

Sequential Spatio-temporal Symbol-level Precoding enabling Faster-than-Nyquist Signaling for Multi-user MISO Systems

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Abstract—This paper addresses the problem of the interference between multiple co-channel transmissions in the downlink of a multi-antenna wireless system. In this context, symbol-level precoding achieves a constructive interference effect which results in SINR gains at the receivers side. Usually the constructive interference is exploited in the spatial dimension (multi-user interference), however in this work we consider a spatio-temporal precoding model which allows to exploit the interference also in the temporal dimension (inter-symbol interference). The proposed method, which optimizes the oversampled transmit waveforms by minimizing the per-antenna transmit power, allows faster-than-Nyquist signaling over multi-user MISO systems without imposing additional complexity at the user terminals. The optimization is performed in a sequential fashion, by splitting the data streams in blocks and handling the inter-block interference. Numerical results are presented to assess the gains of the scheme in terms of effective rate and energy efficiency.

I. INTRODUCTION

Precoding has been a prolific research area in the last two decades, due to the promise of breaking the capacity gridlock of many interference-limited systems. The precoding performance gains arise from the combination of aggressive frequency reuse and suitable interference management strategies in the context of multi-user MISO systems.

The conventional precoding strategies aim at mitigating the multi-user interference (MUI) by exploiting the knowledge of the channel state information (CSI), through the design of a precoding weight matrix (or precoder) to be applied to the multiple data streams [1], [2]. More recently, several works have proposed a different precoding rationale, where the main objective is not to eliminate the MUI, but rather to control it so as to achieve a constructive interference effect at each receiver [3]–[10]. This novel strategy is referred to as symbol-level precoding (SLP), since the knowledge of the data information (data symbols) is used together with the CSI to constructively exploit the interference. Different optimization strategies have been considered in the literature for SLP. In [5] the sum power minimization and the *max-min fair* problem were solved for PSK modulations. Extensions of these schemes include optimization strategies for multi-level modulations [6], more flexible approaches for exploiting the constructive interference [7], and SLP strategies for non-linear channels [9], [10]. A more detailed review of SLP can be found in [11].

It should be highlighted that the SLP schemes mentioned above work on a symbol-by-symbol basis, as they optimize the transmitted signals separately for each symbol slot. As a consequence, in the optimization of the transmit signals they do not take into account the temporal dimension of the waveforms. In this direction, the authors have proposed in [12]–[14] a new SLP model, referred to as spatio-temporal SLP. This new model introduces the temporal dimension in the design of the transmit signals, by jointly optimizing temporal blocks of symbols for each transmit antenna and therefore stretching the potential of SLP. In particular, a spatio-temporal SLP formulation has been used in [12] for minimizing the peak-to-average power ratio (PAPR) of the waveforms, while [13], [14] propose spatio-temporal SLP methods which enable faster-than-Nyquist (FTN) signaling over multi-user MISO systems. The key idea of FTN signaling is to increase the data rate by accelerating the transmitted pulses in the temporal dimension (time packing), thus introducing controlled inter-symbol interference (ISI). The FTN concept was introduced in the mid 70s by Mazo in [15] for binary sinc pulses, and it has been widely investigated more recently, considering squared root raised cosine (SRRC) pulses [16] and also extensions in the frequency domain [17], [18]. A review of the work on FTN signaling can be found in [19]. The main problem of FTN signaling is the need to cope with the introduced ISI, which in turn results in complex receivers relying on trellis decoders as well as ad hoc equalization schemes, which are often prohibitive in practical applications. In [13], [14], spatio-temporal SLP is adopted to constructively handle at the transmitter side not only the MUI but also the ISI which arises within the transmit waveforms when FTN is applied.

In this work we address the problem of spatio-temporal SLP enabling FTN signaling. Differently from [13], [14], where sum power minimization schemes are proposed, we optimize the transmit waveforms (accounting for oversampling) by minimizing the average per-antenna transmit power under Quality-of-Service (QoS) constraints. This is particularly important for systems having individual per-antenna amplifiers, which have a lack of flexibility in sharing the energy resources amongst the multiple transmitting antennas. In this respect, the proposed approach can be seen as a spatio-temporal extension of the scheme in [9]. The proposed optimization is performed in a

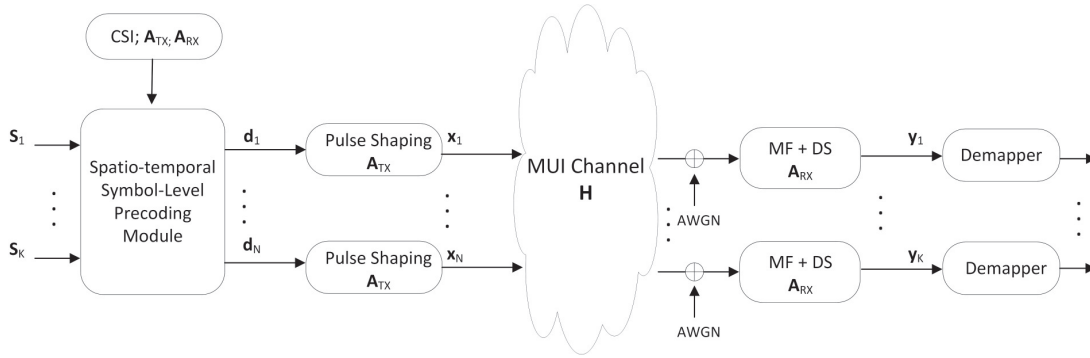


Figure 1: Block scheme of the considered system model relying on spatio-temporal symbol-level precoding.

sequential fashion, by splitting the data streams in blocks and handling the resulting inter-block interference.

Notation: We use upper-case and lower-case bold-faced letters to denote matrices and vectors, respectively. $(\cdot)^T$ denotes the transpose of (\cdot) , while $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ are the real and imaginary parts of (\cdot) , respectively. $\|\cdot\|$ represents the Euclidean norm of a vector, while \otimes is used to denote the Kronecker product. Finally, $\text{vec}(\cdot)$ denotes the columnwise vectorization of a matrix, while $\mathbf{0}_{a \times b}$ and \mathbf{I}_a denote the matrix of all ones of size $a \times b$ and the identity matrix of size $a \times a$, respectively.

II. SYSTEM AND SIGNALS COMMUNICATION MODEL

In this section we introduce a spatio-temporal model for SLP and we highlight how it can be used to enable FTN signaling over multi-user MISO systems, by handling the ISI and the MUI at the transmitter side. We focus on a single-cell multiple-antenna downlink scenario, where a base-station delivers K independent data streams to K single-antenna user terminals through N transmit antennas, with $N \geq K$. The channel is assumed to be quasi-static flat fading, while each data stream is splitted in blocks of S symbols. The symbols to be conveyed to the different users, for one data block, can be aggregated in a data information matrix $\mathbf{S} = [s_1 \dots s_K]^T \in \mathbb{C}^{K \times S}$. Analogously, we aggregate in the matrix $\mathbf{D} = [d_1 \dots d_N]^T \in \mathbb{C}^{N \times S}$ the precoded symbol streams to be transmitted. Each symbol stream undergoes pulse shaping before transmission, which is performed using a unit energy symmetric pulse waveform $\alpha(t)$. Denoting by T the symbol period and by μ the oversampling factor, the transmitted waveform for the generic n -th antenna can be represented through its discrete samples spaced by $t_s = \frac{T}{\mu}$, as follows:

$$x_n[m] = \sum_{j=1}^S d_n[j] \alpha[(m-1)t_s - (j-1)T], \quad m = 1, \dots, \mu S, \quad (1)$$

where $d_n[j]$ is the j -th element of the symbol vector \mathbf{d}_n , with \mathbf{d}_n^T being in turn the n -th row of \mathbf{D} . The output (oversampled) signals from all the antennas can be aggregated in a matrix $\mathbf{X} = [x_1 \dots x_N]^T \in \mathbb{C}^{N \times \mu S}$. This way, the pulse shaping

operation can be represented in a compact matrix form as $\mathbf{X} = \mathbf{D}\mathbf{A}_{\text{TX}}$, with $\mathbf{A}_{\text{TX}} \in \mathbb{R}^{S \times \mu S}$ being a block Toeplitz matrix having as (j, m) -th element:

$$[\mathbf{A}_{\text{TX}}]_{(j,m)} = \alpha[(m-1)t_s - (j-1)T]. \quad (2)$$

The received symbols at the K users for the considered data block can be grouped in a matrix $\mathbf{Y} = [y_1 \dots y_K]^T \in \mathbb{C}^{K \times S}$. Based on the well-known multi-user MISO channel model, the overall communication model can be written as:

$$\mathbf{Y} = \mathbf{H}\mathbf{X}\mathbf{A}_{\text{RX}} + \tilde{\mathbf{Z}}\mathbf{A}_{\text{RX}} = \mathbf{H}\mathbf{D}\mathbf{A} + \mathbf{Z}, \quad (3)$$

where $\mathbf{H} = [\mathbf{h}_1^T \dots \mathbf{h}_K^T]^T \in \mathbb{C}^{K \times N}$ is the channel matrix modeling the interference among the different data streams, $\tilde{\mathbf{Z}} = [\tilde{z}_1 \dots \tilde{z}_K]^T \in \mathbb{C}^{K \times \mu S}$ models the Additive White Gaussian Noise (AWGN) in the oversampled domain, and $\mathbf{A}_{\text{RX}} \in \mathbb{R}^{\mu S \times S}$ is a block Toeplitz matrix modeling the matched filtering and downsampling operation performed at each receiver, which can be defined in the same fashion of (2). Further, $\mathbf{A} = \mathbf{A}_{\text{TX}}\mathbf{A}_{\text{RX}} \in \mathbb{R}^{S \times S}$ represents the combination of the filters at the transmitter and at the receiver, while $\mathbf{Z} = \tilde{\mathbf{Z}}\mathbf{A}_{\text{RX}} \in \mathbb{C}^{K \times S}$ is the noise in the symbol domain. Without loss of generality, the noise power is assumed to be 1. The complete system model is represented in the block scheme of Fig. 1, where it is clear how the symbol matrix \mathbf{D} is obtained as the output of a spatio-temporal precoding module, which takes as input the CSI, i.e. an estimate of \mathbf{H} , the filters matrices \mathbf{A}_{TX} and \mathbf{A}_{RX} and the data information matrix \mathbf{S} . Differently than in previous SLP works [6], [9], the model in (3) represents the signals not only in the spatial dimension (i.e., how they vary across the antennas), but also in the temporal dimension, considering a whole block of S symbols per stream and the oversampled transmitted waveforms through \mathbf{X} . In particular, the introduced model takes into account the interference both in the spatial dimension (the MUI), through the spatial channel matrix \mathbf{H} , and in the temporal dimension (the ISI), through the temporal channel matrix \mathbf{A} . In order to facilitate the formulation of the proposed optimization scheme, discussed in the next section, it is convenient to further manipulate the model of (3) by vectorizing the introduced signal matrices over the temporal dimension (rows first). Hence, we model the

data information streams through the vector $\mathbf{s} = \text{vec}(\mathbf{S}^T) = [s_1^T \dots s_K^T]^T \in \mathbb{C}^{KS \times 1}$, the designed symbol streams through $\mathbf{d} = \text{vec}(\mathbf{D}^T) = [d_1^T \dots d_K^T]^T \in \mathbb{C}^{NS \times 1}$, the transmitted signals through $\mathbf{x} = \text{vec}(\mathbf{X}^T) = [x_1^T \dots x_N^T]^T \in \mathbb{C}^{N\mu S \times 1}$, the noise through $\mathbf{z} = \text{vec}(\mathbf{Z}^T) = [z_1^T \dots z_K^T]^T \in \mathbb{C}^{KS \times 1}$, and the received symbols through $\mathbf{y} = \text{vec}(\mathbf{Y}^T) = [y_1^T \dots y_K^T]^T \in \mathbb{C}^{KS \times 1}$. Accordingly, the communication model can be formalized as:

$$\mathbf{y} = (\mathbf{H} \otimes \mathbf{A}^T)\mathbf{d} + \mathbf{z} = \mathbf{G}\mathbf{d} + \mathbf{z}. \quad (4)$$

This final formulation represents the introduced spatio-temporal system model in a very simple way, formally similar to the spatial model used in the previous SLP literature. The matrix $\mathbf{G} = \mathbf{H} \otimes \mathbf{A}^T \in \mathbb{C}^{KS \times NS}$ is an equivalent representation of the channel matrix in this novel spatio-temporal model, therefore it will be referred to as spatio-temporal channel matrix.

As already mentioned, FTN signaling manages to pack more information in the time domain by reducing the symbol period T below the minimum allowed by the Nyquist criterion, thus introducing controlled ISI. So far, we have not made any assumptions on the symbol-rate. It can be easily seen that if we do not apply FTN, then the \mathbf{A} simply reduces to a scaled identity matrix. In this case there is no ISI and the model in (3) boils down to the classic multi-user MISO case. Now, let us assume that we apply an acceleration factor $\tau \in [0, 1]$, so that the effective symbol period is $T = \tau T_{\text{ny}}$, with T_{ny} indicating the minimum symbol period allowed by the Nyquist criterion. In this case the temporal channel matrix \mathbf{A} is not anymore a scaled identity, but a symmetric Toeplitz matrix modeling the introduced ISI within the considered data block. It can be easily seen that the lower is the acceleration factor τ (i.e., the more we accelerate the transmissions) the larger is the number of non-zero values in the matrix \mathbf{A} , thus the higher is the ISI level in the system.

The model in (3)-(4) accounts for the ISI within the considered data block of S symbols. However, in a practical system several temporal data blocks need to be processed sequentially¹. As a consequence, the inter-block ISI between two adjacent blocks has to be modeled. In particular, if we denote the current block under processing by an index l , we need to model the residual ISI coming from the previous $(l-1)$ -th block, as well as the ISI that the current l -th block is causing to the $(l-1)$ -th one. This inter-block interference can be taken into account by extending the communication model in (4) as follows:

$$\begin{bmatrix} \mathbf{y}^{l-1} \\ \mathbf{y}^l \end{bmatrix} = \begin{bmatrix} \mathbf{G} & \mathbf{G}_U \\ \mathbf{G}_P & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{d}^{l-1} \\ \mathbf{d}^l \end{bmatrix} + \begin{bmatrix} \mathbf{z}^{l-1} \\ \mathbf{z}^l \end{bmatrix}, \quad (5)$$

where $\mathbf{G}_P = \mathbf{H} \otimes \mathbf{A}_P^T \in \mathbb{C}^{KS \times NS}$ and $\mathbf{G}_U = \mathbf{H} \otimes \mathbf{A}_U^T \in \mathbb{C}^{KS \times NS}$ respectively, and the matrices $\mathbf{A}_P \in \mathbb{R}^{S \times S}$ and $\mathbf{A}_U \in \mathbb{R}^{S \times S}$ model the ISI coming from the previous block

¹The precoding scheme described in Section III cannot handle any arbitrary block length S , thus a sequential processing of subsequent blocks is required.

and the ISI caused to the previous block, respectively. \mathbf{A}_P and \mathbf{A}_U are (non-symmetric) Toeplitz matrices that can be straightforwardly obtained from \mathbf{A} .

III. SEQUENTIAL FTN SLP FOR PER-ANTENNA POWER MINIMIZATION

In this section a novel SLP scheme accounting for FTN signaling is presented, which exploits in a constructive fashion [5] the interference both in the spatial and in the temporal domain. Differently from [13], [14], we propose herein a per-antenna power minimization scheme with QoS constraints. The proposed formulation assumes a QAM modulation scheme for the data symbols², and the QoS constraints are given in the form of per-user target signal-to-interference-plus-noise ratio (SINR) values. The scheme targets a constructive interference effect at each receiver and for each symbol slot. Moreover, the inter-block ISI is taken into account based on (5). Specifically, assuming that blocks are serially processed, the ISI caused by the $(l-1)$ -th block to the l -th one is represented by the vector $\mathbf{v} = \mathbf{G}_P \mathbf{d}^{l-1} \in \mathbb{C}^{KS \times 1}$, which is known and is utilized in the optimization scheme³ designing \mathbf{d}^l . Analogously to the other introduced vectorized quantities, \mathbf{v} can also be decomposed by indexing the components related to the different users, i.e., $\mathbf{v} = [\mathbf{v}_1^T \dots \mathbf{v}_K^T]^T$. Similarly to [9], the per-antenna power minimization is pursued by minimizing the maximum transmit power among the antennas, averaged over the block duration. Taking also into account that the average transmit power for the n -th antenna is given by $\frac{\|\mathbf{x}_n\|^2}{\mu S}$, the optimization problem can be written as:

$$\begin{aligned} \mathbf{d}^l &= \arg \min_{\mathbf{d}} \max_n \|\mathbf{x}_n\|^2 \\ &\text{subject to} \\ \text{C1: } \quad &\text{Re}(\mathbf{g}_{ki} \mathbf{d} + \mathbf{v}_k[i]) \gtrless \sqrt{\gamma_k} \text{Re}(s_k[i]), \quad k = 1, \dots, K, \\ &\quad \quad \quad \quad \quad i = 1, \dots, S, \\ \text{C2: } \quad &\text{Im}(\mathbf{g}_{ki} \mathbf{d} + \mathbf{v}_k[i]) \gtrless \sqrt{\gamma_k} \text{Im}(s_k[i]), \quad k = 1, \dots, K, \\ &\quad \quad \quad \quad \quad i = 1, \dots, S, \end{aligned} \quad (6)$$

where \mathbf{g}_{ki} denotes the spatio-temporal channel related to the k -th user for the i -th symbol slot, thus it is the $[(k-1)S+i]$ -th row of \mathbf{G} , and $s_k[i]$ is the i -th element of s_k . Further, γ_k is the target SINR that should be granted for the k -th user. The notation \gtrless denotes a generalized inequality, which shall be read as $>$, $<$ or $=$ depending on the position of the data $s_k[i]$ within the QAM constellation and, accordingly, on its detection region. It should be stressed that the quantity $\mathbf{g}_{ki} \mathbf{d} + \mathbf{v}_k[i]$ represents the noise-free received symbol at the k -th user terminal in the i -th symbol slot, and that the imposed constraints (decoupled along the in-phase and quadrature

²However, it can be straightforwardly tailored to APSK constellations as in [12].

³On the other hand, in the proposed optimization we neglect the ISI induced by the l -th block on the previous one, which is represented by $\mathbf{G}_U \mathbf{d}^l$. Although this choice allows some residual inter-block ISI, it guarantees more degrees of freedom to the optimization problem (see [14] for a detailed analysis of this aspect for the sum power minimization problem).

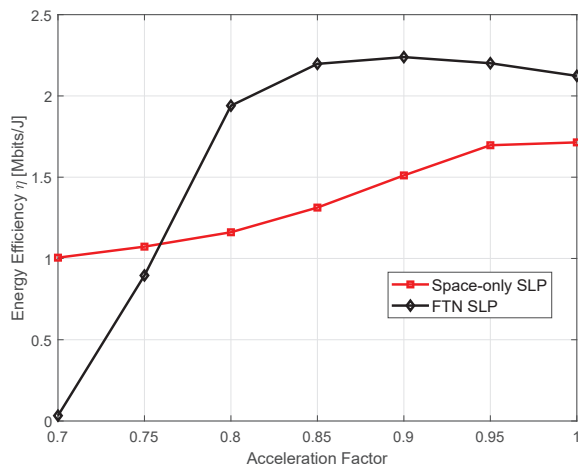


Figure 3: Attained energy efficiency η , in Mbits/J, versus acceleration factor.

captures the achieved effective rate and the peak per-antenna transmit power. The related result is shown in Fig. 3 as a function of τ . It can be seen how the proposed approach outperforms the benchmark for $\tau \geq 0.8$, while for lower values of τ (system more accelerated) the required transmit power P_{\max} becomes too high and η becomes considerably lower. As a final remark, it is interesting to note how the proposed approach outperforms the benchmark even for $\tau = 1$ in terms of energy efficiency. We can conclude that, even when FTN signaling is not applied, the spatio-temporal processing herein proposed is more effective in minimizing the per-antenna transmit power than the scheme of [9], which optimizes the signals separately for each symbol slot.

V. CONCLUSIONS

In this work, a novel SLP strategy has been presented, which constructively handles, at the transmitter side, both the MUI and the ISI. This new precoding method, referred to as spatio-temporal symbol-level precoding, enables faster-than-Nyquist signaling over multi-user MISO systems without imposing additional complexity at the user terminals. In this context, the proposed optimization scheme minimizes the per-antenna transmit power accounting for the oversampled signals, guaranteeing some predefined SINR targets at the users. The proposed optimization is performed in a sequential fashion, by serially processing subsequent data blocks and taking into account the resulting inter-block ISI. Numerical results have been presented to show the effectiveness of the proposed scheme with respect to a space-only SLP benchmark, in terms of improved effective rate and energy efficiency.

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REFERENCES

- [1] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*. CRC Press, 2001.
- [2] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2646–2660, June 2007.
- [3] C. Masouros and E. Alsusa, "Dynamic linear precoding for the exploitation of known interference in MIMO broadcast systems," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1396–1404, March 2009.
- [4] C. Masouros, "Correlation rotation linear precoding for MIMO broadcast communications," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 252–262, Jan. 2011.
- [5] M. Alodeh, S. Chatzinotas, and B. Ottersten, "Constructive multiuser interference in symbol level precoding for the MISO downlink channel," *IEEE Transactions on Signal Processing*, vol. 63, no. 9, pp. 2239–2252, May 2015.
- [6] M. Alodeh, S. Chatzinotas, and B. Ottersten, "Symbol-level multiuser MISO precoding for multi-level adaptive modulation," *IEEE Transactions on Wireless Communications*, vol. 16, no. 8, pp. 5511–5524, Aug 2017.
- [7] M. Alodeh, S. Chatzinotas, and B. Ottersten, "Energy-efficient symbol-level precoding in multiuser MISO based on relaxed detection region," *IEEE Transactions on Wireless Communications*, vol. 15, no. 5, pp. 3755–3767, May 2016.
- [8] C. Masouros and G. Zheng, "Exploiting known interference as green signal power for downlink beamforming optimization," *IEEE Transactions on Signal Processing*, vol. 63, no. 14, pp. 3628–3640, July 2015.
- [9] D. Spano, M. Alodeh, S. Chatzinotas, and B. Ottersten, "Per-antenna power minimization in symbol-level precoding," in *2016 IEEE Global Communications Conference (GLOBECOM)*, Dec 2016, pp. 1–6.
- [10] D. Spano, M. Alodeh, S. Chatzinotas, and B. Ottersten, "Symbol-level precoding for the nonlinear multiuser MISO downlink channel," *IEEE Transactions on Signal Processing*, vol. 66, no. 5, pp. 1331–1345, March 2018.
- [11] M. Alodeh, D. Spano, A. Kalantari, C. Tsinos, D. Christopoulos, S. Chatzinotas, and B. Ottersten, "Symbol-level and multicast precoding for multiuser multiantenna downlink: A state-of-the-art, classification and challenges," *IEEE Communications Surveys & Tutorials*, 2018. [Online]. Available: <https://arxiv.org/pdf/1703.03617.pdf>
- [12] D. Spano, M. Alodeh, S. Chatzinotas, J. Krause, and B. Ottersten, "PAPR minimization through spatio-temporal symbol-level precoding for the non-linear multi-user MISO channel," in *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, April 2018, to appear.
- [13] M. Alodeh, D. Spano, S. Chatzinotas, and B. Ottersten, "Faster-than-Nyquist spatiotemporal symbol-level precoding in the downlink of multiuser MISO channels," in *2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, March 2017.
- [14] D. Spano, M. Alodeh, S. Chatzinotas, and B. Ottersten, "Faster-than-Nyquist signaling through spatio-temporal symbol-level precoding for the multiuser MISO downlink channel," *IEEE Transactions on Wireless Communications*, 2018, submitted. [Online]. Available: <http://hdl.handle.net/10993/35040>
- [15] J. E. Mazo, "Faster-than-Nyquist signaling," *The Bell System Technical Journal*, vol. 54, no. 8, pp. 1451–1462, Oct 1975.
- [16] A. D. Liveris and C. N. Georghiades, "Exploiting faster-than-Nyquist signaling," *IEEE Transactions on Communications*, vol. 51, no. 9, pp. 1502–1511, Sept 2003.
- [17] F. Rusek and J. B. Anderson, "The two dimensional Mazo limit," in *Proceedings. International Symposium on Information Theory, 2005. ISIT 2005.*, Sept 2005, pp. 970–974.
- [18] A. Modenini, F. Rusek, and G. Colavolpe, "Faster-than-Nyquist signaling for next generation communication architectures," in *2014 22nd European Signal Processing Conference (EUSIPCO)*, Sept 2014, pp. 1856–1860.
- [19] J. B. Anderson, F. Rusek, and V. Öwall, "Faster-than-Nyquist signaling," *Proceedings of the IEEE*, vol. 101, no. 8, pp. 1817–1830, Aug 2013.
- [20] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge Univ. Press, 2004.