Compressive Sensing of Temporally Correlated Sources Using Isotropic Multivariate Stable Laws

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***Abstract*—This paper addresses the problem of compressively sensing a set of temporally correlated sources, in order to achieve faithful sparse signal reconstruction from noisy multiple measurement vectors (MMV). To this end, a simple sensing mechanism is proposed, which does not require the restricted isometry property (RIP) to hold near the sparsity level, whilst it provides additional degrees of freedom to better capture and suppress the inherent sampling noise effects. In particular, a reduced set of MMVs is generated by projecting the source signals onto random vectors drawn from isotropic multivariate stable laws. Then, the correlated sparse signals are recovered from the random MMVs by means of a recently introduced sparse Bayesian learning algorithm. Experimental evaluations on synthetic data with varying number of sources, correlation values, and noise strengths, reveal the superiority of our proposed sensing mechanism, when compared against well-established RIP-based compressive sensing schemes.**

***Index Terms*—Compressive sensing, correlated sources, isotropic multivariate stable laws, sparse Bayesian learning**

I. Introduction

At the heart of compressive sensing (CS) is the key idea that the generation of random measurements can be used as an efficient sensing mechanism. The intrinsic randomness is critical not only in deducing important theoretical results, but also in achieving a better tradeoff between the sampling cost and the reconstruction accuracy of signals acquired in a broad range of practical applications.

Given a signal $\mathbf{x} \in \mathbb{R}^N$, the basic mathematical model for generating a reduced set of noisy random measurements is $\mathbf{y} = \mathbf{\Phi}\mathbf{x}+\mathbf{z}$, where $\mathbf{y} \in \mathbb{R}^M$ is the measurement vector ($M \ll N$), $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is a known measurement matrix whose rows are the *sensing vectors*, and $\mathbf{z} \in \mathbb{R}^M$ is an unknown sampling noise. The objective is to estimate $\mathbf{x}$ given $\mathbf{y}$ and $\mathbf{\Phi}$. For this, specific theoretical upper bounds have been obtained for the maximum sparsity level of $\mathbf{x}$, i.e., the number of its nonzero elements, that guarantee perfect and unique reconstruction [1].

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Motivated by several applications, such as multichannel electroencephalographic (EEG) signal processing [2], target localization [3], and direction-of-arrival (DOA) estimation [4], where a set of measurement vectors is available for each source, the basic CS model has been extended to the *multiple measurement vectors* (MMV) model [5], given by

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + \mathbf{Z} \ , \tag{1}$$

where $\mathbf{Y} \doteq [\mathbf{y}_1 \cdots \mathbf{y}_L] \in \mathbb{R}^{M\times L}$ is the matrix whose *columns* are the L measurement vectors, $\mathbf{X} \doteq [\mathbf{x}_1 \cdots \mathbf{x}_L] \in \mathbb{R}^{N\times L}$ is a sparse matrix to be recovered with each *row* representing a source, and $\mathbf{Z} \doteq [\mathbf{z}_1 \cdots \mathbf{z}_L] \in \mathbb{R}^{M\times L}$ is an unknown noise matrix. Although the conventional MMV model assumes that all the columns in $\mathbf{X}$ have identical support (i.e., the indexes of nonzero elements), however, in practice the sparsity profiles may vary and the common sparsity assumption is valid for only a small number L of measurement vectors.

Nevertheless, the source signals of interest often share some common structures in practice. Most of the MMV-based CS reconstruction algorithms exploit spatial dependencies [6], whereas the existence of temporal correlations among the sources has been recently accounted for in [7] to improve the reconstruction of a sparse source matrix $\mathbf{X}$. In this paper, we focus on the design of an efficient, yet simple, CS mechanism for generating MMVs from *temporally correlated sources*. As for the reconstruction, we employ the block sparse Bayesian learning framework proposed in [7], and particularly the fast T-MSBL algorithm[1], which yields an improved recovery performance among existing algorithms for the MMV model.

A key ingredient of any CS scheme is the appropriate selection of a measurement matrix $\mathbf{\Phi}$. The traditional way of addressing this issue is to rely on matrices that satisfy the *restricted isometry property* (RIP) [8], such as those with independent and identically distributed (i.i.d.) Gaussian or Bernoulli entries. However, it is generally very difficult to prove the RIP for generic matrices. To alleviate this issue,

[1]MATLAB code: http://dsp.ucsd.edu/~zhilin/TMSBL.html

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a RIPless theory of CS has been introduced in [9]. In particular, it was proven that if the sensing vectors are drawn independently at random from a probability distribution F that obeys a simple *incoherence* and *isotropy* properties, then we can faithfully recover approximately sparse signals from a minimal number of noisy measurements.

Contrary to the conventional RIP-based selection of $\mathbf{\Phi}$, which guarantees universality with high probability, our proposed sensing mechanism implies a *non-universal stability*. This means that, if we are given an arbitrary sparse (or approximately sparse) signal $\mathbf{x}$ and generate compressed measurements using our proposed scheme, then the recovery of this *fixed* $\mathbf{x}$ will be accurate.

Motivated by the RIPless framework and the efficiency of alpha-stable laws [10] in modelling a broad range of impulsive phenomena, we propose a new sensing mechanism for generating MMVs from temporally correlated sources. Specifically, the measurement vectors are generated by projecting the source matrix $\mathbf{X}$ onto sensing vectors drawn from an *isotropic multivariate stable distribution*. Doing so, we achieve an increased robustness against the presence of additive sampling noise, whilst better capturing the temporal correlations among the sources. An experimental evaluation on synthetic data with varying number of sources, correlation values, and noise strengths, demonstrates the efficiency of our sensing scheme when compared with traditional RIP-based approaches.

The rest of the paper is organized as follows: Section II briefly reviews the family of isotropic multivariate stable laws. In Section III, our proposed sensing mechanism is described in detail, whereas an experimental evaluation of its performance is carried out in Section IV. Finally, Section V summarizes the key messages and gives ideas for further extensions.

II. Isotropic Multivariate Stable Laws

Stable distributions constitute a class of probability distributions that generalize the normal law, allowing heavy (algebraic) tails and skewness that make them attractive in modelling a broad range of statistical behaviors, from linear (i.e., Gaussian) to extremely impulsive ones. Although stable laws are characterized by many attractive theoretical properties, however, their use in practical applications has been restricted by the lack of closed-form expressions for stable densities and distribution functions.

Focusing on multivariate stable distributions, apart from the lack of density functions in closed form, there is an additional difficulty in expressing the complexity of the dependence structures. Fortunately, these limitations have been alleviated significantly via the design and implementation of computationally tractable numerical algorithms for parameter estimation and simulation of classes of multivariate stable densities and distribution functions [11]–[13].

To specify a multivariate stable distribution for a random vector $\mathbf{x} = [x_1, x_2, \ldots, x_N]^\mathsf{T}$ in N dimensions requires an index of stability $\alpha \in (0, 2]$, a finite Borel (spectral) measure Λ on the unit sphere $\mathbb{S} = \{\mathbf{s} \in \mathbb{R}^N : \|\mathbf{s}\|_2 = 1\}$ and a shift vector $\boldsymbol{\delta} \in \mathbb{R}^N$. The general case is beyond current computational capabilities, but several special cases, including isotropic (i.e., radially symmetric), elliptical, independent components, and discrete spectral measure, are computationally accessible. Motivated by the RIPless CS framework in [9], which requires the sensing vectors to be drawn from a distribution that satisfies an incoherence and isotropy properties, hereafter we exploit the isotropic multivariate stable family.

In the isotropic case, the spectral measure is continuous and uniform, leading to radial symmetry for the distribution. The joint characteristic function of a random vector $\mathbf{x}$ following an isotropic multivariate stable law is as follows,

$$\mathbb{E}\{\exp(i\mathbf{u}^\mathsf{T}\mathbf{x})\} = \exp(-\gamma^\alpha \|\mathbf{u}\|_2^\alpha + i\mathbf{u}^\mathsf{T}\boldsymbol{\delta}) \ , \qquad (2)$$

where $\alpha \in (0, 2]$ is the characteristic exponent which controls the thickness of the tails of the density function (the smaller the α, the heavier the tails), $\gamma > 0$ is the dispersion parameter which determines the spread of the distribution around its location, and $\boldsymbol{\delta} \in \mathbb{R}^N$ is a location parameter. Hereafter, the notation $\mathbf{x} \sim S_\alpha(\gamma, \boldsymbol{\delta})$ denotes that a random vector $\mathbf{x}$ follows an isotropic multivariate stable distribution with parameters α, γ, $\boldsymbol{\delta}$. Without loss of generality, we assume $\boldsymbol{\delta} = \mathbf{0}$.

Notice also that for $\alpha = 2$ the distribution reduces to the multivariate normal case with i.i.d. components. However, this is not the case when $\alpha < 2$. Especially for correlated sources, we expect that the dependency among the components of isotropic multivariate stable sensing vectors will better capture the underlying correlation structure of the source signals. Furthermore, such sensing vectors are uniformly spread in all directions, as opposed, for instance, to the case of the commonly used Bernoulli matrix. In the latter case, the random directions of the sensing vectors are always in one of the fixed "diagonal directions" (e.g. in the 2-dimensional case, $\boldsymbol{\phi} = [+1, +1]$ corresponds to the positive diagonal; $\boldsymbol{\phi} = [+1, -1]$ corresponds to the $-45°$ ray, etc.). The uniform coverage of the original signal space via sensing vectors drawn from a $S_\alpha(\gamma, \boldsymbol{\delta})$ law guarantees that, with high probability, the matrix $\mathbf{\Phi}$ is prevented from being rank deficient when sufficiently many measurements are taken.

The radial symmetry of isotropic multivariate stable sensing vectors allows us to characterize the joint distribution in terms of their amplitude $r_\phi = \|\boldsymbol{\phi}\|_2$. The amplitude distribution itself depends on the specific characteristic exponent α and dispersion γ, which control the impulsiveness and spread of the stable density function. This *double control* on the behavior of our sensing vectors is a key aspect of our proposed sensing mechanism. More specifically, it enables a better adaptation to the original signal subspace, subsequently achieving a more accurate discrimination and suppression of the contaminating noise. Fig. 1 illustrates the difference between 2-dimensional isotropic and i.i.d. multivariate stable distributions, for two distinct pairs of parameters, namely, $(\alpha, \gamma) = (1, 1)$ and $(\alpha, \gamma) = (1.5, 0.5)$. Indeed, the isotropic case (Figs. 1a, 1c) yields a uniform coverage of the space, in contrast to the i.i.d. case (Figs. 1b, 1d). Also notice how α and γ affect the shape of the ball around the location (here $\boldsymbol{\delta} = (0, 0)$) in the isotropic case, which will be exploited by our sensing

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mechanism to increase the robustness of the generated random MMVs against the additive sampling noise. We note that all the subsequent numerical calculations involving multivariate stable densities are performed using the STABLE toolbox[2].

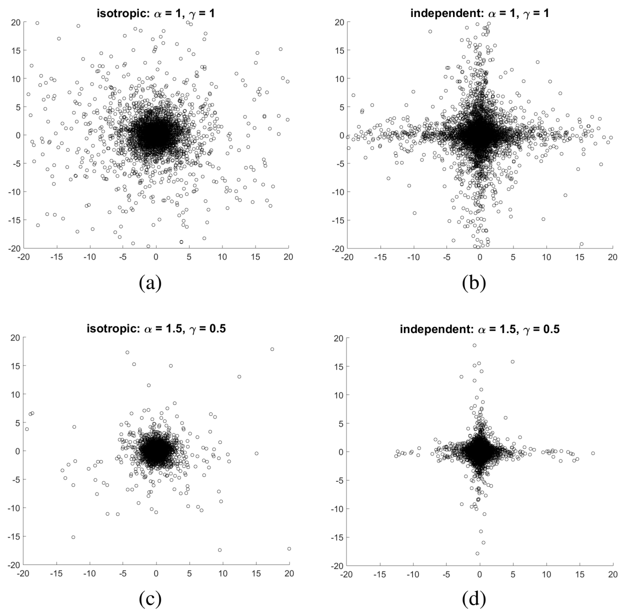

Fig. 1: Simulated isotropic and i.i.d. multivariate stable random samples for i) $(\alpha, \gamma) = (1, 1)$, ii) $(\alpha, \gamma) = (1.5, 0.5)$.

III. Isotropic Stable Sensing of Temporally Correlated Sources

Given an ensemble $\mathbf{X} \in \mathbb{R}^{N \times L}$ of temporally correlated sources, our goal is twofold: i) generate a set of measurement vectors $\mathbf{Y} \in \mathbb{R}^{M \times L}$ ($M \ll N$) capable of encoding the temporal correlations, and ii) achieve increased robustness to the presence of additive sampling noise by appropriately discriminating the noise and original signal subspaces.

To address these tasks, we propose a simple sensing mechanism by operating in a RIPless framework. More specifically, the linear sampling model (1) is employed. However, instead of relying on the commonly used i.i.d. assumption for the elements of the measurement matrix, in our proposed approach the rows of $\mathbf{\Phi} = [\boldsymbol{\phi}_1 \, \boldsymbol{\phi}_2 \cdots \boldsymbol{\phi}_M]^\mathsf{T} \in \mathbb{R}^{M \times N}$ are drawn from an isotropic multivariate stable distribution, i.e., $\boldsymbol{\phi}_m \sim S_\alpha(\gamma, \mathbf{0})$, for $m = 1, \ldots, M$.

As mentioned in Section II, this choice of sensing vectors $\boldsymbol{\phi}_m$ is motivated by two key characteristics, namely, i) the uniform spread of information across all the directions of the original N-dimensional space, and ii) the adaptation to the amplitude (strength) of the source signals, which allows us to better discriminate between the noise and signal subspaces.

The uniform coverage of the original signal space is an intrinsic property of the isotropic multivariate stable family. Concerning the adaptation to the source signals strength, this is achieved by appropriately setting the parameters α and γ of the stable model, which subsequently affects the form of the projection ball (ref. Fig. 1). To this end, we need to quantify the impulsiveness and spread of each source signal (i.e., column of $\mathbf{X}$). For this purpose, a symmetric *univariate* stable distribution is fitted to each signal $\mathbf{x}_l$, $l = 1, \ldots, L$, to estimate the parameters (α_l, γ_l). To mitigate the effects of zeros in the case of strictly sparse signals, as well as the small size effects (small N), the stable parameters are estimated using the empirical characteristic function based method described in [14]. Specifically, let $\tilde{\gamma} = \gamma^\alpha$ in (2). An estimate of the dispersion $\tilde{\gamma}$ is given by

$$\hat{\tilde{\gamma}} = -\ln\left(\frac{1}{N}\left|\sum_{n=1}^{N} e^{ix_n}\right|\right) . \tag{3}$$

On the other hand, an estimate of the characteristic exponent α is obtained by solving the following equation,

$$\hat{\alpha} = \log_{\omega_0}\left(\frac{\ln\left(\left|\frac{1}{N}\sum_{n=1}^{N} e^{i\omega_0 x_n}\right|\right)}{\ln\left(\left|\frac{1}{N}\sum_{n=1}^{N} e^{ix_n}\right|\right)}\right) , \tag{4}$$

where ω_0 is the solution of the nonlinear equation $\hat{\tilde{\gamma}} = \left(\ln(2\omega)/(\omega^2 - \omega)\right)\big|_{\omega_0}$, with $\hat{\tilde{\gamma}}$ given by (3). Finally, the dispersion in the parameterization (2) is calculated by $\hat{\gamma} = \hat{\tilde{\gamma}}^{1/\hat{\alpha}}$.

In our proposed sensing method we implement the following rules of thumb to estimate the parameters (α, γ) for the generation of the sensing vectors $\{\boldsymbol{\phi}_m\}_{m=1}^M \sim S_\alpha(\gamma, \mathbf{0})$:

1) **Estimation of α:** For each column $\mathbf{x}_l$, $l = 1, \ldots, L$, of $\mathbf{X}$ we estimate α_l from (4). Then, the value of α is set to

$$\alpha = (\alpha_1 + \cdots + \alpha_L)/L . \tag{5}$$

In order to avoid numerical instability when the above average is very small or close to 2, we bound α such that $\alpha = 0.5$ for $\alpha < 0.5$, and $\alpha = 1.95$ for $\alpha \in (1.95, 2]$.

2) **Estimation of γ:** For each column $\mathbf{x}_l$, $l = 1, \ldots, L$, of $\mathbf{X}$ we estimate the dispersion $\hat{\gamma}_l$. Then, the value of γ is set to

$$\gamma = c_\gamma \cdot (\hat{\gamma}_1 + \cdots + \hat{\gamma}_L)/L . \tag{6}$$

The factor $c_\gamma > 0$ controls the spread of the projection ball (ref. Fig. 1), and subsequently the capability to discriminate between the signal and noise subspaces. Our empirical results showed that by setting c_γ such that $\gamma \approx 1$ we achieve faithful reconstruction for a broad range of sparsity levels, correlation values, and signal-to-noise ratios (SNR). Nevertheless, the optimal selection of c_γ is still an open question. Notice that estimating α and γ requires access to the original signals, which is the case in several applications, especially those related with streaming sensor data.

Having estimated the model parameters according to (5) and (6), the sensing vectors $\{\boldsymbol{\phi}_m\}_{m=1}^M \in \mathbb{R}^N$ are drawn from the corresponding $S_\alpha(\gamma, \mathbf{0})$ distribution. Then, the rows of our proposed sensing matrix $\mathbf{\Phi}$ consist of the M *orthonormalized sensing vectors*. The orthonormalization step, which is carried out using the modified Gram-Schmidt algorithm, aims at yielding an incoherence parameter $\mu(\mathbf{\Phi})$ at the order of 1, while further simplifying the calculations involving the inverse of $\mathbf{\Phi}$ during the reconstruction process. Given the generated $\mathbf{\Phi}$, the noisy random MMVs are produced according to (1), where

[2]Robust Analysis Inc., STABLE toolbox v.5.3 (www.robustanalysis.com).

$\mathbf{Z}$ is assumed to be a sampling noise of bounded energy, i.e., $\|\mathbf{Z}\|_F \leq \epsilon$ (with $\|\cdot\|_F$ denoting the Frobenius matrix norm).

Interestingly, the computational cost for generating an N-dimensional isotropic stable sensing vector is roughly half the cost for generating N i.i.d. elements. Indeed, in the former case we need N uniform $(0, 1)$ random variables to simulate N i.i.d. Gaussian entries plus 2 uniform variables to simulate one positive stable variable for scaling, based on the Box-Muller algorithm (ref. [12]). In the latter case, $2N$ uniform $(0, 1)$ inputs are required to simulate N i.i.d. symmetric stable elements using the Chambers-Mallows-Stuck algorithm (ref. [10]). A detailed analysis of the computational complexity for generating the proposed sensing vectors is important for their application in real scenarios, and is left as a future work.

Given $\mathbf{Y}$ and $\mathbf{\Phi}$ the sparse matrix $\mathbf{X}$ is recovered using the T-MSBL algorithm [7]. T-MSBL first transforms the MMV problem into a block single measurement vector problem,

$$\mathbf{y}_v = \mathbf{A}\mathbf{x}_v + \mathbf{z}_v \ , \tag{7}$$

where $\mathbf{A} = \mathbf{\Phi} \otimes \mathbf{I}_{L\times L}$, $\mathbf{y}_v = \text{vec}(\mathbf{Y}^\mathsf{T}) \in \mathbb{R}^{ML\times 1}$, $\mathbf{x}_v = \text{vec}(\mathbf{X}^\mathsf{T}) \in \mathbb{R}^{NL\times 1}$, and $\mathbf{z}_v = \text{vec}(\mathbf{Z}^\mathsf{T}) \in \mathbb{R}^{ML\times 1}$. In the previous expressions, $\otimes$ denotes the Kronecker product of two matrices, $\mathbf{I}_{L\times L}$ is the $L \times L$ identity matrix, and $\text{vec}(\mathbf{B})$ denotes the vectorization of a matrix $\mathbf{B}$ formed by stacking its columns into a single column vector. Then, the original signal ensemble $\mathbf{x}_v$ is given by the maximum a posteriori (MAP) estimate of a posterior probability $p(\mathbf{x}_v|\mathbf{y}_v; \Theta)$ via the Bayesian rule, where Θ is the set of all the hyperparameters. The hyperparameters, which are related to the parameters of a Gaussian approximation for the densities of the source signals and the Gaussian likelihood of the random measurements, are estimated from the data by marginalizing over $\mathbf{x}_v$ and performing evidence maximization or Type-II maximum likelihood.

Notice that although T-MSBL is based on a Gaussian assumption for the statistics of the temporally correlated source signals, however, our proposed measurement matrix $\mathbf{\Phi}$ is constructed by also accounting for the impulsiveness of the source signals, as expressed by their estimated characteristic exponents α_l, $l = 1, \ldots, L$.

IV. Experimental Evaluation

In this section, we evaluate the efficiency of the proposed sensing mechanism for compressively sampling an ensemble of temporally correlated synthetic signals. Specifically, a set of representative test cases are presented that demonstrate the superiority of isotropic multivariate stable laws, in terms of better capturing the underlying signal subspace and dependence structure among the source signals, against well-established i.i.d. and multivariate generators of the measurement matrix $\mathbf{\Phi}$. In all the subsequent experiments, the results are averaged over 500 independent Monte Carlo runs. In each run, the signal length is fixed to $N = 128$, while the number of measurements varies, $M = \lfloor \delta_M \cdot N \rfloor$, with $\delta_M \in [0.10, 0.50]$. The source matrix $\mathbf{X}$ is randomly generated with $S = \lfloor \delta_S \cdot N \rfloor$ nonzero rows (i.e., sources), where $\delta_S \in [0.02, 0.15]$. Each source is generated as an AR(1) process, with the AR coefficient of the i-th source, denoted by β_i, indicating its temporal correlation. In the following, we assume that all the sources have equal AR coefficients, and compare five different temporal correlation levels, $\beta \in \{-0.9, -0.5, 0, 0.5, 0.9\}$. Notice that no prior assumption is made for the statistical distribution of the source signals. In the general noisy case we consider herein, the ℓ_2 norm of each source was rescaled to be uniformly distributed between 0.5 and 2.5. Regarding the additive noise $\mathbf{Z}$, it is drawn from a zero-mean homoscedastic Gaussian distribution with variance adjusted to have a targeted SNR value, which is defined by $\text{SNR} \doteq 20\log_{10}(\|\mathbf{\Phi X}\|_F/\|\mathbf{Z}\|_F)$ (in dB). Several noise strengths are tested by varying SNR in $[5, 20]$ dBs. Finally, the number of measurement vectors is fixed to $L = 5$.

Our proposed measurement matrix, hereafter denoted by $\mathbf{\Phi}_{mvs}$ is compared against the following commonly used measurement matrices: i) $\mathbf{\Phi}_{bnl}$: entries are i.i.d. Bernoulli random variables $(+1/-1)$; ii) $\mathbf{\Phi}_{orth}$: orthonormal basis for the range of an $N \times M$ Gaussian matrix; iii) $\mathbf{\Phi}_{use}$: columns are uniformly drawn from the surface of a unit hypersphere in $\mathbb{R}^{M-1}$. The reconstruction accuracy is quantified using two performance measures: i) the *failure rate* (FR), which indicates the percentage of failed Monte Carlo runs over the total runs. Since in the noisy case the recovery of $\mathbf{X}$ cannot be exact, a run is marked as a failure if the indexes of estimated sources with the S largest ℓ_2 norms differ from the true indexes; ii) the *mean squared error* (MSE), defined by $\|\hat{\mathbf{X}} - \mathbf{X}\|_F^2/\|\mathbf{X}\|_F^2$, where $\hat{\mathbf{X}}$ is the estimated source matrix.

As a first illustration, we examine the effect of the number of active sources on the reconstruction accuracy. Specifically, Fig. 2 shows the failure rate as a function of δ_S for two distinct correlation levels, $\beta = -0.5$ and $\beta = 0.9$, for a fixed $\delta_M = 0.35$ and $\text{SNR} = 10\,\text{dB}$. For this specific setup, all the measurement matrices yield an almost perfect recovery as the size of the sparse supports decreases. Nevertheless, as the number of active sources increases, all matrices fail to recover the sparse supports. Notice that this failure is also related to the number of generated random measurements for the given sparsity level. Indeed, as it is proven in [9], we can faithfully recover an S-sparse signal from about $S\ln(N)$ random incoherent measurements, when the coherence parameter $\mu(F)$ of the generating distribution F is at the order of $O(1)$, which is the case for all the matrices $\mathbf{\Phi}$ considered herein. As such, for $S = \lfloor 0.15 \cdot 128 \rfloor = 19$ at least $M = 19\ln(128) = 92$ measurements are required to guarantee faithful reconstruction, which is double than the $M = 44$ measurements used in this example. Notably, our proposed $\mathbf{\Phi}_{mvs}$ matrix demonstrates a robust performance against its competitors for the whole range of sparsity levels.

As a second illustration, we explore the effect of temporal correlation β on the reconstruction accuracy. To this end, Fig. 3 shows the FR $(\%)$ and MSE $(\%)$ as a function of β for the four measurement matrices, by fixing $\delta_S = 0.10$, $\delta_M = 0.35$, and $\text{SNR} = 10\,\text{dB}$. As it can be seen, the higher the absolute correlation level is, the more difficult it becomes to recover the sparse supports in this noisy scenario. An explanation for this behavior is that the sampling noise induces errors in the

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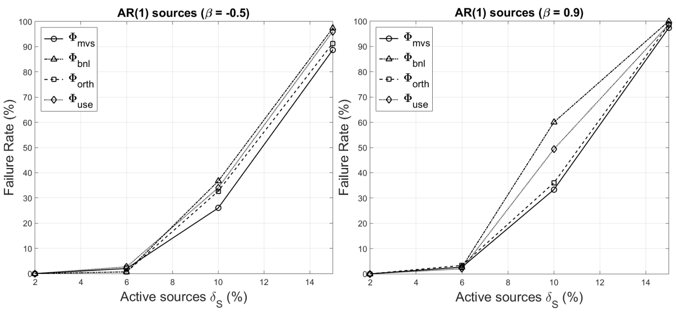
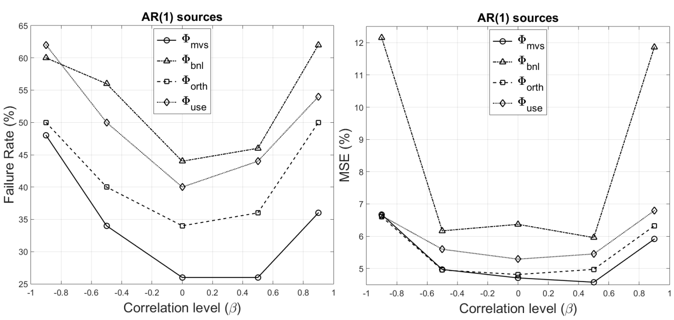
estimated sparse supports, which are spread out among the source signals because of the underlying temporal correlation. Nevertheless, sensing using our proposed $\mathbf{\Phi}_{mvs}$ matrix yields significantly lower failure rates and reduced reconstruction errors over the whole range of correlation values, when compared against the other three measurement matrices.



(a) (b)

Fig. 2: Failure rate $(\%)$ as a function of the percentage of active sources $\delta_S\,(\%)$, for the four measurement matrices and for two correlation levels ($\beta = -0.5$, $\beta = 0.9$).



(a) (b)

Fig. 3: FR $(\%)$ and MSE $(\%)$ as a function of temporal correlation β, for the four measurement matrices ($\delta_S = 0.10$, $\delta_M = 0.35$, SNR $= 10\,$dB).

Lastly, we evaluate the reconstruction performance in terms of the FR $(\%)$ as a function of the SNR, by fixing $\beta = 0.8$, $\delta_S = 0.10$ and $\delta_M = 0.35$. Clearly, $\mathbf{\Phi}_{mvs}$ and $\mathbf{\Phi}_{orth}$ recover the sparse supports with the highest accuracy, with the former matrix yielding an improved performance for the whole range of SNR values.

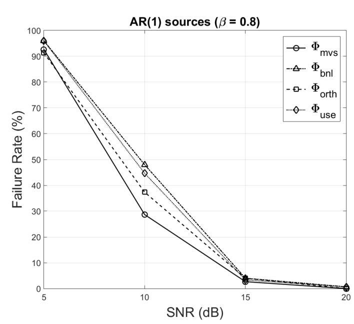


Fig. 4: FR $(\%)$ as a function of SNR, for the four measurement matrices ($\beta = 0.8$, $\delta_S = 0.10$, $\delta_M = 0.35$).

V. Conclusions

This paper introduced a simple, yet efficient, sampling mechanism for the generation of compressed measurements from temporally correlated sources. The proposed approach adopted a RIPless assumption, with the only requirement being the satisfaction of two properties, namely, incoherence and isotropy. To this end, our measurement matrix is generated by drawing vectors from an isotropic multivariate stable distribution, which is characterized by two parameters that control the shape and spread of the distribution. This double control of the projection ball yielded an increased robustness against the presence of additive sampling noise, as well as an improved discriminative capability between the signal and noise subspaces, thus resulting in a higher reconstruction accuracy when compared against well-established measurement matrices.

In this work, the isotropic multivariate stable family was considered, which assigns equal weights (i.e., dispersions) to all the sources (activated or not). However, we expect that the reconstruction accuracy can improve by incorporating side information regarding the sparse supports. To this end, we will investigate the generation of sensing vectors by assigning a distinct dispersion to each source according to the available prior information. Furthermore, the more general noisy model, including both observation and sampling noise, will be investigated, along with a thorough analysis of the computational cost for generating the proposed sensing vectors.

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