Decision statistics for noncoherent signal detection in multi-element antenna arrays

Olesya Bolkhovskaya
Lobachevsky University of Nizhny Novgorod,
Nizhny Novgorod, Russia
Email: obol@rf.unn.ru

Alexander Maltsev
Lobachevsky University of Nizhny Novgorod
and Intel Corporation,
Nizhny Novgorod, Russia
Email: alexander.maltsev@intel.com

***Abstract*—This article gives a detailed analysis of the characteristics of the exact statistic for the optimal noncoherent signal detection in multi-element antenna arrays. This task is important for reliable initial detection of User Equipment (UE) signals coming from an unknown direction in LTE random access procedure. It is shown that application of the exact statistic is complex because the thresholds of the Neyman-Pearson (NP) criterion depend on the SNR. A detailed comparison of the exact and various approximate decision statistics has been carried out. The results show that the choice of the statistic can make noticeable impact on the probability of missed detection for multi-element antenna array. A new combined decision statistic has been proposed, whose characteristics are close to the exact statistic.**

***Index Terms*—noncoherent detection, decision statistics, multi-element antenna arrays,**

I. Introduction

In this paper, we present a detailed theoretical and numerical analysis of the characteristics of the exact decision statistic and its approximations for noncoherent detection of the useful signal in multi-element antenna arrays. In the past decade, noncoherent detection schemes have attracted considerable interest because massive antenna array technologies were introduced in different wireless communication systems [1]- [2]. Noncoherent detection does not require *a priori* information about the wavefront of the useful signal in contrast with coherent detection in which the optimal antenna array radiation pattern is formed in accordance with the wavefront of the incoming useful signal. Therefore, the implementation of noncoherent signal detection scheme needs less a priori information and, it is commonly used for the initial detection of new unknown signal sources. For example, different robust noncoherent detection schemes were investigated with respect to cognitive radio and satellite broadcasting systems, working in conditions of a priori uncertainty about signal and interference parameters [3] - [5].

In the classical works [6]- [8] a Bayesian approach to the noncoherent detection problem was usually used, where the conditional likelihood ratio (LR) was averaged over unknown random parameters of the signal (phases and amplitudes). However another more complex general method, the so-called Generalized Likelihood Ratio Test (GLRT), may be also exploited for detection of the signals with unknown wavefront spatial characteristics [9]- [11]. In the GLRT approach, maximum likelihood estimations of the unknown parameters are inserted in the LR to form a decision statistic.

In the problem statement, considered here, it is assumed that the phases of the useful signal on the antenna elements are a priori unknown, and the amplitudes are constant. This assumption fairly well describes a few important practical cases of initial detection of signals coming from an unknown direction, signals having an arbitrary shape of the wavefront, signals received by a distributed antenna system (D-MIMO) whose geometry is not precisely known, etc. The considered case is realized not only for the simple line-of-sight (LOS) channel but this assumption can also be applied to multipath channels, if the detection is performed on the base of the most powerful channel ray. As far as we know, until now there have been neither an accurate theoretical analysis of the exact statistic in the considered case nor a detailed comparison of the exact statistic performance characteristics with the approximate statistics widely used in practice. That is because, when the amplitudes of the signals in the antenna array elements (or pulses in the train in radars) are constant, the use of the exact statistic is very difficult because it is expressed as a product of special functions, and its thresholds, even when using the NP criterion, depend on the SNR value. Therefore, in all publications on the noncoherent detection problem, known to the authors, only characteristics of different approximate decision statistics were investigated.

Unlike in previous works, in this paper we consider the exact statistic behavior in the observation space of the signals at the outputs of noncoherent matched filters which provide the initial signal processing in each element of the antenna array. The deformation of the multidimensional hypersurface dividing the observation space into two decision regions is investigated for the NP criterion depending on the probability of false alarm and the SNR value.

We present the problem statement in Section II and give a detailed analysis of performance characteristics of the exact statistic for noncoherent signal detection in Section III. Next, approximate desision statistics widely used in practice are described in Section IV,where the new combined statistic is also introduced. A comparison of all considered statistics is done in Section V and in Section VI final conclusions are made.

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II. PROBLEM STATEMENT

Consider the classical problem of detecting a deterministic useful signal with unknown parameters by an antenna array system with M elements. Assume that the useful signal is narrowband in the sense that the propagation time of the signal at the aperture of the antenna system is much less than the duration of one signal symbol. Consider the case when the useful signal amplitudes at the antenna elements are the same, but the phases ψ_m, $m = 1, ..., M$ are random IID values with uniform probability density function $W(\psi_m)$.

Following the Bayesian approach [12]- [15], the exact statistic may be derived by averaging the conditional LR over the random phases $\boldsymbol{\psi} = (\psi_1, \psi_2, ...\psi_M)^T$. It is assumed that a complex vector of the received signals in the antenna elements $\boldsymbol{x}[n] = \{x_1[n], x_2[n], ..., x_M[n]\}^T$ is a discrete-time $(n = 1, ..., N)$ data set. Then the conditional LR for an M-element antenna array can be represented in the following form

$$\Lambda(\boldsymbol{x}/\boldsymbol{\psi}) = \\ = \frac{\frac{1}{\pi^{MN}\sigma^{2MN}} exp[\frac{1}{\sigma^2}\sum_{n=1}^{N}(\boldsymbol{x}[n]-\boldsymbol{s}[n,\boldsymbol{\psi}])^{\dagger}(\boldsymbol{x}[n]-\boldsymbol{s}[n,\boldsymbol{\psi}])]}{\frac{1}{\pi^{MN}\sigma^{2MN}} exp[\frac{1}{\sigma^2}\sum_{n=1}^{N}\boldsymbol{x}[n]^{\dagger}\boldsymbol{x}[n]]} = \\ = \prod_{m=1}^{M}\Lambda_m(\mathbf{x_m}/\psi_m), \tag{1}$$

where $\boldsymbol{s}[n,\boldsymbol{\psi}] = (s_1[n,\psi_1], s_2[n,\psi_2], ..., s_M[n,\psi_M])^T$ is a complex useful signal vector with phases ψ_m, σ^2 is a variance of internal Gaussian IID noise in the antenna elements, † is the transpose conjugate operator. The useful signal in the m-th antenna element can be represented as

$$s_m[n,\psi_m] = Aa[n]e^{j\psi_m}, \quad m = 1, ..., M, \tag{2}$$

where $a[n]$ is a normalized modulation function of the useful signal and A is the amplitude coefficient characterizing the energy of the received signal. It is easy to show that the unconditional (averaged over the random phases) multidimensional LR can be written as follows:

$$\Lambda = \Lambda(\boldsymbol{x}) = \prod_{m=1}^{M}\int\Lambda_m(\boldsymbol{x}_m/\psi_m)W(\psi_m)d\psi_m \\ = \prod_{m=1}^{M}\Lambda_m(\boldsymbol{x}_m) \tag{3}$$

where the one-dimensional m-th LR is equal to

$$\Lambda_m(\boldsymbol{x}_m) = e^{-\frac{A^2}{\sigma^2}\sum_{n=1}^{N}a^2[n]}I_0(\frac{A}{\sigma^2}Y_m), \; Y_m = |\sum_{n=1}^{N}x_m[n]a^*[n]|. \tag{4}$$

Here $I_0(z)$ is the modified Bessel function of zero order of the argument z, and the Y_m are the signals amplitudes in the outputs of noncoherent matched filters (or of correlators of the known reference signal $a[n]$ with received signals $x_m[n]$). Taking into account (3) and (4) the optimal solution of the noncoherent detection problem [15] is reduced to calculation of the unconditional LR (Λ-*statistic*) and comparing it with the threshold Λ_{th}, in accordance with the given optimality criterion:

$$\Lambda(x) = \prod_{m=1}^{M}\Lambda_m(\boldsymbol{x}_m) = e^{-\frac{A^2M}{\sigma^2}}\prod_{m=1}^{M}I_0(\frac{A}{\sigma^2}Y_m)\gtrless\Lambda_{th}. \tag{5}$$

The following sections provide the results of detailed investigations of this Λ-statistic characteristics by using the NP criterion.

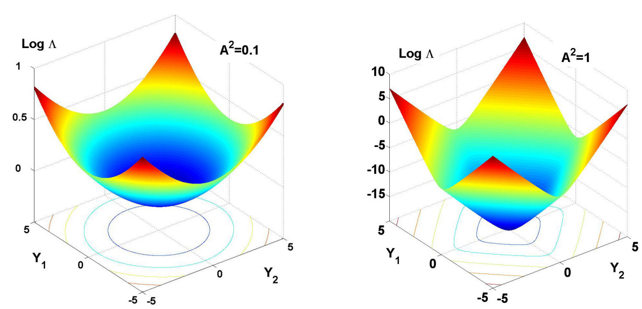
III. Λ-STATISTIC PERFORMANCE CHARACTERISTICS

It is convenient to investigate the exact statistic $\Lambda(x)$ (5) not in the space of the signals at the input of the antenna array $\mathbf{x}[n] = \{x_1[n], x_2[n], ..., x_M[n]\}^T$ but in the space of variables $\mathbf{Y} = \{Y_1, Y_2, ..., Y_M\}^T$, observed at the noncoherent matched filter outputs. Then Eq. (5) can be rewritten in the form:

$$\Lambda = \Lambda(Y) = \prod_{m=1}^{M}\Lambda_m(Y_m) = \prod_{m=1}^{M}\frac{W(Y_m/H_1)}{W(Y_m/H_0)}, \tag{6}$$

where $W(Y_m/H_0)$ and $W(Y_m/H_1)$ are Rayleigh and Rice distributions respectively in accordance with made assumptions.

The Λ-statistic depends on the energy parameter A, and depending on the value of this parameter, its behavior changes qualitatively. Fig. 1 shows the surfaces and level curves of the logarithm of Λ-statistic (Log Λ) as a function of Y_m arguments for two-dimensional case (M=2) for different $SNR = A^2/\sigma^2$.



a) b)

Fig. 1. Λ-statistic surfaces for different SNRs: a). SNR=-10 dB, b). SNR=10 dB. For illustrative purposes the functions are given for the full space of variables Y_1 and Y_2 including negative values.

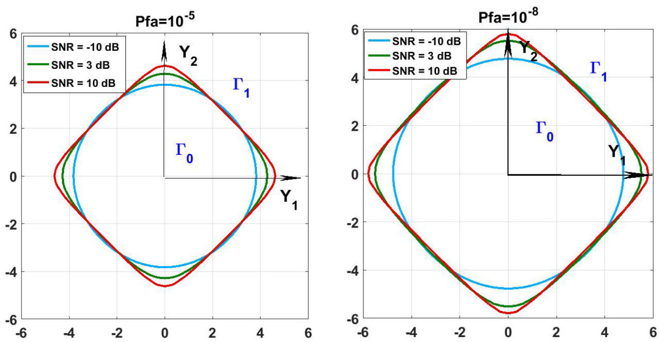
It is seen that the Λ-statistic in the space $\mathbf{Y}$ for low SNR is well approximated by an axial symmetric function depending on the sum of the squares of the amplitudes $\sum_{m=1}^{M}Y_m^2$, and for large SNR by a certain function depending on the sum of their modules $\sum_{m=1}^{M}|Y_m|$. Indeed, in the general case of arbitrary dimension M, using the well-known power-series expansions [16] of the Bessel function $I_0(z)$ for small and large values of z it is possible to obtain the following approximate expressions from Eq. (5):

$$\Lambda \approx e^{-\frac{A^2}{\sigma^2}\sum_{i=1}^{n}a^2[n]}\sum_{m=1}^{M}Y_m^2 \sim \sum_{m=1}^{M}Y_m^2 \tag{7}$$

$$\Lambda \approx e^{-\frac{A^2}{\sigma^2}\sum_{i=1}^{n}a^2[n]}\frac{e^{\frac{A}{\sigma^2}\sum_{m=1}^{M}Y_m}}{(2\pi\frac{A}{\sigma^2})^{\frac{m}{2}}\prod_{m=1}^{M}\sqrt{Y_m}} \sim \sum_{m=1}^{M}Y_m, \tag{8}$$

where (7) applies for small, and (8) - for large values of SNR.

To solve the optimal detection problem for the NP criterion it is necessary to find the thresholds Λ_{th} for the Λ-statistic for a given false alarm probability P_{FA}. However, the analysis shows that in the present case, these thresholds depend on the SNR values which, in general, is untypical for the NP criterion [15]. To explain the causes of this dependence let us consider the simplest two-dimensional case (M=2) as an illustrative example. Fig. 2 shows the division of the space $\mathbf{Y}$ into the decision regions: Γ_0 - signal presence and Γ_1 - signal absence for different SNRs and fixed values $P_{FA} = 10^{-5}, 10^{-8}$. It can be clearly seen from the graphs that to maximize the probability of the right detection (P_{RD}) at a fixed P_{FA}, it is necessary to change the shape of the division boundary lines L_Λ when the SNR is changing. Similar behavior of the division boundary will be also observed for the multidimensional case.



a) b)

Fig. 2. The division boundary lines L_Λ for two-dimensional observation space $\mathbf{Y}$ ($M = 2$) for different SNR values: -10dB, 3dB, 10dB.

Using the exact Λ-statistic, detection curves (dependencies P_{RD} on the SNR) have been obtained for antenna arrays with a different number of elements. It should be noted, that the threshold values for the Λ-statistic can be found only numerically for a given SNR due to the complexity of Eq. (5). Fig. 3 shows how the P_{RD} depends on the increase in the number of antenna elements for the $P_{FA} = 10^{-5}$. It is seen that the gain for the $P_{RD} \approx 0.9$ used in practice increases by 2 dB as the number of elements doubles. This coincides with the approximate estimates given in the known works for noncoherent detection in radar applications [8], [17]. However, more accurate calculations show that increasing the number of elements in the antenna array reduces this gain from 2.2 dB, for doubling the number of elements from 2 to 4, to 1.9 dB , for doubling the number of elements from 32 to 64, see Fig.2. Also, graphs clearly show that increasing antenna elements can significantly increase the P_{RD}, even for noncoherent detection (without beamforming to DoA of the useful signal). For example, as seen in Fig.2, for $SNR = 0dB$, the right detection probabilities are: $P_{RD} = 0.15$ for a 16-element antenna array, $P_{RD} = 0.55$ for a 32-element and $P_{RD} = 0.97$ for a 64-element.

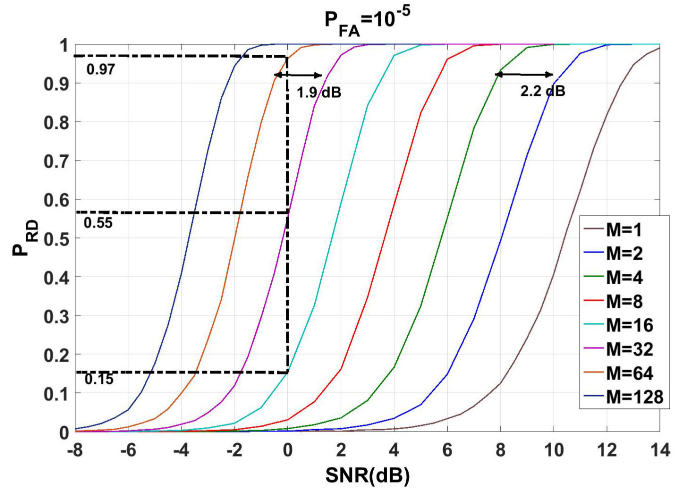


Fig. 3. Detection curves for the Λ-statistic for different number M of antenna array elements ($P_{FA} = 10^{-5}$)

IV. The approximate decision statistics

As it can be seen from the previous section, it is difficult to apply the Λ-statistic in the considered scenario because the statistic itself and a decomposition of the observation space $\mathbf{Y}$ on the decision regions depend on the energy parameter (see Fig.2). Therefore, in majority of practical tasks noncoherent detection of signals by the NP criterion, one uses simplified decision statistics for which threshold values do not depend on the energy of the useful signal. Detection characteristics of such decision statistics most widely used in practical applications will be studied in the present work. We consider three decision statistics T_1, T_2 and T_3:

$$\begin{aligned}
\sum_{m=1}^{M} Y_m &= \sum_{m=1}^{M} \Big| \sum_{n=1}^{N} x_m[n]a^*[n]\Big| = T_1 \geq T_{1th},\\
\sum_{m=1}^{M} Y_m^2 &= \sum_{m=1}^{M} \Big| \sum_{n=1}^{N} x_m[n]a^*[n]\Big|^2 = T_2 \geq T_{2th},\\
\sum_{m=1}^{M} \ln Y_m &= \sum_{m=1}^{M} \sum_{n=1}^{N} ln|x_m[n]a^*[n]| = T_3 \geq T_{3th},
\end{aligned} \tag{9}$$

where the parameters T_{1th}, T_{2th}, T_{3th} are thresholds, depending only on the given level of P_{FA} and do not depend on the amplitude A in the case of H_0 hypothesis. The first of these statistics T_1 is the sum of the amplitudes of noncoherent matched filter outputs, it gives a good approximation of the Λ-statistic for large SNR , see (8). The second statistic T_2 is an approximate expression for the Λ-statistic at low SNR, see (7), and the third T_3 (logarithmic) is used in some practical tasks with a large dynamic range of variation of the signal power [17]. The boundary surfaces L_1, L_2 and L_3 that divide the observation space $\mathbf{Y}$ into Γ_0 and Γ_1 decision regions for the statistics T_1, T_2 and T_3 for M-dimensional case will be hypersurfaces: the hyperplane, the hypersphere and the hyperboloid, respectively. From the analysis of this M-dimensional boundaries it follows that the division boundary L_Λ for the exact Λ-statistic lies in the region between the

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boundaries L_1 and L_2, but the hypersurface L_3 is outside this region. Therefore, it seems reasonable to consider a combined statistic T_{comb} representing a combination of statistics T_1 and T_2. To justify the introduction of such a combined decision statistic let us note that the normalized statistics T_1/M, T_2/M can be viewed as the estimates of *the mean* and *the mean square* of the signals at the outputs of noncoherent matched filters. It is well known that in accordance with the Chebyshev theorem [18] these estimates are consistent, i.e. with increasing M the statistics T_1/M and T_2/M are converging to the true values of these parameters:

$$\begin{aligned} \frac{T_1}{M} &= \frac{1}{M}\sum_{m=1}^{M} Y_m \xrightarrow{M\to\infty} < Y >, \\ \frac{T_2}{M} &= \frac{1}{M}\sum_{m=1}^{M} Y_m^2 \xrightarrow{M\to\infty} < Y^2 > \end{aligned} \tag{10}$$

where the operator <...> denotes statistical averaging. Since under the hypothesis H_0 the signals at the outputs of noncoherent matched filters have Rayleigh distributions (6), then the values of these parameters are bound by the deterministic relation [14]:

$$< Y >= \sqrt{\pi}/2\sqrt{< Y^2 >} \tag{11}$$

The analysis has shown that the estimates of the parameters in (11) have the same variances, therefore, it seems logical to introduce a combined *ad hoc* statistic T_{comb} as a weighted arithmitic mean of the normalized statistics T_1/M and T_2/M:

$$\begin{aligned} T_{comb}[\mathbf{Y}] &= \frac{T_1}{M} + \frac{\sqrt{\pi}}{2}\sqrt{\frac{T_2}{M}} = \\ &= \frac{1}{M}\sum_{m=1}^{M} Y_m + \frac{\sqrt{\pi}}{2}\sqrt{\frac{1}{M}\sum_{m=1}^{M} Y_m^2} \end{aligned} \tag{12}$$

and explore its performance alongside with others.

V. Comparison of decision statistics characteristics

To compare the characteristics of the exact Λ-statistic and approximate statistics considered above we have performed simulations in the Matlab environment. An antenna array with M elements was modeled under the assumptions made in Section II. Length 112 Zadoff-Chu sequence used in the random access channel of LTE-Advanced networks, was taken as a useful signal. As shown by the preceding analysis, the characteristics of the detection system do not depend on the modulation law $a[n]$, and are defined by the SNR only.

A detailed comparative analysis of the detection characteristics has been done for all the considered statistics for antenna arrays with different number of antenna elements and for different values of P_{FA}. As expected, the approximate statistic T_1 shows visible performance degradation in the negative SNR region, but demonstrates near optimal performance for the larger SNR values. In this work, due to shortage of place, we present only the detection curves behavior in the most interesting region of the large hight detection probabilities P_{RD} ($P_{RD} > 0.9$). In this case, it is convenient to go to the miss detection probabilities $P_{Miss} = 1 - P_{RD}$ and plot their dependencies on the SNR for different P_{FA}.

Fig. 4 illustrates the P_{Miss} behavior in the most interesting, from a practical point of view, region of values of $P_M = 0.1 - 0.001$. The graphs show that for antenna arrays with small number of elements ($M = 2$) the P_{Miss} weakly depends on the decision statistic used (see Fig.3a) and curves are practically indistinguishable. However, for large antenna arrays ($M = 16$) the impact of the statistic on the P_{Miss} becomes more significant (see Fig.3b). An especially big performance degradation is demonstrated by the logarithmic statistic T_3 . The relative increase of the P_{Miss} for this statistic at a fixed value of the SNR can reach 3-4 times in comparison with the exact Λ-statistic. For all other decision statistics (except logarithmic) for large P_{FA} (about 0.1) the difference in the P_{Miss} is rather small (not more than 5%). However, for low P_{FA} ($10^{-5} - 10^{-8}$) the use of the quadratic statistic T_2 in large antenna arrays (M=16) can lead to a relatively large increase in the P_{Miss} by 30-40% in comparison with other statistics Λ, T_1, T_{comb} (see Fig.3b). It is also interesting to note that the characteristics of the combined decision statistic T_{comb}

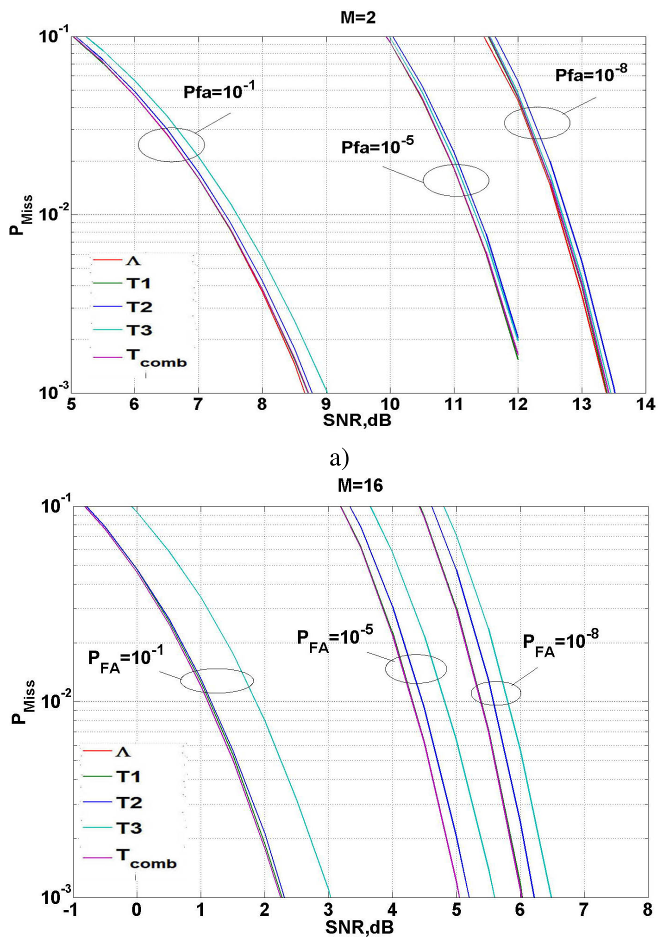

Fig. 4. Detection characteristics ($P_{Miss} = 1 - P_{RD}$) of decision statistics Λ, T_1, T_2, T_3, T_{comb} for noncoherent detection in antenna arrays with different numbers of elements (M=2, M=16) and different P_{FA}.

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practically coincide with the characteristics of the exact Λ-statistic (increase of the P_{Miss} relatively to the Λ-statistic does not exceed 2-3% for all the considered levels of P_{FA} and the SNR). Therefore, the combined statistic of T_{comb} can be recommended for practical use in various systems of noncoherent detection for all SNR values.

VI. Conclusion

For the first time a detailed analysis of the characteristics of the exact statistic for the optimal noncoherent signal detection in multi-element antenna arrays has been carried out. The analysis of the exact decision statistic behavior was fulfilled in the space of variables observed at the outputs of noncoherent matched filters engaged for the primary signal processing in each element of the antenna array.

It is shown that threshold values for the decision statistic depend on the SNR for the NP criterion, and this dependence is caused by the deformation of the division boundary between decision regions Γ_0 and Γ_1 of the observation space. The decision statistic detection curves for antenna arrays with different numbers of elements have been obtained and analysed.

A detailed comparison of various approximate decision statistics and the exact one has been done. It is shown that for large multi-element antenna arrays and small false alarm probabilities the decision statistic choice can have a sizeable impact on the detection scheme performance. For example, the use of the traditional quadratic statistic in the antenna arrays with the number of elements more than 16 can increase the miss probability by 30-40%.

To improve initial detection of UE signals at Base Stations with large antenna arrays in the LTE random access procedure a combined decision statistic for which the NP criterion thresholds do not depend on the SNR has been proposed. The characteristics of this statistic practically coincide with the characteristics of the decision test-statistic for all levels of the false alarm probability and the SNR.This task is important for reliable initial detection of UE signals.

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