A Convex-Combined Step-Size-Based Normalized Modified Filtered-x Least Mean Square Algorithm for Impulsive Active Noise Control Systems

Muhammad Tahir Akhtar
Department of Electrical and Computer Engineering, School of Engineering,
Nazarbayev University, Kabanbay Batyr Avenue 53, Astana, Republic of Kazakhstan.
Emails: muhammad.akhtar@nu.edu.kz, akhtar@ieee.org

***Abstract*—The celebrated filtered-x least mean square (FxLMS) algorithm does not work well for active noise control (ANC) of impulsive source. In previous attempts, the robustness of FxLMS algorithm has been improved by thresholding the reference and/or error signals used in the ANC system. However, estimating these thresholds is not any easy task in most of the practical scenarios. The need for appropriate thresholds is avoided in a previously proposed improved normalized-step-size FxLMS (INSS-FxLMS) algorithm, however, there is a trade-off situation between the convergence speed and steady-state performance as a fixed step-size needs to be selected properly. In this paper, we propose a novel algorithm for impulsive ANC (IANC) systems. The proposed algorithm is based on the previously proposed INSS-FxLMS. The main idea to employ a convex-combined step-size which automatically converges to a large value to improve the convergence speed during the transient state, and to a small value as the IANC system converges at the steady-state. Extensive simulation results are presented to demonstrate the effective performance of the proposed algorithm.**

***Index Terms*—adaptive algorithm, active noise control, impulsive noise, convex combination, variable step-size**

I. Introduction

During the last decades, acoustic noise pollution has become a serious threat to a comfortable living. The passive means of reducing noise prove to be very expensive and even inefficient when targeting low frequency noise control, particularly in the large or open spaces. In such scenarios, active noise control (ANC) is a preferred choice, where the target noise is canceled or at least reduced by generating and acoustically combing an anti-noise canceling signal. A block diagram for ANC in a single-channel configuration, well suited for duct applications for example [1] is shown in Fig. 1. Here $\boldsymbol{p}(n)$ and $\boldsymbol{s}(n)$ denote the impulse responses of the so-called primary and secondary acoustic paths, respectively. Without loss of generality, the acoustic paths are hereby assumed to be modeled as finite impulse response (FIR) filters. The adaptive ANC filter $\boldsymbol{h}(n)$ is excited by the reference signal $x(n)$, and its output $y(n)$ is used to provide the cancelation of the noise around the location of the error microphone. On the basis of the residual error signal $e(n)$, the coefficients of $\boldsymbol{w}(n)$ are updated by the filtered-x least mean square (FxLMS) algorithm [1], [2] as

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu e(n)\boldsymbol{x}_f(n), \qquad (1)$$

where μ is a fixed step-size parameter, and $\boldsymbol{x}_f(n)$ is the vector for the filtered-reference signal being computed as $x_f(n) = \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}(n)$ where $\hat{\boldsymbol{s}}(n)$ denotes the coefficient vector of a filter modeling the secondary path $\boldsymbol{s}(n)$. Although the FxLMS algorithms has a low computational complexity, it does not perform well for the impulsive sources as considered in this paper. The impulsive noise can be modeled as standard symmetric α-stable (SαS) distribution having characteristic function of the form $\varphi(t) = e^{-|t|^{\alpha}}$ [3]. Here the parameter α is the so-called characteristics exponent, with a small valued α indicating a heavy tailed distribution.

Broadly speaking there are two approaches for developing impulsive ANC (IANC) algorithms: the adaptive algorithm is derived using a robust cost function, or the FxLMS algorithm be modified to improve its robustness against the impulsive sources. In the first approach, the filtered-x least mean p-power (FxLMP) algorithm [4] is considered as a benchmark algorithm

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu p|e(n)|^{p-1}\text{sgn}(e(n))\boldsymbol{x}_f(n), \qquad (2)$$

where $\text{sgn}(\cdot)$ computes sign of the quantity inside. The FxLMP algorithm with $p < \alpha$ shows better robustness as compared with the FxLMS algorithm. However, its convergence speed is very slow. Many variants have been proposed to improve performance of FxLMP algorithm. Nevertheless, the estimation of p in relation to α remains a challenge [5]–[7]. In this paper, therefore, we stick to FxLMS-based algorithms for IANC systems.

The Sun's algorithm [8] ignores the large-valued samples of the reference signal $x(n)$ before being used in the update equation of the FxLMS algorithm (1). This simple truncation idea improves the robustness of FxLMS algorithm; however, the slow convergence speed, and stability (especially for strongly impulsive sources) remain issues to be resolved. An improved version of Sun's algorithm, hereafter referred as thresholding FxLMS (Th-FxLMS) algorithm, has been suggested in [9], where saturation non-linear functions are being employed both with the reference $x(n)$ and error signals $e(n)$ in (1). These algorithms [8], [9] require estimation of appropriate thresholding parameters, which may depend upon the statistical properties of the noise source. These statistics, however, may not be available in some situations particulary during when the IANC system is in operation. In order to avoid computing thresholding parameters, a normalized step-size-based FxLMS (INSS-FxLMS) algorithm has been proposed in [10]. Here the step-size is normalized with respect to the input as well as the error signal. However, a proper tuning of step-size is needed to get an adequate performance. In this paper, we propose an

ISBN 978-90-827970-1-5

improved performance algorithm for IANC systems, where the step-size is automatically tuned by a convex mixing parameter. Furthermore, the proposed algorithm has been developed in the frame work of modified FxLMS (MFxLMS) structure [11], which has better convergence properties as compared with the standard FxLMS algorithm.

The rest of the paper is organized as follows. Section II gives a brief overview of INSS-FxLMS algorithm [10], and details on the derivation of the proposed algorithm. A few remarks regarding selection of various parameters and computational complexity of the proposed algorithms are also presented. Section III presents results of computer simulations followed by the concluding remarks presented in Section IV.

II. Proposed Algorithm for Impulsive Active Noise Control Systems

A. Summary of Previous INSS-FxLMS Algorithm

In order to improve the robustness of the standard FxLMS algorithm for IANC systems, the following INSS-FxLMS algorithm has been proposed in [10]

$$\boldsymbol{h}(n+1)=\boldsymbol{h}(n)+\frac{\mu}{\|\boldsymbol{x}_f(n)\|^2+\sigma_e^2(n)+\delta}e(n)\boldsymbol{x}_f(n), \quad (3)$$

where $\|\cdot\|$ denotes the Euclidean norm, δ is small positive number added to avoid division by zero, and $\sigma_e(n)$ can be estimated as

$$\sigma_e(n)=\beta\sigma_e(n-1)+(1-\beta)|e(n)|, \quad (4)$$

where $|\cdot|$ is the absolute value of quantity, and $0.9<\beta<1$ is the forgetting factor. This algorithm is based on the idea that in IANC systems the error signal is also peaky in nature, e.g., during the transient state of IANC system, and its effect must also be taken into account. Nevertheless, the INSS-FxLMS algorithm is using a fixed-step-size which must be tuned properly to reach a compromise between the convergence speed and the steady-state performance. Furthermore, our experience shows that the choice of step-size depends on the impulsiveness of the noise sources: a small value must be selected for strongly-impulsive noise sources, and a large one may be used for mildly- or low-impulsive noise sources. These limitations set motivation for the proposed algorithm.

B. Development of Proposed Algorithm

A block diagram of the proposed IANC system is shown in Fig. 2, which is based on MFxLMS structure. In MFxLMS structure, the filter $\boldsymbol{h}(n)$ is adapted using an internally generated error signal $g(n)$. After adaptation the coefficients are copied to the main noise control filter to generate the control signal $y(n)$. The MFxLMS algorithm has better convergence properties than the conventional FxLMS algorithms (for details reader is referred to [11], and references there in). In Fig. 2, the control signal $y(n)$ is computed as

$$y(n)=\boldsymbol{h}^T(n)\boldsymbol{x}(n), \quad (5)$$

where $\boldsymbol{h}(n)=[h_0(n),h_1(n),\cdots,h_{L-1}(n)]^T$ is the coefficient vector of the main noise control filter being considered as an

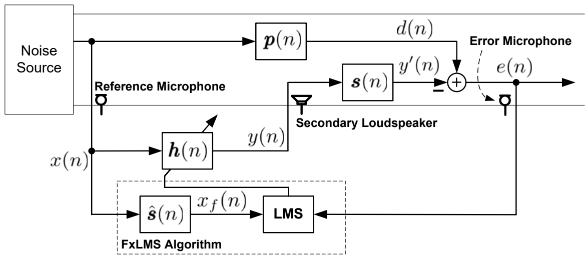

Fig. 1. Block diagram of FxLMS algorithm based single-channel feedforward ANC systems.

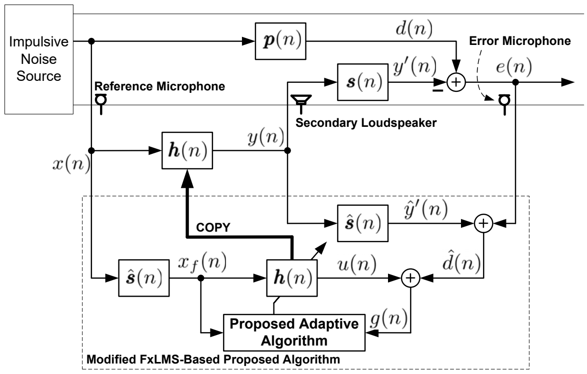

Fig. 2. Block diagram of modified FxLMS-based proposed algorithm for impulsive active noise control (IANC) systems.

FIR filter of length L, and $\boldsymbol{x}(n)=[x(n),x(n-1),\cdots,x(n-L+1)]^T$ is the corresponding reference signal vector. The residual error signal $e(n)$, being picked up by the error microphone, is given as

$$e(n)=d(n)-y'(n), \quad (6)$$

where $d(n)=p(n)*x(n)$ is the primary disturbance signal, and $y'(n)=s(n)*y(n)$ is the secondary canceling signal. Here $*$ denotes the linear convolution. The output signal $y(n)$ is filtered through the secondary-path-modeling filter $\hat{\boldsymbol{s}}(n)$ to get an estimate of $d(n)$ as

$$\hat{d}(n)=e(n)+\hat{\boldsymbol{s}}^T(n)\boldsymbol{y}(n). \quad (7)$$

The filtered reference signal $x_f(n)$ is used as an input signal for the adaptive filter $\boldsymbol{h}(n)$, whose error signal is computed as

$$g(n)=\hat{d}(n)-u(n)=\hat{d}(n)-\boldsymbol{h}^T(n)\boldsymbol{x}_f(n). \quad (8)$$

The coefficients of the adaptive filter $\boldsymbol{h}(n)$ are updated as

$$\boldsymbol{h}(n+1)=\boldsymbol{h}(n)+\frac{\mu}{\|\boldsymbol{x}_f(n)\|_2^2+\sigma_g^2(n)+\delta}g(n)\boldsymbol{x}_f(n), \quad (9)$$

where $\sigma_g(n)$ is estimated as in (4). Finally, the updated coefficients are copied to the main noise control filter generating $y(n)$. It is well known that choosing step-size is a trade-off between the convergence speed and steady-state performance.

Many variable step-size approaches have been proposed to solve this trade-off issue, where the key idea is to use a large step-size initially for fast convergence, and reduce gradually to a small value for an improved steady-state performance [12]–[14]. Another approach is a convex combination of two adaptive filters [15]–[19], being adapted using two different step-sizes. This has obvious disadvantage of increased computational complexity as the two adaptive filters are adapted simultaneously. Recently, a new convex-combined step-size strategy has been proposed in [20], [21]. Motivated by their results, we hereby employ this strategy to develop a convex-combined step-size-based normalized MFxLMS algorithm for IANC systems. In order to develop the proposed algorithm, assume that two copies of $\boldsymbol{h}(n)$ are updated according to (9) as

$$\boldsymbol{h}_1(n+1) = \boldsymbol{h}_1(n) + \mu_1 \Delta\boldsymbol{h}(n), \tag{10}$$

$$\boldsymbol{h}_2(n+1) = \boldsymbol{h}_2(n) + \mu_2 \Delta\boldsymbol{h}(n), \tag{11}$$

where

$$\Delta\boldsymbol{h}(n) = \frac{g(n)\boldsymbol{x}_f(n)}{\|\boldsymbol{x}_f(n)\|_2^2 + \sigma_g^2(n) + \delta}, \tag{12}$$

is the same increment vector (as in (9)) to update the two copies of adaptive filter $\boldsymbol{h}_1(n)$ and $\boldsymbol{h}_2(n)$. Selecting $0 < \mu_2 < \mu_1 \leq 1$, $\boldsymbol{h}_1(n)$ shows fast convergence speed and $\boldsymbol{h}_2(n)$ gives good steady-state behavior. These two performance behaviors can be combined by considering an overall adaptive filter $\boldsymbol{h}(n)$ being obtained as a convex combination of $\boldsymbol{h}_1(n)$ and $\boldsymbol{h}_2(n)$ as

$$\boldsymbol{h}(n+1) = \lambda(n)\boldsymbol{h}_1(n+1) + (1-\lambda(n))\boldsymbol{h}_2(n+1), \tag{13}$$

where $\lambda(n)$ is a (time-varying) scalar between $0 \leq \lambda(n) \leq 1$. Substituting (10) and (11) in (13), we get

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \mu(n)\Delta\boldsymbol{h}(n), \tag{14}$$

where $\boldsymbol{h}(n) = \lambda(n)\boldsymbol{h}_1(n) + (1-\lambda(n))\boldsymbol{h}_2(n)$ is the convex combination of currently available filter coefficients, and $\mu(n)$ is the convex-combined step-size being given as

$$\mu(n) = \lambda(n)\mu_1 + (1-\lambda(n))\mu_2. \tag{15}$$

It is evident from (14) and (15) that $\lambda(n) = 1$ results in adaptive filter being adapted using a large step-size, and $\lambda(n) = 0$ corresponds to adaptive filter updated using a small step-size. We would like to have a mixing parameter such that, $\lambda(n) = 1$ at the start-up at $n = 0$ and $\lambda(n) \to 0$ at the steady-state. As per literature on convex-combination adaptive filters, it is customary to consider $\lambda(n)$ being output of a sigmoidal activation function [19]

$$\lambda(n) = \frac{1}{1+e^{-a(n)}}, \tag{16}$$

where $a(n)$ is the adaptive parameter. In the proposed algorithm, $a(n)$ is adapted to minimize the IANC system error $e(n)$ as follows

$$a(n) = a(n-1) - \mu_a \frac{\partial |e(n)|}{\partial a(n-1)}, \tag{17}$$

where μ_a is a fixed step-size parameter, and $|e(n)|$ is used to achieve robustness against impulsive sources. The residual error signal $e(n)$ in (6) can be written as

$$\begin{aligned} e(n) &= d(n) - s(n) * y(n), \\ &= d(n) - s(n) * (h(n) * x(n)), \\ &= d(n) - (s(n) * x(n)) * h(n). \end{aligned} \tag{18}$$

It is worth to mention here that $e(n)$ is directly available at the error microphone, and its component signals are not accessible. Assuming that secondary-path-modeling filter $\hat{\boldsymbol{s}}(n)$ is available, the convolution $s(n) * x(n)$ can be approximated as

$$s(n) * x(n) \approx \hat{s}(n) * x(n) \equiv \hat{\boldsymbol{s}}^T(n)\boldsymbol{x}(n) = x_f(n). \tag{19}$$

Thus, the residual error signal $e(n)$ can be approximated as

$$e(n) \approx d(n) - \boldsymbol{h}^T(n)\boldsymbol{x}_f(n) = d(n) - \boldsymbol{x}_f^T(n)\boldsymbol{h}(n), \tag{20}$$

and (17) becomes

$$a(n) = a(n-1) + \mu_a \text{sgn}(e(n))\boldsymbol{x}_f^T(n)\frac{\partial \boldsymbol{h}(n)}{\partial a(n-1)}, \tag{21}$$

where the gradient vector is computed by applying the following chain rule

$$\frac{\partial \boldsymbol{h}(n)}{\partial a(n-1)} = \frac{\partial \boldsymbol{h}(n)}{\partial \mu(n-1)} \cdot \frac{\partial \mu(n-1)}{\partial \lambda(n-1)} \cdot \frac{\partial \lambda(n-1)}{\partial a(n-1)}, \tag{22}$$

where three gradients on the right hand side can be computed by considering (14), (15) and (16), respectively. Finally, the update equation for the parameter $a(n)$ is given as

$$\begin{aligned} a(n) \;\;=\;\; & a(n-1) + \mu_a \text{sgn}(e(n))(\mu_1 - \mu_2) \cdot \\ & (\lambda(n-1)(1-\lambda(n-1)))\, \boldsymbol{x}_f^T(n)\Delta\boldsymbol{h}(n-1). \end{aligned} \tag{23}$$

C. A Few Remarks

1) In order to avoid freezing the adaptation of $a(n)$ when $\lambda(n) \to 0$ or $\lambda(n) \to 1$, the value of $a(n)$ is restricted between $[-a^+, a^+]$. Thus, following procedure is used to update the mixing parameter $\lambda(n)$

$$\boxed{\begin{array}{lll} \text{compute } a(n) \text{ using (23)} & & \\ \texttt{if} & a(n) \leq -a^+ + \epsilon, & a(n) = -a^+ + \epsilon \\ \texttt{elseif} & a(n) \geq a^+ - \epsilon, & a(n) = a^+ - \epsilon \\ \text{compute } \lambda(n) \text{ using (16)} & & \end{array}}, \tag{24}$$

where ϵ is a small positive constant. By plotting $\lambda(n)$ vs $a(n)$ in (16), it is straightforward to show that $a^+ = 4$ is a reasonable choice.

2) A small modification is needed in the adaptive algorithm for $a(n)$ derived in (23). It is important to note that the gradient information used to update the noise control filter is also used here in adaptation of $a(n)$ (see $\Delta\boldsymbol{h}(n-1)$ in (23)). Furthermore, the residual error signal acts as an error signal for the adaptation of $a(n)$. In order to align these two, it is suggested to replace sgn$(e(n))$ in (23) with sgn$(e(n-1))$.

3) Since the proposed algorithm can be considered as a variable-step-size adaptive algorithm, based on this the theoretical results previously studied would equally apply for the selection of step-size parameters μ_1 and μ_2 in (15). Since the proposed algorithm essentially is based on the normalization of the step-size, the large value μ_1 may be selected close to 1. The other parameter μ_2 can be selected as a small positive number.

4) In (23), $(\mu_1 - \mu_2)$ is a difference of two fixed numbers, and can be assumed to be combined with μ_a.

5) A detailed computational complexity analysis is omitted for the sake of space. However, the proposed algorithm, being based on structure of MFxLMS algorithm, has a somewhat increased computational complexity as compared with the existing FxLMS-based algorithms. However, this increased computational cost can be considered as a price paid for the greatly improved performance as demonstrated in the next section.

III. Results of Computer Simulations

This section provides the simulation results to verify the effectiveness of the proposed algorithm in comparison with the previously proposed INSS-FxLMS [10]. The Sun's algorithm [8], and Th-FxLMS algorithm [9] are also in included in the performance comparison. The experimental data provided with [1] is used to model the primary and secondary acoustic paths as FIR filters $\boldsymbol{p}(n)$ and $\boldsymbol{s}(n)$ of lengths 256 and 128, respectively. It is assumed that the secondary path modeling filter $\hat{\boldsymbol{s}}(n)$ is exactly identified as $\boldsymbol{s}(n)$. The ANC filter $\boldsymbol{h}(n)$ is selected as an FIR filter of length 192. All simulation results presented below are averaged over 100 realizations.

In the first experiment, the reference signal $x(n)$ is modeled by a standard SαS process with $\alpha = 1.15$ which corresponds to a strongly impulsive noise source. The simulation results are shown in Fig. 3, where step-size parameters are mentioned in the caption. The thresholding parameters are selected as [0.01, 99.99] percentile in Sun's algorithm, and [1,99] percentile in Th-FxLMS algorithm [9]. The rest of the simulation parameters are selected as: $\beta = 0.99$, $\epsilon = \delta = 1 \times 10^{-4}$, $\mu_a = 0.1$, $a^+ = 4$. Fig. 3(a) shows curves for mean noise reduction (MNR) being defined as

$$\text{MNR}(n) = \mathbb{E}\left\{\frac{\sigma_e(n)}{\sigma_d(n)}\right\}, \tag{25}$$

where $\sigma_e(n)$ and $\sigma_d(n)$ are estimated using the low pass estimator as in (4). We see that Sun's algorithm is not stable even for a very small step-size. Furthermore, Th-FxLMS and INSS-FxLMS algorithms indeed offer a trade-off for selection of step-size: the steady-sate performance severely degrades when a large step-size is selected to achieve a fast convergence speed. The proposed algorithm demonstrates the best performance among the algorithms considered in this paper. Fig. 3(b) plots variation of the norm $||\boldsymbol{h}(n)||$ of the IANC filter coefficients, which shows superior convergence of the proposed algorithm in comparison with the other algorithms. The performance gain is due to the variable mixing parameter $\lambda(n)$ (plotted in a small window in Fig. 3(b)). As seen, $\lambda(n)$ initially is close to 1 resulting in a large step-size initially for fast convergence, and $\lambda(n) \to 0$ at the steady-state giving the steady-state performance as good as by the slowly converging previous algorithms.

The above experiment is repeated for SαS processes with $\alpha = 1.45$ (moderately impulsive) and $\alpha = 1.75$ (mildly impulsive). The corresponding results are presented in Figs. 4 and 5, respectively. The simulation parameters are kept same as in the previous experiment for $\alpha = 1.15$, except for the step-size parameters in the previous methods. The proposed method gives the best performance, and does not require further tuning of step-size or other parameters. In the last experiment, we consider a scenario of time varying acoustic environment. For this purpose, experiment for $\alpha = 1.45$ is repeated by considering a sudden change in the primary path at the middle of simulation. For a change in the acoustic path, the impulse response coefficients $\boldsymbol{p}(n)$ are multiplied by -1. Fig. 6 presents the corresponding simulation results, which show that the proposed algorithm outperforms the other methods.

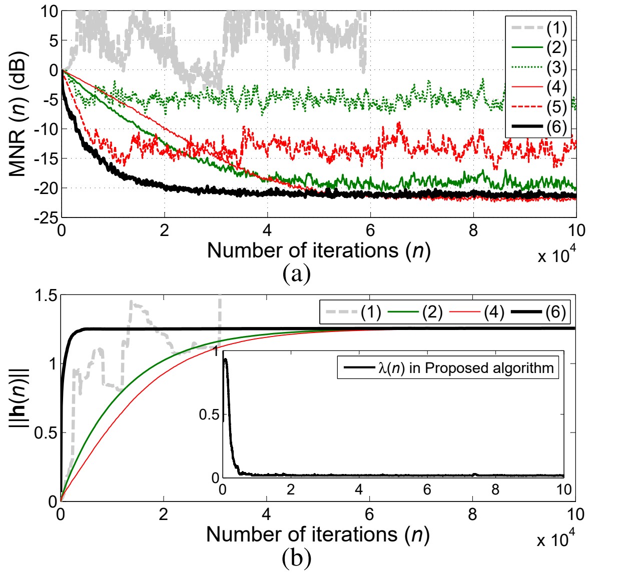

Fig. 3. Simulation results for strongly impulsive noise with $\alpha = 1.15$. (a) Mean noise reduction (MNR) curves for various algorithms, and (b) convergence of norm of noise control filter coefficients, $||\boldsymbol{h}(n)||$. [Step-size parameters: (1) Sun's algorithm ($\mu = 1 \times 10^{-8}$), (2) Th-FxLMS algorithm ($\mu = 1 \times 10^{-6}$), (3) Th-FxLMS algorithm ($\mu = 5 \times 10^{-6}$) (4) INSS-FxLMS algorithm ($\mu = 1 \times 10^{-4}$), (5) INSS-FxLMS algorithm ($\mu = 5 \times 10^{-4}$), and (6) Proposed algorithm ($\mu_1 = 1.0, \mu_2 = 1 \times 10^{-2}$).]

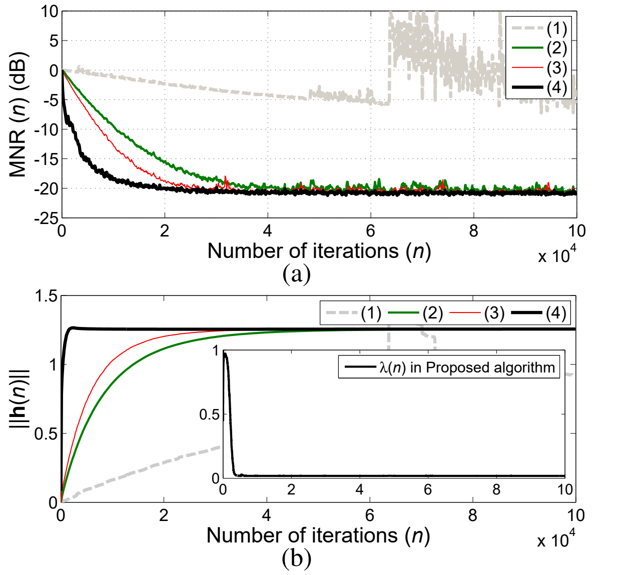

Fig. 4. Simulation results for moderately impulsive noise with $\alpha = 1.45$. (a) Mean noise reduction (MNR) curves for various algorithms, and (b) convergence of norm of noise control filter coefficients, $||\boldsymbol{h}(n)||$. [Step-size parameters: (1) Sun's algorithm ($\mu = 5 \times 10^{-8}$), (2) Th-FxLMS algorithm ($\mu = 5 \times 10^{-6}$), (3) INSS-FxLMS algorithm ($\mu = 5 \times 10^{-4}$), and (4) Proposed algorithm ($\mu_1 = 1.0, \mu_2 = 1 \times 10^{-2}$).]

IV. Conclusions

In this paper, we have proposed a novel IANC algorithm, which is simple to implement and holds promise for practical IANC systems. Among the algorithms considered in this paper, the proposed algorithm gives best convergence speed for strongly impulsive noise sources (see Fig. 3) as well as

ISBN 978-90-827970-1-5

for mildly impulsive ones which are more towards Gaussian distribution (see Fig. 5). Furthermore, the proposed algorithm shows robust performance against a sudden change in the primary path $\boldsymbol{p}(n)$ (see Fig. 6). As stated earlier, the secondary path modeling filter is assumed to be known and in fact fixed in this study. Performing secondary path modeling during the online operation of ANC system is an interesting research topic [11], and is left as a task of future work for the proposed algorithm. Furthermore, it would be interesting to explore application of the proposed algorithm for impulsive noise sources that are not α stable, for example transient sinusoids as considered in [22]. Last but not least, extension to multi-channel scenarios requiring array for microphones and loudspeakers, is another possible direction for future work.

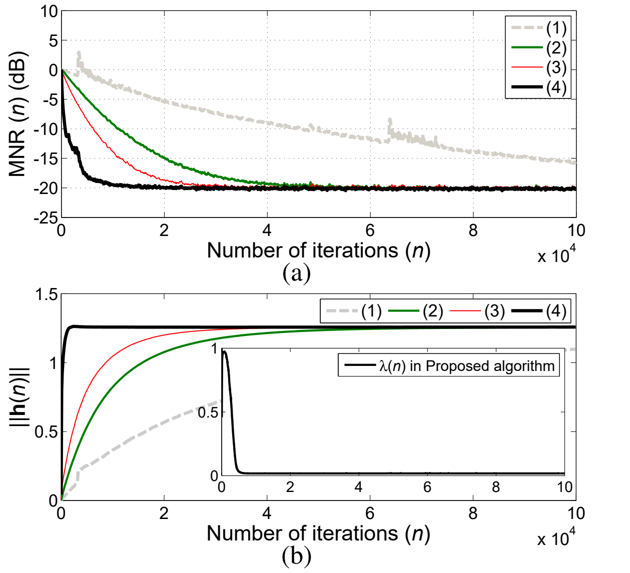

Fig. 5. Simulation results for mildly impulsive noise with $\alpha = 1.75$. (a) Mean noise reduction (MNR) curves for various algorithms, and (b) convergence of norm of noise control filter coefficients, $\|\boldsymbol{h}(n)\|$. [Step-size parameters: (1) Sun's algorithm ($\mu = 1 \times 10^{-6}$), (2) Th-FxLMS algorithm ($\mu = 1 \times 10^{-5}$), (3) INSS-FxLMS algorithm ($\mu = 1 \times 10^{-3}$), and (4) Proposed algorithm ($\mu_1 = 1.0, \mu_2 = 1 \times 10^{-2}$).]

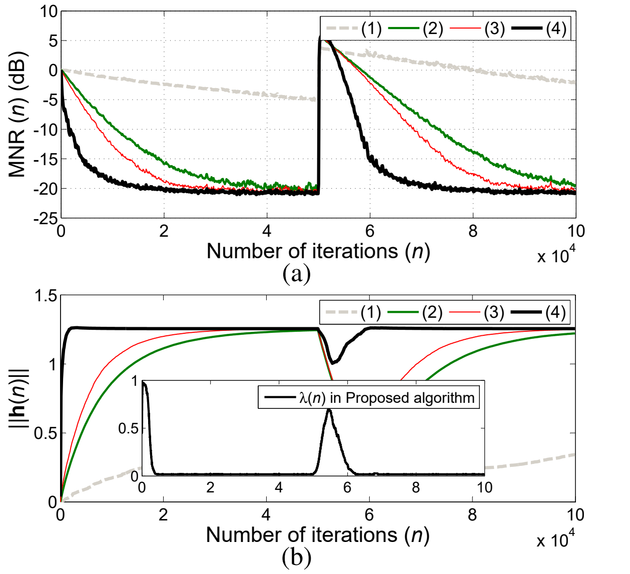

Fig. 6. Simulation results for moderately impulsive noise with $\alpha = 1.45$ with sudden change in the acoustic path. (a) Mean noise reduction (MNR) curves for various algorithms, and (b) convergence of norm of noise control filter coefficients, $\|\boldsymbol{h}(n)\|$. [Step-size parameters are selected the same values as given in the caption for Fig. 4.]

References

[1] S. M. Kuo, and D. R. Morgan, *Active Noise Control Systems-Algorithms and DSP Implementations*, New York: Wiley, 1996.

[2] ———, "Active Noise Control: A tutorial review," *Proc. IEEE*, vol. 87, pp. 943–973, Jun., 1999.

[3] M. Shao, and C. L. Nikias, "Signal processing with fractional lower order moments: stable processes and their applications," *Proc. IEEE*, vo. 81, no. 7, pp. 986–1010, Jul. 1993.

[4] R. Leahy, Z. Zhou, and Y. C. Hsu, "Adaptive filtering of stable processes for active attenuation of impulsive noise," in *Proc. IEEE ICASSP 1995*, vol. 5, May 1995, pp. 2983–2986.

[5] M. T. Akhtar, and W. Mitsuhashi, "Improving robustness of filtered-x least mean p-power algorithm for active attenuation of standard symmetric-$alpha$-stable impulsive noise", *Appl. Acoust.*, vol. 72, pp. 688694, 2011.

[6] M. T. Akhtar, "Fractional lower order moment based adaptive algorithms for active noise control of impulsive noise sources," *Journal Acoustical Society America*, vol. 132, no. 6, pp. EL456-EL462, Dec. 2012.

[7] P. Song, and H. Zhao, "Filtered-x generalized mixed norm (FXGMN) algorithm for active noise control, *Mechanical Systems and Signal Processing*, vol, 107, pp. 93104, 2018.

[8] X. Sun, S. M. Kuo, and G. Meng, "Adaptive algorithm for active control of impulsive noise," *Jr. Sound Vibr.*, vol. 291, no. 1-2, pp. 516–522, Mar. 2006.

[9] M. T. Akhtar, and W. Mitsuhashi, "Improving performance of FxLMS algorithm for active noise control of impulsive noise," *Jr. Sound Vibr.*, vol. 327, no. 3-5, pp. 647–656, Nov. 2009.

[10] M. T. Akhtar, and W. Mitsuhashi, "A modified normalized FxLMS algorithm for active control of impulsive noise," in *Proc. EUSIPCO 2010*, Aug 23-27, 2010, Denmark, pp. 1-5.

[11] M. T. Akhtar, M. Abe, and M. Kawamata, "A New variable step size LMS algorithm-based method for improved online secondary path modeling in active noise control systems," *IEEE Trans. Audio Speech Language Processing*, vol. 14, no. 2, pp. 720-726, Mar. 2006.

[12] J. Benesty, H. Rey, L. R. Vega, and S. Tressens, "A nonparameteric VSS NLMS algorithm," *IEEE Signal Lett.*., vol. 13, no. 10, pp. 581-584, 2006.

[13] L. R. Vega, H. Rey, J. Benesty, and S. Tressens, "A new robust variable step-size NLMS algorithm," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1878-1893, 2008.

[14] S. Zhang, and J. Zhang, "New steady-state analysis of variable step-size LMS algorithm with different noise disctributions," *IEEE Signal Lett.*., vol. 21, no. 6, pp. 653-657, 2014.

[15] S. Kozat, A. T. Erdogan, A. C. Singer, and A. H. Sayed, "Steady-state MSE performance analysis of mixture approaches to adaptive filtering," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 40504063, Aug. 2010.

[16] R. Candido, M. T. M. Silva, and V. H. Nascimento, "Transient and steady-state analysis of the affine combination of two adaptive filters," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4064-4078, 2010.

[17] M. Ferrer, A. Gonzalez, M. deDiego, and G. Piñero, "Convex combination filtered-x algorithms for active noise control systems," *IEEE Trans. Audio Speech Language Processing*, vol. 21, no. 1, pp. 156-167, Jan. 2013.

[18] N. V. George a, and A. Gonzalez, "Convex combination of nonlinear adaptive filters for active noise control", *Appl. Acoust.*, vol. 76, pp. 157161, 2014.

[19] L. A. Azpicueta-Ruiz, M. T. M. Silva, V. H. Nascimento, and A. H. Sayed, "Combinations of adaptive filters: performance and convergence properties," *IEEE Signal Process. Mag.*, vol. 33, no. 1, pp. 120-140, 2016.

[20] F. Huang, J. Zhang and Y. Pang, "A novel combination scheme of proportionate filter," *Signal Processing*, vol. 143, pp. 222-231, 2018.

[21] S. Zhang, W. X. Zheng, and J. Zhang , "A new combined-step-size normalized least mean square algorithm for cyclostationar inputs," *Signal Processing*, vol. 141, pp. 261-272, 2017.

[22] G. Pinte, W. Desmet, and P. Sas, "Active control of repititive transient noise," *Jr. Sound Vibr.*, vol. 307, pp. 513–526, 2007.