Iterative Weighted Least Squares Frequency Estimation for Harmonic Sinusoidal Signal in Power Systems

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***Abstract*—In this paper, a two-level iterative weighted least squares (TIWLS) method is proposed to estimate the voltage frequency in a balanced three-phase (3PH) power system with harmonic distortion. The novel TIWLS estimator exploits the weighted least squares (WLS) technique to reuse the discarded information of the previous harmonic Aboutanios and Mulgrew (HAM) algorithm. Consequently, the TIWLS method can reduce the main estimation error in HAM estimator caused by the maximum bin search. The entire two-step estimation scheme has the same order complexity as the fast Fourier transform (FFT) algorithm, which is computationally efficient. Simulation results are presented to test the TIWLS algorithm, demonstrating that the new TIWLS estimator always outperforms HAM method with less oscillation.**

***Index Terms*—Fundamental frequency estimation, harmonic distortion, weighted least squares, Fourier interpolation, three-phase power systems.**

I. Introduction

The voltage frequency is an important parameter for assuring the health of power networks. If the frequency deviates from the normalized value, the power flows between different generators and loads are redistributed to protect electronic equipments and maintain Power Quality (PQ) [1]. Consequently, the development of robust and accurate frequency estimators in power systems is necessary.

Based on the signal models, the state-of-art estimation methods in power systems can be categorised into two types: 3PH algorithms, such as augmented complex least mean squares (ACLMS) [2], total least squares (TLS) [3] and augmented complex Kalman filters (ACKFs) [4], and single-phase (1PH) algorithms, such as the zero-crossing method [5]. Since 3PH algorithms use all the information existing in three dimensions, they are more robust than 1PH methods. However, the aforementioned estimators only work well in the absence of harmonics. Due to the increasing use of non-linear loads, different levels of harmonics are inevitably embedded in the sampled voltage data. Therefore, algorithms that only consider the fundamental frequency cannot capture the full information in the actual power signal.

Using Clarke's α, β transformation that converts the 3PH signal model into a 1PH complex signal, popular multi-tone parametric estimators, such as MUltiple SIgnal Classification (MUSIC) [6] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [7], can obtain unbiased frequency estimation of the transformed exponential signal [8]. Yet the performance of these estimators can still be improved as they fail to exploit the harmonic structure of the power signals. Thus the improved weighted least squares (IWLS) algorithm [9], [10] is proposed to act as a further polishing step to reduce the estimation variance of MUSIC and ESPRIT estimators. Nevertheless, heavy computational cost ($O(N^3)$) for ESPRIT or more for MUSIC in order to calculate a dense spectral grid is the significant drawback of IWLS. On the other hand, most multi-tone parametric estimators impose heavy demands, such as numerous sampling points and high signal to noise ratio (SNR), which also severely limit the application.

In our previous work, a harmonic A&M (HAM) estimator [11] was proposed to estimate the voltage frequency by extending the single-tone frequency estimator proposed by Aboutanios and Mulgrew [12] (the A&M algorithm). Nevertheless, the major error of the HAM method stems from the rough estimation of each harmonic waveform at the initialized step. As a solution, in this paper we put forward a new two-level iterative weighted least squares (TIWLS) method to compensate the error caused by the HAM estimator. The new estimation framework combines the former HAM estimator and WLS technique to exploit the harmonic structure of the balanced power signal to yield an excellent performance.

The rest of the paper is organised as follows. We review the original HAM estimator in Section II. Next in Section III the novel TIWLS method is developed. The simulation results are given in Section IV and conclusions are drawn in Section V.

II. Harmonic A&M Estimator

In this section, we briefly review the HAM estimator [11]. The harmonically distorted observation model of a balanced

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3PH power signal can be expressed as [1], [13]

$$v_a(n) = \sum_{k=1}^{K} V_k \cos(2\pi k \frac{f_0}{f_s} n + k\phi) + w_a(n),$$
$$v_b(n) = \sum_{k=1}^{K} V_k \cos(2\pi k \frac{f_0}{f_s} n + k\phi - \frac{2\pi}{3}k) + w_b(n),$$
$$v_c(n) = \sum_{k=1}^{K} V_k \cos(2\pi k \frac{f_0}{f_s} n + k\phi + \frac{2\pi}{3}k) + w_c(n), \quad (1)$$

where $n = 0, 1, \ldots, N-1$ is the sampling time index and V_k is the magnitude of the kth harmonic component. The fundamental voltage frequency is f_0 with f_s being the sampling rate. $\phi \in [0, 2\pi)$ is the phase value. The noise terms $\{w_a(n), w_b(n), w_c(n)\}$ are assumed to be i.i.d. real Gaussian noise with zero mean and variance σ^2. The 3PH signal model in (1) can be transformed into a $\{\alpha, \beta, 0\}$ reference frame by an orthogonal transformation matrix as follows

$$\begin{bmatrix} v_0(n) \\ v_\alpha(n) \\ v_\beta(n) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(n) \\ v_b(n) \\ v_c(n) \end{bmatrix}. \quad (2)$$

(2) is widely recognised as Clarke's α, β transformation. In the balanced case, the direct-axis component $v_\alpha(n)$ and the quadrature-axis component $v_\beta(n)$ can be further combined to form a complex harmonic exponential signal $x(n)$,

$$x(n) = v_\alpha(n) + j v_\beta(n),$$
$$= \sum_{k=1}^{K} A_k e^{j l_k (2\pi f n + \phi)} + w(n), \; n = 0, \ldots, N-1, \quad (3)$$

where $l_k = [(-1)^{k-1}(6k-3)+1]/4$. $f = f_0/f_s \in [-0.5, 0, 5]$ is the normalised frequency and $A_k = V_{|l_k|}$. $w(n)$ are complex Gaussian noise terms with zero mean and variance $4\sigma^2/3$.

The HAM method is summarized in Table I, where we use $\hat{\lambda}$ to denote the estimate of λ. The algorithm starts by obtaining a coarse estimate, $\hat{f}$, of the normalised frequency through the maximum bin, $\hat{m}_0$, of the signal periodogram [14], given by

$$\hat{f} = \frac{\hat{m}_0}{N}, \quad \text{where} \quad \hat{m}_0 = \arg\max_m |X(m)|^2, \quad (4)$$

and $X(m)$ is the N-point fast Fourier transform (FFT) of $x(n)$. Next, we interpolate two new Fourier coefficients $\tilde{X}_\pm$ at frequencies $\hat{f} \pm 0.5/N$, which turns out

$$\tilde{X}_\pm = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n(\hat{f} \pm \frac{0.5}{N})},$$
$$= A_1 e^{j\phi} \frac{1 + e^{j2\pi N(f-\hat{f})}}{1 - e^{j2\pi(f-\hat{f})} e^{\mp j\frac{\pi}{N}}}$$
$$+ \sum_{k=2}^{K} A_k e^{j l_k \phi} \frac{1 + e^{j2\pi N(l_k f - \hat{f})}}{1 - e^{j2\pi(l_k f - \hat{f})} e^{\mp j \frac{\pi}{N}}} + W_\pm,$$
$$= X_\pm + \sum_{k=2}^{K} X_{\pm,k} + W_\pm. \quad (5)$$

Here $W_\pm$ are the Fourier coefficients of the noise term $w(n)$ at the interpolation locations and $X_\pm$ are the fundamental Fourier coefficients. $\{X_{\pm,k}\}_{k=2}^{K}$ are the later $K-1$ harmonic spectral leakages that can be reconstructed by replacing the true normalised frequency f with $\hat{f}$, given by

$$\hat{X}_{\pm,k} = \hat{A}_k e^{j l_k \hat{\phi}} \frac{1 + e^{j2\pi N \hat{f}(l_k - 1)}}{1 - e^{j2\pi \hat{f}(l_k-1)} e^{\mp j\frac{\pi}{N}}}, \; k = 2, \ldots, K. \quad (6)$$

In noiseless case, $X_\pm$ in (5) can be recovered by subtracting $\{\hat{X}_{\pm,k}\}_{k=2}^{K}$ from $\tilde{X}_\pm$. Finally, $\hat{f}$ can be refined by $\Im\{\ln(\hat{z})\}/(2\pi) + \hat{f}$, where

$$\hat{z} = \left[\cos\frac{\pi}{N} - j \frac{\hat{X}_+ + \hat{X}_-}{\hat{X}_+ - \hat{X}_-} \sin\frac{\pi}{N} \right]^{-1}, \quad (7)$$

and $\Im\{\bullet\}$ is the imaginary part of $\bullet$. On the other hand, $\{A_k\}_{k=1}^{K}$ and ϕ can be easily estimated by solving the following Least Squares (LS) problem:

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} ||\mathbf{x} - \mathbf{Z}(\hat{f})\mathbf{a}||,$$
$$= [\mathbf{Z}^H(\hat{f})\mathbf{Z}(\hat{f})]^{-1}\mathbf{Z}^H(\hat{f})\mathbf{x}, \quad (8)$$

where

$$\mathbf{a} = [A_1 e^{j\phi}, A_2 e^{j l_2 \phi}, \cdots, A_K e^{j l_K \phi}]^T,$$
$$\mathbf{x} = [x(0), x(1), \cdots, x(N-1)]^T,$$
$$\mathbf{Z}(f) = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_K],$$

and

$$\mathbf{z}_k = [1, e^{j2\pi l_k f}, \cdots, e^{j2\pi(N-1) l_k f}]^T.$$

The estimated values of amplitudes and the fundamental phase are given by $|\hat{\mathbf{a}}|$ and $\angle\hat{\mathbf{a}}(1)$ respectively. To further improve the estimation performance, the refined process is implemented for Q ($Q \geq 2$) iterations.

TABLE I
THE HAM ESTIMATOR [11]

Given	A length-N complex harmonic signal $x(n)$;
Calculate	$X(m) = \text{FFT}\{x(n)\}$, $m = 0, 1, \ldots, N-1$;
Find	$\hat{m}_0 = \arg\max_m |X(m)|^2$;
Initialise	$\hat{f} = \frac{\hat{m}_0}{N}, \{\hat{A}_k\}_{k=1}^{K} = \hat{\phi} = 0$;
Do	For $q = 1$ to Q, loop:
	1. $\tilde{X}_\pm = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n(\hat{f} \pm \frac{0.5}{N})}$;
	2. $\hat{X}_{\pm,k} = \hat{A}_k e^{j l_k \hat{\phi}} \frac{1 + e^{j2\pi N \hat{f}(l_k-1)}}{1 - e^{j2\pi \hat{f}(l_k-1)} e^{\mp j \frac{\pi}{N}}}$;
	3. $\hat{X}_\pm = \tilde{X}_\pm - \sum_{k=2}^{K} \hat{X}_{\pm,k}$;
	4. $\hat{z} = \left[\cos\left(\frac{\pi}{N}\right) - j\frac{\hat{X}_+ + \hat{X}_-}{\hat{X}_+ - \hat{X}_-} \sin\left(\frac{\pi}{N}\right)\right]^{-1}$;
	5. $\hat{f} = \frac{\Im\{\ln(\hat{z})\}}{2\pi} + \hat{f}$;
	6. $\hat{\mathbf{a}} = [\mathbf{Z}^H(\hat{f})\mathbf{Z}(\hat{f})]^{-1}\mathbf{Z}^H(\hat{f})\mathbf{x}$;
Finally	$\hat{f}_0 = \hat{f} f_s$, $\hat{A}_1 = |\hat{\mathbf{a}}(1)|$ and $\hat{\phi} = \angle\hat{\mathbf{a}}(1)$.

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III. The Two-level Iterative Weighted Least Squares estimator

The main drawback of the HAM estimator is that it is developed only based on the fundamental frequency f_0. Specifically, the unexpected leakage $\{X_{k,\pm}\}_{k=2}^{K}$ in the first iteration is just roughly estimated by searching the maximum bin of the signal periodogram. However, discarding the useful information involved in the latter $K-1$ harmonics prevents the algorithm for achieving the full performance. Consequently, the HAM estimator requires relatively large amounts of data and high SNR values to maintain accuracy frequency estimation, which restricts its application. To solve this problem, a two-level iterative weighted least squares (TIWLS) estimator is proposed and summarized in Table II. The modified method combines a variant HAM estimator with the weighted least squares (WLS) technique, which makes full use of the information existing in all harmonics.

Let us start from the $\tilde{K}$th outer loop. In order to obtain the independent frequency estimate of the $\tilde{K}$th harmonic, a reduced signal $x_{\tilde{K}}(n)$ is generated by removing the previously estimated $(\tilde{K}-1)$th harmonic component. The frequency index set $\{l_k\}$ is also updated by deleting $\{l_1, l_2, \ldots, l_{\tilde{K}-1}\}$ and normalizing the remaining indices by $l_{\tilde{K}}$ to give $\breve{l}_p = l_k / l_{\tilde{K}}$, where $k = \tilde{K}, \tilde{K}+1, \ldots, K$ and $p = 1, \ldots, K-\tilde{K}+1$. Similarly, the updated amplitude $\breve{A}_p$ and phase $\phi_{\tilde{K}}$ can form a new complex amplitude vector $\mathbf{a}_{\tilde{K}} = [\breve{A}_1 e^{j\phi_{\tilde{K}}}, \breve{A}_2 e^{j\breve{l}_2\phi_{\tilde{K}}}, \cdots, \breve{A}_{K-\tilde{K}+1} e^{j\breve{l}_{K-\tilde{K}+1}\phi_{\tilde{K}}}]^T$. Treating the component $f_{\tilde{K}} = l_{\tilde{K}} f$ as the "new fundamental frequency", the inner loops starts by implementing the HAM estimator to $x_{\tilde{K}}(n)$ for Q $(Q \geq 2)$ iterations to obtain $\hat{f}_{\tilde{K}}$. Combining what we get in the past $\tilde{K}-1$ outer loops, the independently estimated frequency vectors are represented as $\hat{\boldsymbol{f}} = \left[\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_{\tilde{K}}\right]^T$.

Now, the estimated harmonics can form a "new exponential signal" $v(n)$, satisfying

$$v(n) = \sum_{k=1}^{\tilde{K}} A_k e^{j l_k (2\pi f n + \phi)} + w(n), \; n = 0, \ldots, N-1. \quad (9)$$

Based on the multiple relation given by the index set $\{l_k\}$, the estimated fundamental frequency $\hat{f}$ in last $(\tilde{K}-1)$th outer loop can be further refined by $\hat{\boldsymbol{f}}$. Furthermore, noting that each harmonic component has a different amplitude and therefore a different SNR, we combine the estimated frequency vectors $\hat{\boldsymbol{f}} = \left[\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_{\tilde{K}}\right]^T$ by a WLS matrix. The corresponding cost function is given as

$$\begin{aligned} J(\boldsymbol{\eta}) &= \| \hat{\boldsymbol{f}} - \boldsymbol{\eta} \|_{\mathbf{M}}^2, \\ &= \left[\hat{\boldsymbol{f}} - \boldsymbol{\eta}\right]^T \mathbf{M} \left[\hat{\boldsymbol{f}} - \boldsymbol{\eta}\right], \end{aligned} \quad (10)$$

where $\boldsymbol{\eta} = [l_1 f, l_2 f, \cdots, l_{\tilde{K}} f]^T$ is the vector of true harmonic frequencies and $\mathbf{M}$ is the weighting matrix. In this paper, we choose the inverse version of harmonic Cramér-Rao Lower Bound (CRLB) as the weighting matrix to reflect the varying quality of the elements of $\hat{\boldsymbol{f}}$. The final refined frequency f is given by

$$f = \arg\min_{f} J(\boldsymbol{\eta}). \quad (11)$$

Recall that the signal $v(n)$ in (9) can be rewritten in the noiseless case as

$$\mathbf{v} = \mathbf{R}(\boldsymbol{\theta}), \quad (12)$$

and that $\mathbf{v} = [v(0), \cdots, v(N-1)]^T$ and $\mathbf{R} = \mathbf{D}\tilde{\mathbf{a}}$, where $\mathbf{D} = [\mathbf{d}_1, \cdots, \mathbf{d}_{\tilde{K}}]$ and $\mathbf{d}_k = [1, e^{j l_k 2\pi f}, \cdots, e^{j l_k (N-1) 2\pi f}]^T$. $\tilde{\mathbf{a}} = [A_1 e^{j\phi}, \cdots, A_{\tilde{K}} e^{j l_{\tilde{K}} \phi}]^T$. Based on Slepian-Bangs formula [15], the weighting matrix $\mathbf{M}$ can be calculated as

$$\mathbf{M} = \mathrm{CRB}^{-1}(\boldsymbol{f}) = \frac{3}{2\sigma^2} \Re\left(\frac{\partial \mathbf{R}^H}{\partial \boldsymbol{f}} \frac{\partial \mathbf{R}}{\partial \boldsymbol{f}^T}\right), \quad (13)$$

where $\Re\{\bullet\}$ is the real part of $\bullet$ and $\{\bullet\}^H$ is the Hermitian transpose operator. Based on simple mathematical calculation, it turns out that

$$\hat{\mathbf{M}} = \frac{2}{\sigma^2} \Re \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,\tilde{K}} \\ y_{2,1} & y_{2,2} & \cdots, & y_{2,\tilde{K}} \\ \vdots & \vdots & \ddots & \vdots \\ y_{\tilde{K},1} & y_{\tilde{K},2} & \cdots & y_{\tilde{K},\tilde{K}} \end{bmatrix}, \quad (14)$$

where

$$\begin{aligned} y_{m,n} = \hat{A}_m \hat{A}_n e^{j\hat{\phi}(l_n - l_m)} \Big[& e^{j2\pi \hat{f}(l_n - l_m)} + 2^2 e^{j4\pi \hat{f}(l_n - l_m)} \\ & + \cdots + (N-1)^2 e^{j2(N-1)\pi \hat{f}(l_n - l_m)} \Big]. \end{aligned} \quad (15)$$

Finally, the refined fundamental frequency $\hat{f}$ can be obtained by solving the equation $\partial J(\boldsymbol{\eta}) / \partial f = 0$, which turns out:

$$\hat{f} = \frac{\sum_{u=1}^{\tilde{K}} \hat{f}_u \sum_{k=1}^{\tilde{K}} \Re\{y_{u,k}\} l_k}{\sum_{u=1}^{\tilde{K}} l_u \sum_{k=1}^{\tilde{K}} \Re\{y_{u,k}\} l_k}. \quad (16)$$

The above process is implemented for K loops after all available harmonics are exploited, and the estimated parameters including $\hat{f}$, $\{\hat{A}\}_{k=1}^{K}$ and $\hat{\phi}$ are renewed at the end of each outer loop.

IV. Simulation

Simulation results are presented in this section to verify the performance of the proposed TIWLS algorithm.

Firstly, we apply TIWLS estimator to a set of generalised harmonic signals $y(n)$ for performance demonstration:

$$y(n) = \sum_{k=1}^{K} \frac{e^{j2\pi k f n}}{2^{(k-1)}} + w(n), \; n = 0, \ldots, 63, \quad (17)$$

where $w(n)$ are complex Gaussian noise terms with zero mean and variance σ^2. f is chosen randomly in $[4/64, 5/64]$ and SNR is fixed as 10dB. In this case, asymptotic WLS (AWLS) algorithm [9] is chosen as the contrast method. The root mean square errors (RMSEs) of $\hat{f}$ are shown in Fig. 1 with 10,000 Monte Carlo (MC) runs. Here TIWLS is run for 2, 4 and 6 iterations, and HAM is run for 2 and 4 iterations. Note that AWLS becomes unreliable when the number of harmonic components in the test signal is larger than 3, whereas the

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TABLE II
THE PROPOSED TIWLS ESTIMATOR

Given	A length-N complex harmonic signal $x(n)$;
Initialise	$x_0(n) = x(n), \{\hat{A}_k\}_{k=0}^{K} = \hat{\phi} = \hat{f} = l_0 = 0$;
Loop	Outer loop starts. For $\tilde{K} = 1$ to K, do:
	1. $x_{\tilde{K}}(n) = x_{\tilde{K}-1}(n) - \hat{A}_{\tilde{K}-1} e^{j l_{K-1}(2\pi \hat{f} n + \hat{\phi})}$;
	2. $\check{l}_p = l_k / l_{\tilde{K}}$, $k = \tilde{K}, \tilde{K}+1, \ldots, K$ and $p = 1, \ldots, K - \tilde{K} + 1$;
	3. $X(m) = \text{FFT}\{x_{\tilde{K}}(n)\}$, $m = 0, 1, \ldots, N-1$;
	4. $\hat{f}_{\tilde{K}} = \frac{\hat{m}_0}{N}$, where $\hat{m}_0 = \arg\max_{m} \lvert X(m)\rvert^2$ and $\{\check{A}_k\}_{k=1}^{K-\tilde{K}+1} = \hat{\phi}_{\tilde{K}} = 0$;
	5. Inner loop starts. For $q = 1$ to Q, do:
	5.1. $\tilde{X}_{\pm} = \sum_{n=0}^{N-1} x_{\tilde{K}}(n) e^{-j2\pi n(\hat{f}_{\tilde{K}} \pm \frac{0.5}{N})}$;
	5.2. $\hat{X}_{\pm,k} = \check{A}_k e^{j \check{l}_k \hat{\phi}_{\tilde{K}}} \frac{1 + e^{j2\pi N \hat{f}_{\tilde{K}}(\check{l}_k - 1)}}{1 - e^{j2\pi \hat{f}_{\tilde{K}}(\check{l}_k - 1)} e^{\mp j \frac{\pi}{N}}}$
(**If** $\tilde{K} < K$)	5.3. $\hat{X}_{\pm} = \tilde{X}_{\pm} - \sum_{k=2}^{K-\tilde{K}+1} \hat{X}_{\pm,k}$;
	5.4. $\hat{z} = \left[\cos\left(\frac{\pi}{N}\right) - j \frac{\hat{X}_+ + \hat{X}_-}{\hat{X}_+ - \hat{X}_-} \sin\left(\frac{\pi}{N}\right)\right]^{-1}$;
	5.5. $\hat{f}_{\tilde{K}} = \frac{\Im\{\ln(\hat{z})\}}{2\pi} + \hat{f}_{\tilde{K}}$;
	5.6. $\hat{\mathbf{a}}_{\tilde{K}} = [\hat{\mathbf{Z}}_{\tilde{K}}^H(\hat{f}_{\tilde{K}}) \hat{\mathbf{Z}}_{\tilde{K}}(\hat{f}_{\tilde{K}})]^{-1} \hat{\mathbf{Z}}_{\tilde{K}}^H(\hat{f}_{\tilde{K}}) \mathbf{x}_{\tilde{K}}$;
	Inner loop ends;
(**If** $\tilde{K} > 1$)	6. $\hat{f} = \frac{\sum_{u=1}^{\check{K}} \hat{f}_u \sum_{k=1}^{\check{K}} \Re\{y_{u,k}\} l_k}{\sum_{u=1}^{\check{K}} l_u \sum_{k=1}^{\check{K}} \Re\{y_{u,k}\} l_k}$;
	7. $\hat{\mathbf{a}} = [\mathbf{Z}^H(\hat{f}) \mathbf{Z}(\hat{f})]^{-1} \mathbf{Z}^H(\hat{f}) \mathbf{x}$;
	Outer loop ends;
Finally	$\hat{f}_0 = \hat{f} f_s$, $\hat{A}_1 = \lvert\hat{\mathbf{a}}(1)\rvert$ and $\hat{\phi} = \angle \hat{\mathbf{a}}(1)$.

estimation error of TIWLS and HAM do remain flat. In addition, TIWLS can always outperform HAM, which meets the discussion at the beginning of section III. We also see that TIWLS requires more number of iterations than HAM to show no improvement, which is due to the fact that SNRs of high harmonic orders are low. Thus more number of iterations are needed by the variant HAM of TIWLS to reach best estimation.

In the second set of simulations, we construct the power signal based on the Australian standard AS/NZS 61000.2.2 [16] and the power signal model in (1) to test the proposed TIWLS algorithm. The fundamental frequency f_0 is 50Hz and the sampling frequency f_s is $4,000$Hz, which means that there are 80 samples in a single cycle. The phase ϕ is chosen as $10°$. The comparative amplitudes for individual harmonic voltages in each phase are shown in Table III. Two exponential estimation methods in power systems are chosen as the comparison, namely subspace [8] and IWLS methods [10]. Figs. 2 to 4 show the RMSEs of $\hat{f}_0$, $\hat{\phi}$ and $\hat{V}_1$ versus SNR obtained by various methods in balanced 3PH signal model when $N = 64$. Here we set $Q = 4$ for both TIWLS and HAM estimators. Observe that the RMSEs of TIWLS set on the CRLB at SNR $\geq$ 0dB. The other methods, on the contrary, exhibit high SNR thresholds below which the estimates are not reliable. This is because that the fundamental amplitude is much larger than that of the other $K-1$ harmonics. And subspace and IWLS depend on all harmonic components, while TIWLS and HAM start by utilising the Fourier coefficients close to the fundamental frequency. Furthermore, since TIWLS reuses the discarded information of HAM, the RMSEs of TIWLS are lower than that of HAM.

Next, simulations are presented employing a balanced 3PH power system where the available data points are limited within one cycle. The SNR is fixed to 60dB and the other configurations are kept to be the same as the previous test. In Fig. 5, we find the RMSEs of HAM approaches CRLB quicker than subspace or IWLS methods, especially when the data record N is short. Meanwhile, TIWLS converges even faster than HAM with less estimation bias. In other words, it means that HAM has a good tracking performance and such property is further strengthened by TIWLS.

Finally, the frequency tracking performance of TIWLS and HAM is shown in Fig. 6 for $Q = 4$, where the signal frequency is affected by a ± 0.5 Hz changing. Since both TIWLS and HAM are block based algorithm, we introduce a sliding window with 128 samples and the unknown frequency is estimated based on this sliding window. Fig. 6 shows that both TIWLS and HAM can achieve acceptable frequency estimation. However, there are clear oscillations when using HAM, which can be reduced in the performance of TIWLS.

TABLE III
THE HARMONIC AMPLITUDES IN EACH 1PH POWER SIGNAL

Order	1	5	7	11	13	17
Amplitude	1	0.06	0.05	0.032	0.03	0.02

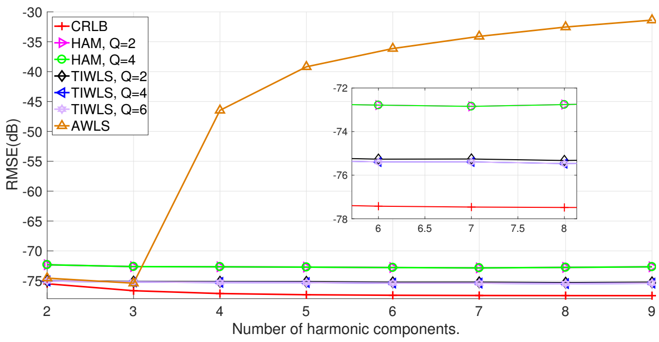

Fig. 1. RMSEs of $\hat{f}$ versus the number of harmonic components.

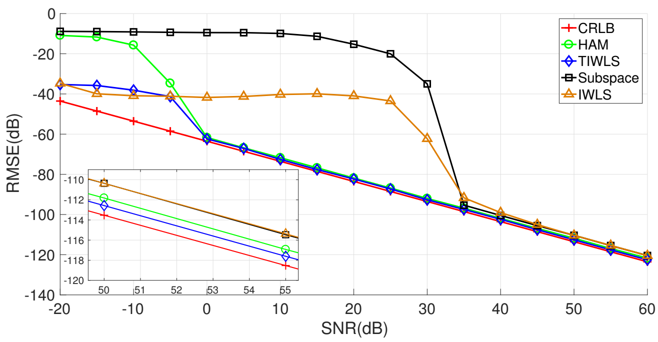

Fig. 2. RMSEs of $\hat{f}_0$ versus SNR with 10,000 MC runs.

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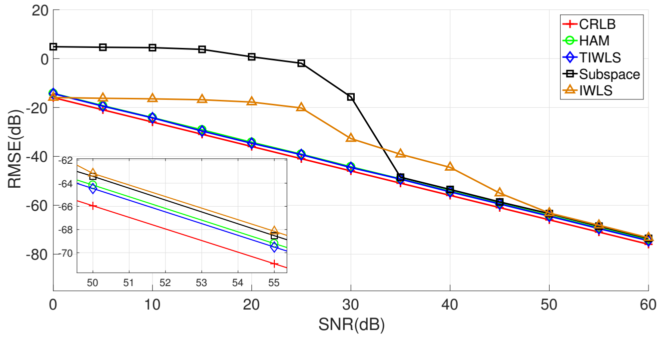

Fig. 3. RMSEs of $\hat{\phi}$ versus SNR with 10,000 MC runs.

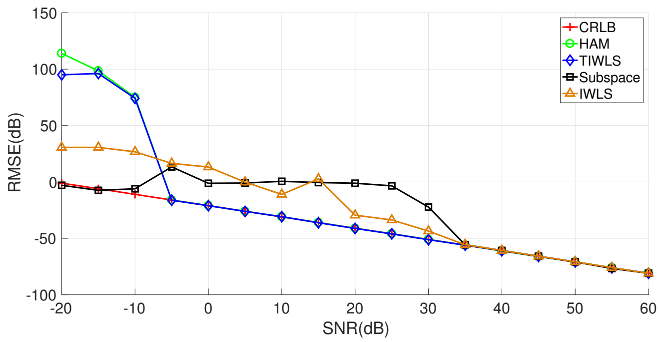

Fig. 4. RMSEs of $\hat{V}_1$ versus SNR with 10,000 MC runs.

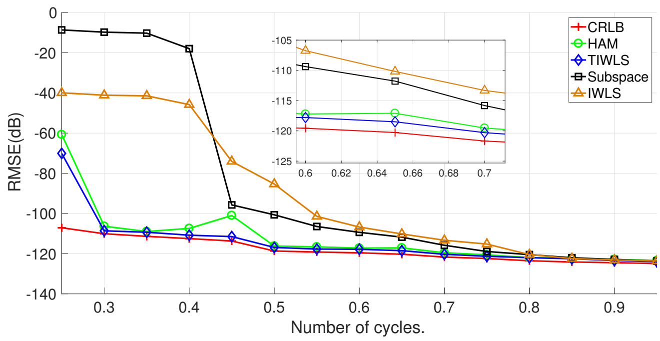

Fig. 5. RMSEs of $\hat{f}_0$ versus the number of cycles with 10,000 MC runs.

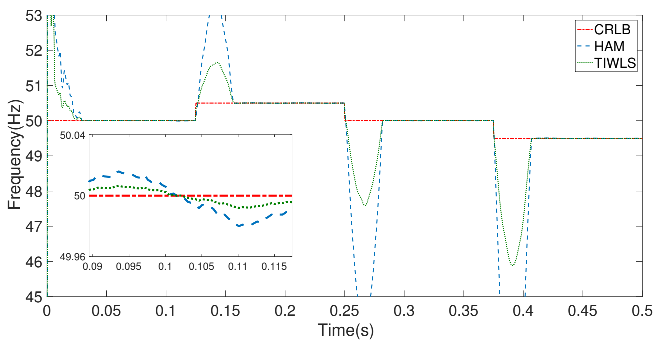

Fig. 6. Frequency tracking performance for $\sigma^2 = 10^{-6}$.

V. Conclusion

A TIWLS estimator is developed in this paper to estimate the fundamental voltage frequency in balanced 3PH power systems with harmonic distortion. We begin by estimating all harmonic frequencies separately. Then a refinement is performed to combine the resulting independent harmonic frequency estimates by the CRLB based weighting matrix. The TIWLS method can eliminate the estimation errors caused by interfering harmonics to yield an excellent performance. Meanwhile, the two-level Iterative scheme has the same order complexity as the FFT algorithm, which is computationally efficient. Simulation results show that the proposed TIWLS estimator can obtain better performance than the former HAM method.

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