New Doppler Processing for the Detection of Small and Slowly-Moving Targets in Highly Ambiguous Radar Context

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***Abstract*—This paper presents a novel Doppler processing scheme for pulse Doppler radars operating in at intermediate Pulse Repetition Frequency (PRF) and suffering from range and Doppler ambiguities. One of the main drawbacks of the classical Doppler processing approach concerns the suppression of small and slowly-moving targets when rejecting ground clutter returns. In order to address this problem a two-step Doppler method is proposed in this paper. The first step uses a new iterative algorithm that resolves ambiguities and detects fast (exo-clutter) targets. The detection of slow (endo-clutter) targets is then performed by an adaptive detection scheme that uses a new covariance matrix estimation technique. Pulse trains with different characteristics are then associated for enhanced detection performance.**

I. Introduction

Moving target separation from ground clutter is generally performed by Doppler processing. A bank of Doppler filters is used to reject clutter returns around zero-Doppler frequency. The consequence of clutter rejection is the suppression of small and slowly-moving targets whose responses are buried into clutter returns. Pulse Doppler radars operating at an intermediate Pulse Repetition Frequency (PRF) suffer from range and Doppler ambiguities. A conventional solution to resolve ambiguities consists in transmitting successive pulse trains with different repetition frequencies. Each PRF value results in distinct unambiguous range and velocity, and ambiguity resolution is generally achieved by searching for coincident returns between burst responses. The transmission of successive pulse trains results in shorter train duration and hence a poorer Doppler resolution. Conventional estimation methods showing good resolution capabilities, like Capon's method [1] and MUSIC [2] can not be applied to resolve ambiguities from burst signals with PRF diversity, they need to estimate the covariance matrix from several observations and only a single observation is available per burst. The well-appreciated Maximum Likelihood (ML) approach [3] is based on a criterion that can jointly handle differently sampled bursts signals and resolve ambiguities, however, its implementation can be extremely complex.

In this paper, a new Doppler processing approach for a better detection of small and slowly-moving target is proposed. It includes a fast iterative ML estimator that allows to resolve the Doppler ambiguities and provides high resolution estimates of exo-clutter target velocities. An adaptive detector is than used to detect slowly-moving targets. The realistic hypothesis of compound-Gaussian environments is considered and the Adaptive Normalized Matched Filter(ANMF) which has been proved to be the most appropriate in this cases is applied. In practice, the ANMF uses an estimate of the covariance matrix obtained from a set of training data. In compound-Gaussian clutter environment, the Fixed Point FP estimator [4] is generally used to estimate the signal covariance matrix and its performance depend on the number of available independent and identically distributed (i.i.d.) training data sharing the same distribution as the cell under test. Numerous research efforts has been devoted to provide an accurate covariance matrix estimate from a reduced number of training data [5], [6]. However, doing so increases the sensibility to the presence of targets in the training support, which may significantly degrades estimation performance. In this paper a method that allows to estimate covariance matrix from the differently sampled burst-observations is proposed. It increases the number of training data and therefore enhance the detection performance.

II. Signal model

This paper considers a ground-based pulse Doppler radar operating at an intermediate PRF with a maximum unambiguous velocity $v_{amb} \simeq 200m/s$ with c the speed of the light and f_c the carrier frequency. In practice, target radial velocities may vary from $5m/s$ (domestic aerial unnamed vehicle) to $2000m/s$ (fighter aircraft). The Doppler ambiguity factor $|\frac{v_i}{v_{amb}}|$ is therefore much greater than one, it reach values up to 10. For the sake of simplicity, only the Doppler axes is considered, targets are supposed to be unambiguous in range. To resolve Doppler ambiguities, successive coherent bursts with PRF diversity are transmitted. Each burst consists of uniformly spaced pulses whereas the number of pulses M_k and the pulse repetition frequency PRF_k are different from one burst to the other. The carrier frequency is also generally varied from bursts to burst in order to enhance detection

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performance by forcing target echo decorelation between the different bursts. Considering N_t targets within the radar scene, the received signal is generally formed by the sum of their responses in addition to undesired acquisition noise $\mathbf{n}$ and clutter returns $\mathbf{c}$. After range-compression and by assuming no range migration all through duration of successive bursts, the received signal from the M_k pulses of the k^{th} burst in a single range-cell can be expressed in the following form

$$\mathbf{y}_k = \sum_{i=1}^{N_t} x_{k,i}\mathbf{a}_k(v_i) + \mathbf{z}_k + \mathbf{n}_k \ \in \mathbb{C}^{M_k} \tag{1}$$

Where,

$x_{k,i}$ is the i^{th} target response complex amplitude. Burst-wise frequency diversity forces target echo decorrelation. Under the hypotheses that the burst duration is small, $x_{k,i}$ is supposed to be constant during each burst and varies from one burst to another as a stationary zero-mean Gaussian process.

$\mathbf{a}_k$ denotes the target Doppler signature, or the Doppler vector corresponding to the velocity v_i, $\mathbf{a}_k(v_i) = [1, \ldots, e^{j2\pi(M-1)\frac{v_i}{v_{amb_k}}}]$.

$\mathbf{z}_k$ is modeled as a Spherically Invariant Random Vector (SIRV) [7], $\mathbf{c}_k = \sqrt{\tau_k}\mathbf{g}_k$ is then the product of a positive random variable τ_k (texture) and a complex Gaussian zero-mean vector $\mathbf{g}_k$ (speckle).

$\mathbf{n}_k$ is a Gaussian white noise.

III. New Doppler processing approach

To enhance the detection of small and slowly-moving targets a two-step Doppler processing technique for intermediate PRF pulse Doppler radars is proposed in this paper. A new iterative algorithm is first applied to burst signals in order to resolve Doppler ambiguities and discriminate high speed exo-clutter targets from the Gaussian white noise. An adaptive detector based on a multiple-burst covariance matrix estimate is then used to detect slowly-moving targets in the main lobe clutter region.

A. *Fast iterative multiple-burst ML estimation*

In the proposed processing approach the Relaxed Iterative Multiple-Burst Algorithm (ReIMBA) [8] is used to detect and estimate radial velocities of exo-clutter targets. The ReIMBA is a fast iterative solution to the Deterministic Maximum Likelihood (DML) criterion

$$\sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{A}_k(v)\,\mathbf{x}_k\|^2 \tag{2}$$

Minimizing the DML criterion with respect to v results in the following multi-dimensional non-linear problem

$$\widehat{\mathbf{v}} = \underset{\mathbf{v}}{\operatorname{argmin}} \sum_{k=1}^{K} \|\mathbf{P}^{\perp}_{\mathbf{A}_k(\mathbf{v})}\mathbf{y}_k\|^2 \tag{3}$$

Where $\mathbf{P}^{\perp}_{\mathbf{A}_k(\mathbf{v})} = \mathbf{I}_{Mk} - \mathbf{A}_k(\mathbf{v})\,(\mathbf{A}_k^H(\mathbf{v})\mathbf{A}_k(\mathbf{v}))^{-1}\mathbf{A}_k^H(\mathbf{v})$ denotes the orthogonal projector onto the null space of $\mathbf{A}_k(\mathbf{v})$, $\mathbf{I}_M$ is the identity matrix of size M. The ReIMBA resolves progressively this problem by sequentially identifying the velocity coordinate that correspond to the nearest grid point from its one-dimensional solution, re-estimating together the velocities of all detected targets, then updating the residual signal of each burst. At each iteration the algorithm selects the velocity coordinate that maximize incoherently integrated signals

$$\mathbf{l}_{\mathbf{\Sigma}} = \sum_{k=1}^{K} \mathbf{l}_k = \sum_{k=1}^{K} |\mathbf{A}_k^H(\mathbf{v_s})\,\mathbf{r}_k|^2 \tag{4}$$

Where $\mathbf{v_s}$ represents a grid of potential velocities in the search domain and $\mathbf{r}_k$ is the residual of the k^{th} burst. Initially $\mathbf{r}_k = \mathbf{y}_k$, it is updated at each iteration as follows

$$\mathbf{r}_k = \mathbf{P}^{\perp}_{\mathbf{A}_k(\widehat{\mathbf{v}})}\mathbf{y}_k \tag{5}$$

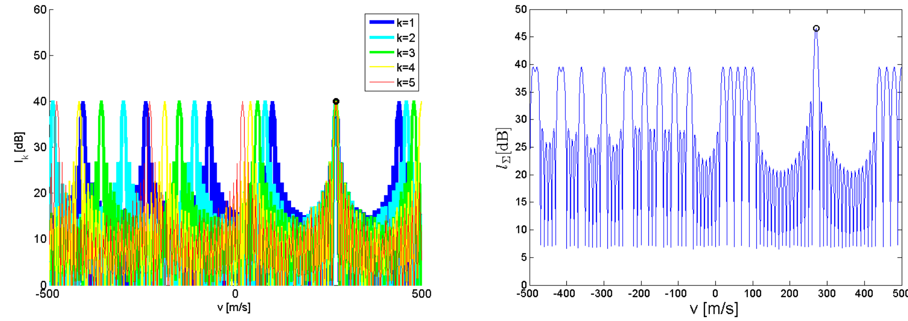
Where $\widehat{\mathbf{v}}$ is the vector of velocity estimates.
Figure 1 illustrates the gain obtained by incoherently integrating five burst signals with PRF and frequency diversity. As showed in table I presenting signal characteristics, burst signals have the same Doppler resolution δv and different unambiguous velocities. They contain a target with a mean power of $40dB$ and a velocity of $0m/s$ in addition to a Gaussian withe noise of $0dB$.

Figure 1a illustrates matched filtered signals. It can be

Parameters	burst 1	burst 2	burst 3	burst 4	burst 5
f_c (GHz)	3	3.01	3.025	3.015	3.02
PRF (Hz)	3400	3812.7	4235	4623	5033.3
M	17	19	21	23	25
δv (m/s)	10	10	10	10	10
v_{amb} (m/s)	170	190	210	230	250

TABLE I
Observed signal characteristics

seen that target responses coincide at the true target velocity (marked with a cross), whereas ambiguous response locations are varying from burst to burst. Incoherent integration of burst signals, in figure 1b, provides a gain of K at true target velocity with respect to ambiguous response power allowing to distinguish the true target response.
The discretization of the search domain in the first step of ReIMBA leads to estimation errors when target velocities do not exactly coincide with the grid. Moreover, as ReIMBA is based on a matched filtering approach, target velocities should be separated by more than the Doppler resolution to be correctly estimated. Estimation errors at one stage affect the next estimates and can lead to the generation of false alarms. In order to improve the estimation accuracy, the ReIMBA re-estimates at each iteration the entire set of selected target velocities before updating the residuals. The computational complexity of this multi-dimensional problem is reduced by relaxing it into multiple one-dimensional problems as proposed in [9].
As long as the maximum of $\mathbf{l}_{\mathbf{\Sigma}}$ exceeds the detection threshold λ, a new contribution (target or clutter response) is detected and the algorithm iterates. The threshold is fixed according to



(a) Matched filters (b) Integrated signal

Fig. 1. Incoherent integration Gain. Ambiguous target with a mean power of $40dB$ and a radiale velocity of $270m/s$.

the desired false alarm rate with respect to the measurement noise. The ReIMBA does not allow to distinguish targets from clutter contributions. Figure 2 illustrates an example of ReIMBA output in case of a complex scenario with two targets, a Gaussian white noise and a compound-Gaussian clutter. The Clutter to Noise Ratio (CNR) is $40dB$. The five burst signals with PRF and frequency diversity of the preceding simulation are considered. The first target is an endo-clutter target with radial velocity $v_1 = 2.5m/s = \frac{\delta v}{4}$ and a mean power of $40dB$ and the second one is a fast exo-clutter target with radial velocity $v_2 = 500m/s = 2v_{amb_5}$ and a mean power of $40dB$.

Simulation results show that the ReIMBA detects the exo-clutter target and estimates its true velocity despite the fact that its response coincides with a clutter side-lobe ($v_2 = 500m/s = 2v_{amb_5}$). Two other contributions are detected at $-0.09m/s$ and $1.5m/s$. These contributions result from the combination of the clutter and the slowly-moving target contribution. Hence, an adaptive detection has to be performed to distinguish slowly-moving target contributions from the clutter response.

B. Multiple-burst adaptive detection

The second step of the proposed approach uses the Adaptive Normalized Matched Filter (ANMF) [10] to detect slowly-moving targets in the main-lobe clutter region

$$\Lambda_{ANMF_k} = \frac{|\mathbf{a}_k^H(v)\mathbf{R}_k^{-1}\mathbf{y}_k|^2}{(\mathbf{a}_k^H(v)\mathbf{R}_k^{-1}\mathbf{a}_k(v))(\mathbf{y}_k^H\mathbf{R}_k^{-1}\mathbf{y}_k)} \gtrless \lambda_{ANMF_k} \quad (6)$$

In practice, the FP estimator usually estimates the unknown covariance matrix $\mathbf{R}_k$ from i.i.d. range cells adjacent to the cell under test

$$\widehat{\mathbf{R}}_{\mathbf{FP}_k} = \frac{M_k}{N_d}\sum_{i=1}^{N_d}\frac{\mathbf{y}_{k,i}\mathbf{y}_{k,i}^H}{\mathbf{y}_{k,i}^H\widehat{\mathbf{R}}_{\mathbf{FP}_k}\mathbf{y}_{k,i}} \quad (7)$$

Where N_d denotes the number of training data. Despite the good statistical properties of this estimator in compound-Gaussian clutter environment, its performance can be strongly degraded when the number of training data is small ($< 2M_k$ [11]). The number of adjacent range cells satisfying the i.i.d. condition with respect to the range under test is generally insufficient to ensure good estimation performance. Associating training data from different burst is then a solution to increase the i.i.d. training support. However, as transmitted bursts have different PRFs and carrier frequencies, a direct association of burst data to estimate a common covariance matrix is impossible.

A method to jointly estimate the covariance matrix from the different bursts data is proposed in this section. It relies on the fact that clutter has generally a continuous power spectral density. In case of a small variation of carrier frequency, the power spectral density of the signals of the different bursts are highly correlated. Thus a spectral interpolation-like transformation is feasible.

The proposed transformation approach matches the training data of burst k' with a number of pulses M'_k, PRF PRF'_k and carrier frequency $f_{c_{k'}}$ matching with the characteristics (M_k, PRF_k and f_{c_k}) of the burst k in the spectral domain. To do so, a transformation matrix $\tilde{\mathbf{A}}_{k'\to k} \in \mathbb{C}^{M_{k'}\times M_k}$ is used to perform a Discrete Fourier Transform (DFT) of the training data $\mathbf{y}_{k',i}$ at the coordinate of burst k

$$\mathbf{y}_{k',i}(f_{d_k}) = \tilde{\mathbf{A}}_{k'\to k}^H\mathbf{y}_{k',i} \quad (8)$$

With

$$\tilde{\mathbf{A}}_{k'\to k} = \frac{1}{\sqrt{M_{k'}}}\begin{pmatrix} 1 & \cdots & 1 \\ 1 & \cdots & e^{j2\pi\frac{(M_k-1)f_{c_{k'}}}{f_{c_k}PRF_{k'}}\delta f_{d_k}} \\ \vdots & & \vdots \\ 1 & \cdots & e^{j2\pi M_{k'}\frac{(M_k-1)f_{c_{k'}}}{f_{c_k}PRF_{k'}}\delta f_{d_k}} \end{pmatrix} \quad (9)$$

An inverse transformation with a classical DFT matrix $\mathbf{A}_k \in \mathbb{C}^{M_{k'}\times M_k}$ is then used to come back to time domain. The resulting signal $\mathbf{y}_{k'\to k,i} \in \mathbb{C}^{M_k}$ is obtained as follows

$$\mathbf{y}_{k'\to k,i} = \boldsymbol{\beta}_{k'\to k}\mathbf{y}_{k',i} \quad (10)$$

With,

$$\boldsymbol{\beta}_{k'\to k} = \mathbf{A}_k\tilde{\mathbf{A}}_{k'\to k}^H \quad (11)$$

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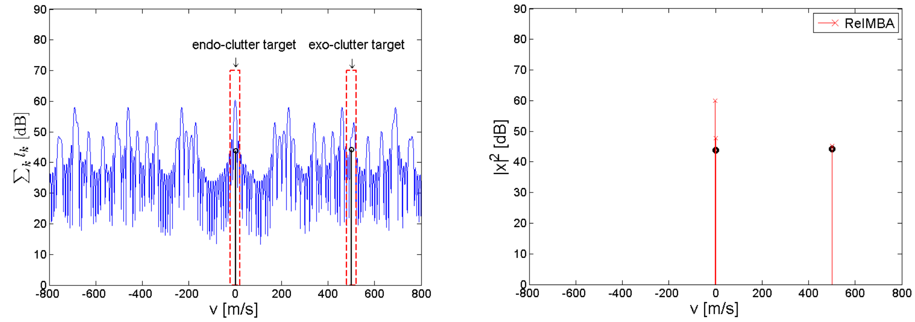

(a) Integrated signal (b) ReIMBA output

Fig. 2. ReIMBA output in presence of endo and exo-clutter targets. Endo-clutter target velocity $2.5m/s = \frac{\delta v}{4}$, exo-clutter target velocity $500m/s = 2v_{amb_5}$, target mean powers $40dB$.

The proposed approach allows to estimate the covariance matrix of each burst k from the training data of bursts $k' = 1, \ldots, K$ as follows

$$\hat{\mathbf{R}}_{\mathbf{CFP}_k} = \frac{M_k}{N_d K} \sum_{i=1}^{N_d} \sum_{k'=1}^{K} \frac{\boldsymbol{\beta}_{k' \to k} \mathbf{y}_{k',i} \mathbf{y}_{k',i}^H \boldsymbol{\beta}_{k' \to k}^H}{\boldsymbol{\beta}_{k' \to k} \mathbf{y}_{k',i} \hat{\mathbf{R}}_{\mathbf{FP}_k}^{-1} \mathbf{y}_{k',i}^H \boldsymbol{\beta}_{k' \to k}^H} \tag{12}$$

Where $\hat{\mathbf{R}}_{\mathbf{CFP}_k}$ is called the Composite Fixed Point matrix (CFP).
Estimated matrices are then injected in the ANMF detector to form a multiple-burst adaptive detection scheme

$$\Lambda_{ANMF} = \sum_{k=1}^{K} \frac{|\mathbf{a}_k^H(v) \mathbf{R}_k^{-1} \mathbf{y}_k|^2}{(\mathbf{a}_k^H(v) \mathbf{R}_k^{-1} \mathbf{a}_k(v))(\mathbf{y}_k^H \mathbf{R}_k^{-1} \mathbf{y}_k)} \gtrless \lambda_{ANMF} \tag{13}$$

IV. Performance assessment

In this section, performance of the proposed Doppler processing approach are evaluated using simulated and real radar data. Performance of the ReIMBA are verified by analyzing the estimation accuracy of target velocity estimates. The proposed transformation approach for the covariance matrix estimation is then investigated. Detection performance of the ANMF detectors associated to multiple-burst CFP estimator are compared against the classical single-burst FP estimator through Monte-Carlo simulations.

A. Performance on simulated Data

Simulated data consist of five burst with PRF and frequency diversity similar to the ones presented in section III.

In the first experiment, the observed signals contains a Gaussian white noise of $0dB$ and two ambiguous and closely spaced targets whose velocities are separated by $\frac{\delta v}{2}$. The Root Mean-Squares Error (RMSE) of one of the two target velocity estimates are compared to the Cramer Rao Bound (CRB) [12] and are illustrated in figure 3. The figure shows that ReIMBA estimates approach the CRB bound. This means that the algorithm resolved Doppler ambiguities and accurately estimated target velocities. The second simulation evaluates

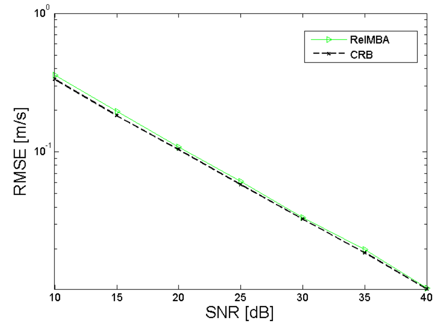

Fig. 3. Estimation accuracy in presence of two closely-spaced targets $|v_1 - v_2| \frac{\delta v}{2}$.

the performance of the proposed multiple-burst covariance matrix estimation approach. A scenario with Gaussian white noise of $0dB$ and a compound-Gaussian clutter of $40dB$ is considered. Clutter samples are generated according the windblown ground clutter model of Billingsley [13] where the wind velocity is set to $w = 8.12m/s$. The texture follows the Weibull distribution with shape parameter $\rho = 0.74$ and scale factor $\psi = 0.65$.
The detection performance of the ANMF-CFP are compared against those of the ANMF-FP. The covariance matrix of the fifth burst, $M_5 = 25$, is estimated by CFP from the five burst training data and by the FP from the fifth burst training data. The number of range cell data considered by each estimator is $N_d = 25$ and $N_d = 10$ for the CFP and $N_d = 125$ and $N_d = 50$ for the FP. Figure 4 represents the Probability of detection (Pd) versus SNR curves. Curves are obtained from 1000 Monte Carlo simulations containing an endo-clutter target with $v = 5m/s$. The figure shows that the ANMF-CFP detector performs as well as the ANMF-FP detector with $K = 5$ times more training cells. The proposed multiple-burst estimation approach allows to obtain a larger training support by combining the signals from different bursts.

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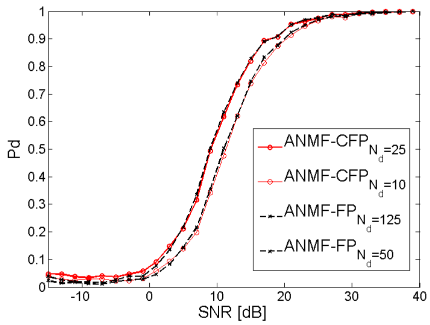

Fig. 4. Comparison of $P_d - SNR$ curves of ANMF-FP and ANMF-CFP on simulated data. Burst of $M_k = 25$ pulses, the CFP matrix is estimated from 5 bursts with $M_1 = 17$, $M_2 = 19$, $M_3 = 21$, $M_4 = 23$ and $M_5 = 25$ pulses, $P_{fa} = 10^{-3}$.

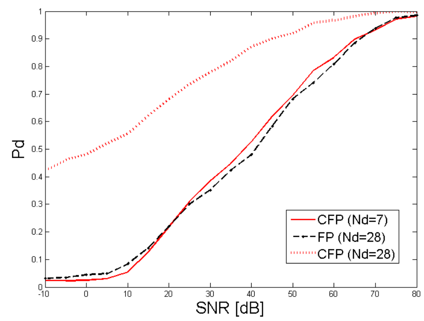

Fig. 5. Comparison of $P_d - SNR$ curves of ANMF-FP and ANMF-CFP on real data. Burst of $M_k = 25$ pulses, the CFP matrix is estimated from 4 bursts with $M_1 = 11$, $M_2 = 12$, $M_3 = 13$ and $M_5 = 14$ pulses, $P_{fa} = 10^{-1}$.

B. Performance on real Data

Detection performance obtained on simulated data are verified on real data collected with a ground-based radar of Thales Air Systems. The radar was illuminating a heavy sea clutter clutter from a height of 60 m at a grazing angle of $0°$. Four successive bursts with different carrier frequencies, different number of pulses $(11, 12, 13, 14)$ and different PRFs are considered.

Figure 5 illustrates the Pd-SNR curves of the ANMF-CFP and ANMF-FP. To obtain these curves a target was generated at the same radial velocity $v = 5m/s$ in 1000 range cells. The covariance matrix of the fourth burst ($M_4 = 14$) is estimated by the CFP and FP estimators. Detection performance of the ANMF-CFP with $N_d = 28$ and $N_d = 7(\frac{28}{4})$ are compared to those of ANMF-FP with $N_d = 28 = 2M_4$. The two detector provides the same performance when the CFP matrix is estimated from $N_d = 7$ rang cells versus $N_d = 28$ for the single burst FP estimator. A significant gain $\simeq 30dB$ is obtained by the ANMF-CFP when the CFP when the CFP matrix is estimated with the same number of rang cells as the FP estimator ($N_d = 28$). This result shows that the proposed multiple-burst approach for the covariance matrix estimation enhance the detection of small and slowly moving targets.

V. Conclusion

In this paper a two-step Doppler processing method that enhances the detection of small and slowly moving target in the context of highly Doppler ambiguous radar has been proposed. It uses a fast iterative ML algorithm that resolves ambiguities, detects principal signal components (targets+ clutter), and provides high resolution estimate of their velocities. After that, an adaptive multiple-burst detector is applied to detect slowly-moving targets in the main-lobe clutter region. A new multiple-burst covariance matrix estimation approach is introduced. It allows to increase the number of i.i.d. training data showing better detection performance. The proposed Doppler processing was tested successfully on real and simulated data showing its effectiveness.

References

[1] J. Capon, “High-resolution frequency-wavenumber spectrum analysis,” *Proceedings of the IEEE*, vol. 57, no. 8, pp. 1408–1418, Aug 1969.

[2] R. Schmidt, *PhD thesis : Signal subspace approach to multiple emitter location and spectral estimation*. Dept. Elect. Eng., Stanford Univ,CA, 1981.

[3] B. Ottersten, M. Viberg, P. Stoica, A. Nehorai, J. L. S. S. Haykin, and T. J. Shepherd, “Exact and large sample ML techniques for parameter estimation and detection in array processing,” *Radar Array Processing,Springer-Verlag*, pp. 99–151, 1993.

[4] F. Pascal, Y. Chitour, J. P. Ovarlez, P. Forster, and P. Larzabal, “Covariance Structure Maximum-Likelihood Estimates in Compound Gaussian Noise: Existence and Algorithm Analysis,” *IEEE Transactions on Signal Processing*, vol. 56, no. 1, pp. 34–48, Jan 2008.

[5] I. P. Kirsteins and D. W. Tufts, “Adaptive detection using low rank approximation to a data matrix,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 30, no. 1, pp. 55–67, Jan 1994.

[6] B. D. Carlson, “Covariance matrix estimation errors and diagonal loading in adaptive arrays,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 24, no. 4, pp. 397–401, Jul 1988.

[7] E. Conte, M. Lops, and G. Ricci, “Adaptive matched filter detection in spherically invariant noise,” *IEEE Signal Processing Letters*, vol. 3, no. 8, pp. 248–250, Aug 1996.

[8] L. Aouchiche, G. Desodt, C. Adnet, and L. Ferro-Famil, “Enhanced OMP algorithm for the detection and estimation of closely spaced moving objects in the presence of Doppler ambiguities,” in *2015 IEEE Radar Conference*, Oct 2015, pp. 260–265.

[9] J. Li and P. Stoica, “Efficient mixed-spectrum estimation with applications to target feature extraction,” *IEEE Transactions on Signal Processing*, vol. 44, no. 2, pp. 281–295, Feb 1996.

[10] S. Kraut and L. L. Scharf, “The CFAR adaptive subspace detector is a scale-invariant GLRT,” in *Ninth IEEE Signal Processing Workshop on Statistical Signal and Array Processing (Cat. No.98TH8381)*, Sep 1998, pp. 57–60.

[11] I. S. Reed, J. D. Mallett, and L. E. Brennan, “Rapid Convergence Rate in Adaptive Arrays,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-10, no. 6, pp. 853–863, Nov 1974.

[12] P. Stoica and A. Nehorai, “MUSIC, maximum likelihood, and Cramer-Rao bound,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 5, pp. 720–741, May 1989.

[13] J. B. Billingsley, *Low-angle radar land clutter: measurements and empirical models*, 1st ed. William Andrew, 2002.

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