{(opt)}} = \max\left\{\mathrm{P_{tx}^{(min)}},\min\left\{\mathrm{P_{tx}^{*}},\mathrm{P_{tx}^{(max)}}\right\}\right\}$, where $\mathrm{P_{tx}^{*}}$ is the only stationary point of the EE in (11) that is obtained as the unique solution of the following equation:

$$\dot{\mathrm{EE}}_{\mathrm{P_{tx}}}\left(\mathrm{P_{tx}^{*}}\right) = \mathrm{P_{idle}} - \mathcal{S_P}\left(\mathrm{P_{tx}^{*}}\right) = 0 \Leftrightarrow \mathcal{S_P}\left(\mathrm{P_{tx}^{*}}\right) = \mathrm{P_{idle}} \quad (13)$$

where $\dot{\mathrm{EE}}_{\mathrm{P_{tx}}}(\cdot)$ is the first-order derivative of EE with respect to $\mathrm{P_{tx}}$.

V. Numerical Results

In this section, we show numerical results in order to validate the proposed analytical framework for computing the PSE and EE, as well as to substantiate the findings originating from the analysis of the system-level EE optimization problem as a function of the transmit power. The simulation setup is reported in Table II. For ease of understanding, the density of the BSs is represented via the inter-site distance ($\mathrm{R_{cell}}$) defined in Table II. A similar comment applies to the density of the MTs that is expressed in terms of their average distance ($\mathrm{R_{MT}}$). The power consumption model agrees with

TABLE II
Setup of parameters (unless otherwise stated).

Parameter	Value
β	3.5
$\kappa = \left(4\pi f_c/3\cdot 10^8\right)^2$	f_c = 2.1 GHz
$\mathrm{N_0}$	-174 dBm/Hz
$\mathrm{B_W}$	20 MHz
$\mathrm{P_{circ}}$	51.14 dBm [12]
$\mathrm{P_{idle}}$	48.75 dBm [12]
$\mathrm{P_{tx}}$	43 dBm [12]
$\lambda_{\mathrm{BS}} = 1/\left(\pi\mathrm{R_{cell}^2}\right)$ BSs/m^2	$\mathrm{R_{cell}}$ = 250 m
$\lambda_{\mathrm{MT}} = 1/\left(\pi\mathrm{R_{MT}^2}\right)$ = 121 MTs/km^2	$\mathrm{R_{MT}}$ = 51.29 m
$\gamma_{\mathrm{D}} = \gamma_{\mathrm{A}}$	5 dB

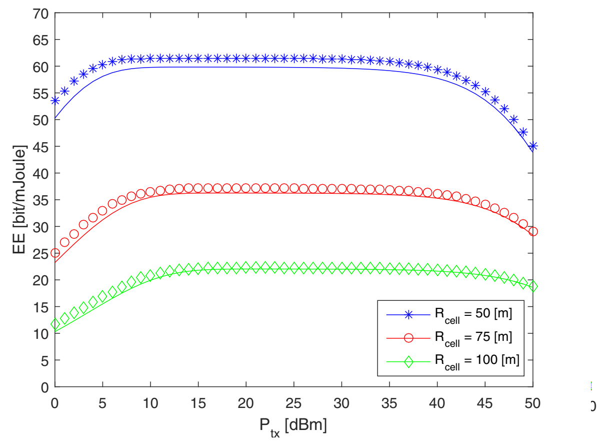

Fig. 1. Energy efficiency versus the transmit power. Solid lines: Framework from (11). Markers: Monte Carlo simulations.

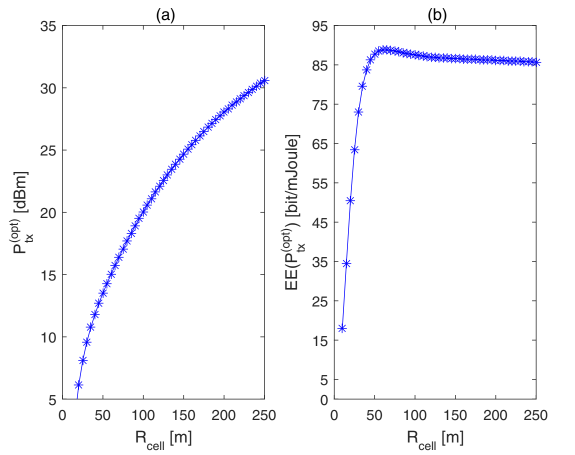

Fig. 2. Optimal transmit power (a) and energy efficiency (b) versus $\mathrm{R_{cell}}$. Solid lines: Optimum from *Theorem 1*. Markers: Optimum from a brute-force search of (12).

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[12], [16].

a) Validation against Monte Carlo Simulations: In Fig. 1, we validate the correctness of (11) against Monte Carlo simulations. Monte Carlo results are obtained by simulating several realizations, according to the PPP model, of the cellular network and by empirically computing the PSE according to its definition in (7), as well as the power consumption based on the operating principle described in *Proposition 2*. It is worth mentioning that, to estimate the PSE, only the definition in the first line of (7) is used. The results depicted in Fig. 1 confirms the good accuracy of the proposed mathematical approach. The small gap between the analytical and simulation results originates from the difficulty of simulating a large number of network realizations. The curves highlight, in addition, the unimodal and pseudo-concave shape of the EE as a function of the transmit power.

b) Validation of Theorem 1: In Fig. 2, we compare the optimal transmit power obtained from *Theorem 1*, i.e., by computing the unique zero of (13), against a brute-force search of the optimum of (12). We observe the correctness of *Theorem 1*. Figure 2, in addition, brings to our attention that an optimal density of the BSs exists if the cellular network is optimized to maximize the EE as a function of the transmit power. This implies that the transmit power and density of BSs can be jointly optimized for enhancing the EE.

VI. Conclusion

In this paper, we have introduced a new closed-form analytical expression of the coverage probability and potential spectral efficiency of cellular networks. The proposed approach is conveniently formulated for optimizing the network planning of cellular networks, by taking into account important system parameters. We have applied the new approach to the analysis and optimization of the energy efficiency of cellular networks. We have mathematically proved that the proposed closed-form expression of the energy efficiency is a unimodal and strictly pseudo-concave function in the transmit power. All analytical derivations and findings have been substantiated with the aid of numerical simulations. We argue that the potential applications of the proposed approach to the system-level modeling and optimization of cellular networks are countless and go beyond the formulation of energy efficiency problems. An extended version of the present work can be found in [19].

Acknowledgment

This work was supported in part by the European Commission through the H2020-MSCA ETN-5Gwireless project under Grant Agreement 641985, the H2020-MSCA IF-BESMART project under Grant Agreement 749336, and the H2020-ERC PoC-CacheMire project under Grant Agreement 727682.

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ISBN 978-90-827970-1-5 © EURASIP 2018