A Novel Recursive Bayesian Weighted Instrumental Variable Estimator for 3D Bearings-only TMA

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***Abstract*—In our previous work [17], we derived a novel Bayesian weighted instrumental variable (WIV) estimator for the three-dimensional bearings-only target motion analysis problem. While the proposed approach has the desirable characteristic of incorporating *a priori* information in the estimation process and is proven to be approximately asymptotically unbiased, this estimator has a batch structure which is generally not suitable for online processing of measurements in practical applications. Therefore, in this paper we develop a recursive Bayesian WIV, which also uses an adaptive selective angle measurement approach to increase its stability. Simulations show that the proposed estimator outperforms the compared Bayesian algorithms with similar computational complexity for poorly observable scenarios.**

***Index Terms*—Estimation theory, bearings-only target motion analysis, Bayesian estimation, pseudolinear estimator, instrumental variables**

Bearings-only tracking, also known as target motion analysis (TMA), involves estimating the state of a target (often position and velocity), from noise-corrupted bearing measurements [1]. The nonlinear nature of the problem and observability issues make this problem challenging and difficult to solve [2].

Several Bayesian and non-Bayesian filtering techniques have been developed to solve this nonlinear problem. Extended Kalman filter (EKF) [3], unscented Kalman filter (UKF) [4], shifted Rayleigh filter (SRF) [5], and particle filters [6] are some of the Bayesian algorithms applied to this problem.

A simple non-Bayesian solution called the pseudolinear estimator (PLE) was developed for this problem in [7]. The PLE does not have the drawbacks of the conventional maximum likelihood estimator (MLE) [8] (high computational complexity and potential divergence). However, it suffers from severe bias problems. Subsequent improvements to the conventional PLE appear to have remedied its bias problem to some extent in two-dimensional (2D) geometries e.g., [8]-[10], and three-dimensional (3D) geometries e.g., [11]-[14]. In particular, the batch weighted instrumental variable estimator (WIV) has been used to remove the bias of the PLE by employing an instrumental variable matrix, which ideally is uncorrelated with the error vector e.g., [10], [11], [15].

In our previous work [17], we noticed that although the 3D WIV has desirable characteristics of being closed-form and highly accurate, this estimator is non-Bayesian and cannot incorporate prior information in the estimation process. Thus, a batch Bayesian WIV estimator was developed [17] that is approximately asymptotically unbiased and outperforms its non-Bayesian counterpart. Nevertheless, as the proposed Bayesian WIV is a batch technique, it is not suitable for sequential processing of measurements. Recursive estimators are generally preferred to their batch counterparts in widespread practical applications as they are more computationally efficient and thus more suitable for online processing of measurements. Therefore, in this paper we develop a recursive form for the Bayesian WIV and its two intermediate estimators: Bayesian weighted PLE (WPLE) and Bayesian bias-compensated WPLE (BCWPLE). Moreover, an adaptive selective angle measurement approach (SAM) is used to increase the stability of the resulting recursive WIV estimator. The performance and computational-complexity of the proposed Bayesian estimator are compared to that of the EKF, UKF, and SRF in a poorly observable scenario. Results indicate that the proposed Bayesian estimator outperforms the compared Bayesian algorithms, while requiring comparable computational resources.

The paper is organised as follows. Section II formulates the 3D bearings-only TMA problem. The batch Bayesian WIV is reviewed in Section III. Section IV presents the proposed recursive Bayesian WIV algorithm. The simulation results in Sections V are followed by concluding remarks in Section VI.

I. Problem Formulation

In this paper, we assume the target travels with a constant velocity during the measurement period. Defining the target state vector at time k as $\boldsymbol{x}_k = [x_k, y_k, z_k, \dot{x}_k, \dot{y}_k, \dot{z}_k]^T$, the target state model is written as

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}\boldsymbol{x}_k, \quad \boldsymbol{F} = \begin{bmatrix} \boldsymbol{I}_3 & T\boldsymbol{I}_3 \\ \boldsymbol{0}_3 & \boldsymbol{I}_3 \end{bmatrix}, \tag{1}$$

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where T denotes the time interval between consecutive measurements and $k = 1, 2, \ldots$ is the measurement time index. Moreover, $\boldsymbol{I}_3$ is the 3×3 identity matrix and $\boldsymbol{0}_3$ is the 3×3 zero matrix. Note that there is no process noise involved in the target motion given in (1) as the target motion model is assumed to be purely deterministic.

The state vector $\boldsymbol{x}_0$ at time $k = 0$ is assumed to be a Gaussian random variable with known mean $\bar{\boldsymbol{x}}_0$ and covariance $\boldsymbol{\Sigma}_0$. i.e., $\boldsymbol{x}_0 \sim \mathcal{N}(\boldsymbol{x}_0; \bar{\boldsymbol{x}}_0, \boldsymbol{\Sigma}_0)$. Furthermore, the range between the target and receiver r_k is constrained by a maximum r_{max} and minimum r_{min} range.

At time index k, noisy bearing $\tilde{\theta}_k$ and elevation $\tilde{\phi}_k$ measurements are collected by a moving observer, with a known state vector $\boldsymbol{x}_k^o = [x_k^o, y_k^o, z_k^o, \dot{x}_k^o, \dot{y}_k^o, \dot{z}_k^o]^T$, where $\boldsymbol{x}_k^{o,p} = [x_k^o, y_k^o, z_k^o]^T$ and $\boldsymbol{x}_k^{o,v} = [\dot{x}_k^o, \dot{y}_k^o, \dot{z}_k^o]^T$ are respectively the observer position and velocity. The measurement model for the bearing and elevation angle measurements is

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{n}_k, \qquad k = 1, 2, \ldots \tag{2}$$

where

$$\boldsymbol{z}_k = \left[\tilde{\theta}_k, \tilde{\phi}_k\right]^T, \qquad \boldsymbol{n}_k = [n_{\theta,k}, n_{\phi,k}]^T,$$

$$\boldsymbol{h}(\boldsymbol{x}_k) = \left[\tan^{-1}\left(\frac{\Delta y_k}{\Delta x_k}\right), \tan^{-1}\left(\frac{\Delta z_k}{\sqrt{\Delta x_k^2 + \Delta y_k^2}}\right)\right]^T. \tag{3}$$

Here $\Delta x_k = x_k - x_k^o, \Delta y_k = y_k - y_k^o, \Delta z_k = z_k - z_k^o$ and $n_{\theta,k}$ and $n_{\phi,k}$ are the measurement noises which are assumed to be zero mean white Gaussian with variances $\sigma_{\theta_k}^2$ and $\sigma_{\phi_k}^2$, respectively. The elevation and bearing measurement noises are also assumed to be uncorrelated.

We are interested in the recursive estimation problem in this paper. The problem of interest is to recursively estimate the state vector $\boldsymbol{x}_k$ from measurements $\{\boldsymbol{z}_1, \ldots, \boldsymbol{z}_k\}$.

II. Background

In our previous work [17], we first developed a batch closed-form pseudolinear estimator. The main idea was to combine the prior information with the likelihood of pseudolinear measurements using Bayes' Theorem. The closed-form Bayesian WPLE estimate at time zero given measurements up to k is given by [17]

$$\hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} = \left(\boldsymbol{A}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{A}_k + \boldsymbol{\Sigma}_0^{-1}\right)^{-1} (\boldsymbol{A}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{Z}_k^s + \boldsymbol{\Sigma}_0^{-1} \bar{\boldsymbol{x}}_0), \tag{4}$$

where $\boldsymbol{A}_k$, $\boldsymbol{\Sigma}_k$, and $\boldsymbol{Z}_k^s$ respectively denote the system matrix, covariance matrix and pseudomeasurement vector

$$\begin{aligned} \boldsymbol{A}_k &= \left[\boldsymbol{a}_1^T, \ldots, \boldsymbol{a}_k^T\right]_{2k\times 6}^T, \boldsymbol{\Sigma}_k = \text{diag}\left\{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_k\right\}_{2k\times 2k} \\ \boldsymbol{Z}_k^s &= \left[(\boldsymbol{z}_1^s)^T, \ldots, (\boldsymbol{z}_k^s)^T\right]_{2k\times 1}^T, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \boldsymbol{a}_k &= \begin{bmatrix} \tilde{\boldsymbol{c}}_{1,k}^T \boldsymbol{M}_k \\ \tilde{\boldsymbol{c}}_{2,k}^T \boldsymbol{M}_k \end{bmatrix}, \boldsymbol{y}_k = \begin{bmatrix} y_{1,k} & 0 \\ 0 & \lambda_k^2 \bar{r}^2 (1 - \cos^2 \tilde{\phi}_k \mu_k^2) \end{bmatrix}, \\ \boldsymbol{z}_k^s &= \begin{bmatrix} \tilde{\boldsymbol{c}}_{1,k}^T \boldsymbol{x}_k^{o,p} \\ \tilde{\boldsymbol{c}}_{2,k}^T \boldsymbol{x}_k^{o,p} \end{bmatrix}, \end{aligned} \tag{6}$$

$$y_{1,k} = \begin{cases} \bar{r}^2 \cos^2(\frac{\pi}{2} - \varepsilon)\, \mu_k^2 & \text{if } \frac{\pi}{2} - \varepsilon < |\tilde{\phi}_k| < \frac{\pi}{2} + \varepsilon, \\ \bar{r}^2 \cos^2 \tilde{\phi}_k\, \mu_k^2 & \text{otherwise}, \end{cases} \tag{7}$$

$$\begin{aligned} \tilde{\boldsymbol{c}}_{2,k} &= [\sin \tilde{\phi}_k \cos \tilde{\theta}_k, \sin \tilde{\phi}_k \sin \tilde{\theta}_k, -\cos \tilde{\phi}_k]^T, \\ \tilde{\boldsymbol{c}}_{1,k} &= [\sin \tilde{\theta}_k, -\cos \tilde{\theta}_k, 0]^T, \boldsymbol{M}_k = \begin{bmatrix} I_3 & kT\boldsymbol{I}_3 \end{bmatrix}. \end{aligned} \tag{8}$$

Here $\bar{r}$ is the prior mean range, $\varepsilon > 0$ is set to a small fixed value and

$$\mu_k^2 = \frac{1}{2}\left(1 - \exp(-2\sigma_{\theta_k}^2)\right), \lambda_k^2 = \frac{1}{2}\left(1 - \exp(-2\sigma_{\phi_k}^2)\right).$$

Despite its simplicity, the WPLE solution is biased. In order to reduce the bias, we proposed a bias-compensated WPLE (BCWPLE) $\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}}$ [17] by estimating the instantaneous bias $\hat{\boldsymbol{\zeta}}_k$ and subtracting it from the estimator.

$$\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}} = \hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} - \hat{\boldsymbol{\zeta}}_k. \tag{9}$$

The instantaneous bias is calculated as

$$\hat{\boldsymbol{\zeta}}_k = -\left(\boldsymbol{A}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{A}_k + \boldsymbol{\Sigma}_0^{-1}\right)^{-1} \boldsymbol{K}_k, \tag{10}$$

where

$$\boldsymbol{K}_k = \frac{1}{\bar{r}} \sum_{i=1}^{k} \boldsymbol{M}_i^T \left(\hat{w}_i \hat{\boldsymbol{\psi}}_i + \hat{\boldsymbol{f}}_i + \hat{\boldsymbol{\kappa}}_i\right). \tag{11}$$

Here we have

$$\hat{w}_k = \frac{\mu_k^2 \sin(2\hat{\phi}_k)}{4\lambda_k^2 (1 - \cos^2 \hat{\phi}_k \mu_k^2)}, \hat{\boldsymbol{\kappa}}_k = \frac{1}{\cos \hat{\phi}_k} \left[\cos \hat{\theta}_k, \sin \hat{\theta}_k, 0\right]^T, \tag{12a}$$

$$\hat{\boldsymbol{\psi}}_k = \left[-\sin \hat{\phi}_k \cos \hat{\theta}_k, -\sin \hat{\phi}_k \sin \hat{\theta}_k, \cos \hat{\phi}_k\right]^T, \tag{12b}$$

$$\hat{\boldsymbol{f}}_k = [\cos \hat{\phi}_k \cos \hat{\theta}_k, \cos \hat{\phi}_k \sin \hat{\theta}_k, \sin \hat{\phi}_k]^T. \tag{12c}$$

The estimated parameters $\hat{\theta}_k$ and $\hat{\phi}_k$ are obtained using the WPLE as an initial estimate:

$$\hat{\theta}_k = \tan^{-1}\left(\frac{\Delta \hat{y}_k}{\Delta \hat{x}_k}\right), \hat{\phi}_k = \tan^{-1}\left(\frac{\Delta \hat{z}_k}{\sqrt{(\Delta \hat{x}_k)^2 + (\Delta \hat{y}_k)^2}}\right), \tag{13}$$

where $[\Delta \hat{x}_k, \Delta \hat{y}_k, \Delta \hat{z}_k]^T = \boldsymbol{M}_k \hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} - \boldsymbol{x}_k^{o,p}$. The bias-compensated estimator can significantly reduce the bias; however, it might still be biased as it uses estimated parameters instead of true ones. To address this drawback, a batch Bayesian WIV estimator was proposed, which uses an instrumental variable matrix $\boldsymbol{G}_k$ with desirable characteristics to reduce the bias. The Bayesian WIV, which was shown to be approximately asymptotically unbiased, is given by [17]

$$\hat{\boldsymbol{x}}_{0|k}^{\text{WIV}} = \left(\boldsymbol{G}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{A}_k + \boldsymbol{\Sigma}_0^{-1}\right)^{-1} \left(\boldsymbol{G}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{Z}_k^s + \boldsymbol{\Sigma}_0^{-1} \bar{\boldsymbol{x}}_0\right), \tag{14}$$

where

$$\boldsymbol{G}_k = \left[\boldsymbol{g}_1^T, \ldots, \boldsymbol{g}_k^T\right]_{2k\times 6}^T, \boldsymbol{\Sigma}_k = \text{diag}\left\{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_k\right\}_{2k\times 2k}, \tag{15}$$

$$\boldsymbol{g}_k = \begin{bmatrix} \hat{\boldsymbol{c}}_{1,k}^T \boldsymbol{M}_k \\ \hat{\boldsymbol{c}}_{2,k}^T \boldsymbol{M}_k \end{bmatrix}, \boldsymbol{y}_k = \begin{bmatrix} y_{1,k} & 0 \\ 0 & \lambda_k^2 \hat{r}_k^2 (1 - \cos^2 \hat{\phi}_k \mu_k^2) \end{bmatrix}, \tag{16}$$

$$\hat{\boldsymbol{c}}_{2,k} = [\sin\hat{\phi}_k \cos\hat{\theta}_k, \sin\hat{\phi}_k \sin\hat{\theta}_k, -\cos\hat{\phi}_k]^T, \quad \hat{\boldsymbol{c}}_{1,k} = [\sin\hat{\theta}_k, -\cos\hat{\theta}_k, 0]^T, \tag{17}$$

$$y_{1,k} = \begin{cases} \hat{r}_k^2 \cos^2(\pi/2-\varepsilon)\ \mu_k^2 & \text{if } \frac{\pi}{2}-\varepsilon < |\hat{\phi}_k| < \frac{\pi}{2}+\varepsilon, \\ \hat{r}_k^2 \cos^2\hat{\phi}_k\ \mu_k^2 & \text{otherwise,} \end{cases} \tag{18}$$

Note that $\boldsymbol{G}_k$ is constructed by substituting the estimated bearing $\hat{\theta}_k$ and elevation $\hat{\phi}_k$ angles for the corresponding noisy counterparts in the system matrix $\boldsymbol{A}_k$. The estimated bearing, elevation, and range are obtained using the $\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}}$ as

$$\hat{\theta}_k = \tan^{-1}\left(\frac{\Delta\hat{y}_k}{\Delta\hat{x}_k}\right), \hat{\phi}_k = \tan^{-1}\left(\frac{\Delta\hat{z}_k}{\sqrt{(\Delta\hat{x}_k)^2+(\Delta\hat{y}_k)^2}}\right),$$
$$\hat{r}_k^x = \|\boldsymbol{M}_k\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}} - \boldsymbol{x}_k^{o,p}\|, \hat{r}_k = \begin{cases} r_{\max} & \text{if } \hat{r}_k^x > r_{\max}, \\ r_{\min} & \text{if } \hat{r}_k^x < r_{\min}, \\ \hat{r}_k^x & \text{otherwise.} \end{cases}$$
$$[\Delta\hat{x}_k, \Delta\hat{y}_k, \Delta\hat{z}_k]^T = \boldsymbol{M}_k\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}} - \boldsymbol{x}_k^{o,p}. \tag{19}$$

III. The Proposed Recursive Bayesian Estimator

Since recursive estimators are more suitable for practical applications, in this section we derive a recursive counterpart (see subsection III-C) for the batch Bayesian WIV [17] summarised in the previous subsection. The recursive WIV uses recursive BCWPLE as an initial estimate in each recursion. Therefore we also obtained the recursive forms for the intermediate estimators, Bayesian WPLE and Bayesian BCWPLE in Subsections III-A and III-B, respectively.

A. Recursive Bayesian Weighted Pseudolinear Estimator

The aim of this subsection is to derive a recursive solution for the batch WPLE solution given in (4). For this purpose, first a recursive form is driven for the $\boldsymbol{P}_{0|k}$ defined as

$$\boldsymbol{P}_{0|k} = \left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{A}_k + \boldsymbol{\Sigma}_0^{-1}\right)^{-1}. \tag{20}$$

Remark 1: To be consistent with the terminology used in [16], we refer to $\boldsymbol{P}_{0|k}$ as covariance matrix. However, note that this is not strictly the covariance associated with the WPLE.

The covariance matrix $\boldsymbol{P}_{0|k+1}$ at time zero given measurements up to $k+1$ can be written as

$$\boldsymbol{P}_{0|k+1} = \left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{A}_k + \boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}\boldsymbol{a}_{k+1} + \boldsymbol{\Sigma}_0^{-1}\right)^{-1}, \tag{21a}$$

$$= \left((\boldsymbol{P}_{0|k})^{-1} + \boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}\boldsymbol{a}_{k+1}\right)^{-1}, \tag{21b}$$

where we have used (20). Using the matrix inversion lemma [18] in (21b), we have

$$\boldsymbol{P}_{0|k+1} = \left(\boldsymbol{I} - \boldsymbol{W}_{k+1}\boldsymbol{a}_{k+1}\right)\boldsymbol{P}_{0|k}, \tag{22}$$

where

$$\boldsymbol{W}_{k+1} = \boldsymbol{P}_{0|k}\boldsymbol{a}_{k+1}^T\left(\boldsymbol{a}_{k+1}\boldsymbol{P}_{0|k}\boldsymbol{a}_{k+1}^T + \boldsymbol{y}_{k+1}\right)^{-1}. \tag{23}$$

Rewriting (4) for $k+1$ measurements and using (22) in it, the state vector at time zero can be written as

$$\hat{\boldsymbol{x}}_{0|k+1}^{\text{WPLE}} = \boldsymbol{P}_{0|k+1}\left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{Z}_k^s + \boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}\boldsymbol{z}_{k+1}^s + \boldsymbol{\Sigma}_0^{-1}\bar{\boldsymbol{x}}_0\right), \tag{24a}$$

$$= \boldsymbol{P}_{0|k+1}\left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{Z}_k^s + \boldsymbol{\Sigma}_0^{-1}\bar{\boldsymbol{x}}_0\right) + \boldsymbol{P}_{0|k+1}\boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}\boldsymbol{z}_{k+1}^s. \tag{24b}$$

Using (22) in (24b) results in

$$\hat{\boldsymbol{x}}_{0|k+1}^{\text{WPLE}} = (\boldsymbol{I} - \boldsymbol{W}_{k+1}\boldsymbol{a}_{k+1})\boldsymbol{P}_{0|k}\left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{Z}_k^s + \boldsymbol{\Sigma}_0^{-1}\bar{\boldsymbol{x}}_0\right) + \boldsymbol{P}_{0|k+1}\boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}\boldsymbol{z}_{k+1}^s. \tag{25}$$

Using the alternative expression for $\boldsymbol{W}_{k+1}$ as [16]

$$\boldsymbol{W}_{k+1} = \boldsymbol{P}_{0|k+1}\boldsymbol{a}_{k+1}^T\boldsymbol{y}_{k+1}^{-1}, \tag{26}$$

and observing that

$$\hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} = \boldsymbol{P}_{0|k}\left(\boldsymbol{A}_k^T\boldsymbol{\Sigma}_k^{-1}\boldsymbol{Z}_k^s + \boldsymbol{\Sigma}_0^{-1}\bar{\boldsymbol{x}}_0\right), \tag{27}$$

(25) can be expressed as

$$\hat{\boldsymbol{x}}_{0|k+1}^{\text{WPLE}} = \hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} + \boldsymbol{W}_{k+1}(\boldsymbol{z}_{k+1}^s - \boldsymbol{a}_{k+1}\hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}}). \tag{28}$$

B. Recursive Bayesian Bias-Compensated Weighted Pseudolinear Estimator

Given the recursive form for the WPLE in the previous subsection, in order to derive a recursive version of the batch BCWPLE, we just need to develop a recursive form for the estimated instantaneous bias. Using (20), the instantaneous bias given in (10) can equivalently be expressed as

$$\hat{\boldsymbol{\zeta}}_k = -\boldsymbol{P}_{0|k}\boldsymbol{K}_k. \tag{29}$$

Assuming the terms $\boldsymbol{P}_{0|k}$ and $\boldsymbol{K}_k$ are available and have been used for calculating the instantaneous bias at time k, we would like to obtain $\boldsymbol{P}_{0|k+1}$ and $\boldsymbol{K}_{k+1}$ to calculate the instantaneous bias at time $k+1$. Note that the recursion for $\boldsymbol{P}_{0|k+1}$ is given in (22) and the recursion for $\boldsymbol{K}_{k+1}$ can simply be written as

$$\boldsymbol{K}_{k+1} = \boldsymbol{K}_k + \frac{\boldsymbol{M}_{k+1}^T}{\bar{r}}\left(\hat{w}_{k+1}\hat{\boldsymbol{\psi}}_{k+1} + \hat{\boldsymbol{f}}_{k+1} + \hat{\boldsymbol{\kappa}}_{k+1}\right). \tag{30}$$

Hence the recursive Bayesian BCWPLE is summarised as: Given the estimated state at time zero given k measurements, $\boldsymbol{P}_{0|k}$ and the vector $\boldsymbol{K}_k$, the state estimate of BCWPLE at time zero given $k+1$ measurements is obtained as:

$$\begin{aligned}
\boldsymbol{W}_{k+1} &= \boldsymbol{P}_{0|k}\boldsymbol{a}_{k+1}^T\left(\boldsymbol{a}_{k+1}\boldsymbol{P}_{0|k}\boldsymbol{a}_{k+1}^T + \boldsymbol{y}_{k+1}\right)^{-1}, \\
\hat{\boldsymbol{x}}_{0|k+1}^{\text{WPLE}} &= \hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}} + \boldsymbol{W}_{k+1}(\boldsymbol{z}_{k+1}^s - \boldsymbol{a}_{k+1}\hat{\boldsymbol{x}}_{0|k}^{\text{WPLE}}), \\
\boldsymbol{P}_{0|k+1} &= (\boldsymbol{I} - \boldsymbol{W}_{k+1}\boldsymbol{a}_{k+1})\boldsymbol{P}_{0|k}, \\
\boldsymbol{K}_{k+1} &= \boldsymbol{K}_k + \frac{\boldsymbol{M}_{k+1}^T}{\bar{r}}\left(\hat{g}_{k+1}\hat{\boldsymbol{\psi}}_{k+1} + \hat{u}_k\hat{\boldsymbol{f}}_{k+1} + \hat{\boldsymbol{\kappa}}_{k+1}\right), \\
\hat{\boldsymbol{x}}_{0|k+1}^{\text{BCWPLE}} &= \hat{\boldsymbol{x}}_{0|k+1}^{\text{WPLE}} + \boldsymbol{P}_{0|k+1}\boldsymbol{K}_{k+1}.
\end{aligned} \tag{31}$$

The estimated bearing $\hat{\theta}_{k+1}$ and elevation $\hat{\phi}_{k+1}$ angles used in (31) can be approximated similar to (19), by using the BCWPLE at previous time index $\hat{\boldsymbol{x}}_{0|k}^{\text{BCWPLE}}$.

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C. Recursive Bayesian WIV Estimator

In this section a recursive form is presented for the batch Bayesian WIV estimator given in (14). Denoting

$$\boldsymbol{C}_{0|k} = \left(\boldsymbol{G}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{A}_k + \boldsymbol{\Sigma}_0^{-1}\right)^{-1} \quad (32)$$

and using a similar approach to the one used in Subsection III-A (Deriving the recursion for the $\boldsymbol{C}_{0|k}$ using matrix inversion lemma and then calculating the recursive form for the $\hat{\boldsymbol{x}}_{0|k}^{\text{WIV}}$.), the recursive WIV can be written as

$$\boldsymbol{W}_{k+1} = \boldsymbol{C}_{0|k} \boldsymbol{g}_{k+1}^T \left(\boldsymbol{a}_{k+1} \boldsymbol{C}_{0|k} \boldsymbol{g}_{k+1}^T + \boldsymbol{y}_{k+1}\right)^{-1}, \quad (33a)$$

$$\hat{\boldsymbol{x}}_{0|k+1}^{\text{WIV}} = \hat{\boldsymbol{x}}_{0|k}^{\text{WIV}} + \boldsymbol{W}_{k+1}(\boldsymbol{z}_{k+1}^s - \boldsymbol{a}_{k+1} \hat{\boldsymbol{x}}_{0|k}^{\text{WIV}}), \quad (33b)$$

$$\boldsymbol{C}_{0|k+1} = (\boldsymbol{I} - \boldsymbol{W}_{k+1} \boldsymbol{a}_{k+1}) \boldsymbol{C}_{0|k}. \quad (33c)$$

Here $\boldsymbol{y}_{k+1}$ and $\boldsymbol{g}_{k+1}$ are calculated using (16). The estimated bearing, elevation and range are estimated similar to (19), but using the BCWPLE at time $k+1$ as an initial estimate. Similar to Remark 1, note that the $\boldsymbol{C}_{0|k}$ is not strictly the covariance associated with the WIV. Due to the assumption of constant velocity, the target state at time $k+1$ is given by

$$\hat{\boldsymbol{x}}_{k+1|k+1}^{\text{WIV}} = \boldsymbol{F}^{k+1} \hat{\boldsymbol{x}}_{0|k+1}^{\text{WIV}}.$$

Similar equations can be written for the WPLE and BCWPLE.

When the observability is poor, the estimated bearing and elevation angles used in the WIV may be erratic and have large errors, which can have an adverse effect on the performance of the WIV. To address this problem, recently a selective angle measurement (SAM) approach has been proposed [12]. Using a similar approach, to increase the stability of the recursive WIV and to avoid using inaccurate estimated values, the estimated bearing and elevation angles will respectively be replaced by the noisy bearing and elevation measurements, if the estimated angles deviate too much from their noisy counterparts:

$$\hat{\theta}_k = \begin{cases} \hat{\theta}_k & \text{if } |\hat{\theta}_k - \tilde{\theta}_k| < \alpha_{1,k}, \\ \tilde{\theta}_k & \text{otherwise}, \end{cases} \quad (34a)$$

$$\hat{\phi}_k = \begin{cases} \hat{\phi}_k & \text{if } |\hat{\phi}_k - \tilde{\phi}_k| < \alpha_{2,k}, \\ \tilde{\phi}_k & \text{otherwise}. \end{cases} \quad (34b)$$

Selecting the thresholds $\alpha_{1,k}$ and $\alpha_{2,k}$ plays an important role in the performance of the WIV estimator. If these thresholds are chosen to be very large, the estimator reduces to the basic WIV, whereas if they are too small, the WIV estimator reduces to the WPLE, increasing the bias of the estimator. Usually $\alpha_{1,k}$ and $\alpha_{2,k}$ are chosen proportional to their measurement noise standard deviations [12], i.e.,

$$\alpha_{1,k} = \beta_k \sigma_{\theta_k}, \quad \alpha_{2,k} = \beta_k \sigma_{\phi_k}. \quad (35)$$

In this work, the parameter β_k is chosen adaptively using the norm of the difference between the input and output of the recursive WIV in each recursion $e_k = \|\hat{\boldsymbol{x}}_{0|k}^{\text{WIV}} - \hat{\boldsymbol{x}}_{0|k-1}^{\text{WIV}}\|$, according to the following scheme:

$$\beta_k = \begin{cases} 5 & e_k < 1.5 \text{ km}, \\ 4 & \text{if } 1.5 \leq e_k < 2 \text{ km}, \\ 3 & \text{if } 2 \leq e_k < 2.5 \text{ km}, \\ 2 & \text{if } e_k > 2.5 \text{ km}. \end{cases} \quad (36)$$

When the observability is very low, the estimated values can be erratic, thus, it is better to use the noisy counterparts and therefore smaller values of β_k are used. The low observability implies that the difference between the current and previous WIV estimate, i.e., e_k is usually large and thus β_k is chosen to be small. However, when the observability of the scenario is good, the estimated values might be more reliable and the error between input and output e_k is relatively small implying that larger values can be chosen for β_k.

IV. Simulation Results

This section compares the performance of the proposed recursive Bayesian WIV to that of the EKF, UKF, and SRF using 4000 Monte Carlo (MC) runs. The first performance metric is the root-mean-squared (RMS) position error [19]. The second evaluates the number of divergent tracks where a track is considered to be divergent if its estimated target location error at two consecutive measurement times exceeds the threshold of 15000 m.

Suppose the mean range in each MC trial is chosen according to $\bar{r} = \mathcal{N}(r_t, \sigma_r^2)$, where r_t is the true range and $\sigma_r = 3435$ m is the initial range standard deviation. The minimum and maximum ranges are respectively 100 and 12000 m. The maximum target velocity for the x and y axes re set to $V_{1,\max} = 5.14$ m/s (10 knots) and for the z axis is set to $V_{2,\max} = 0.51$ m/s (0.1 knots). The mean $\bar{\boldsymbol{x}}_0$ and covariance $\boldsymbol{\Sigma}_0$ are obtained in a manner similar to [17].

The simulated target-observer geometry is depicted in Fig. 1. Initially located at the origin, the observer travels in a horizontal plane along the x-axis with a constant speed of 2.57 m/s (5 knots), while collecting the first 15 measurements. After executing a coordinated-turn with a turn rate of 1 deg/s for 2 scans, it collects the final 15 measurements. Starting at $[8000, 6000, -400]^T$ m, the target follows a linear path with a constant speed of 2.57 m/s (5 knots) at $-135°$ course with respect to x-axis throughout the observation period. Note that the difference between the depth of the target and observer is 400 m. This scenario is poorly observable as the ownship trajectory has only one manoeuvre and its second leg is almost collinear with the target trajectory.

The RMS position error of the recursive WIV is compared to that of the EKF, UKF, and SRF in Fig. 2. As this figure shows, the WIV outperforms all the compared algorithms and meets the CRLB towards the end of the scenario. Moreover, comparing the number of divergent tracks in Table I shows that EKF, UKF, and SRF produce divergent tracks (Although the number of divergent tracks for the UKF is significantly lower than that of the SRF and EKF.), while the proposed WIV produces none.

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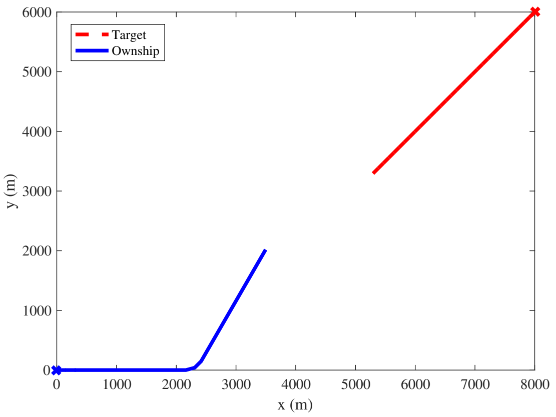

Fig. 1. Scenario for the results shown in Fig. 2.

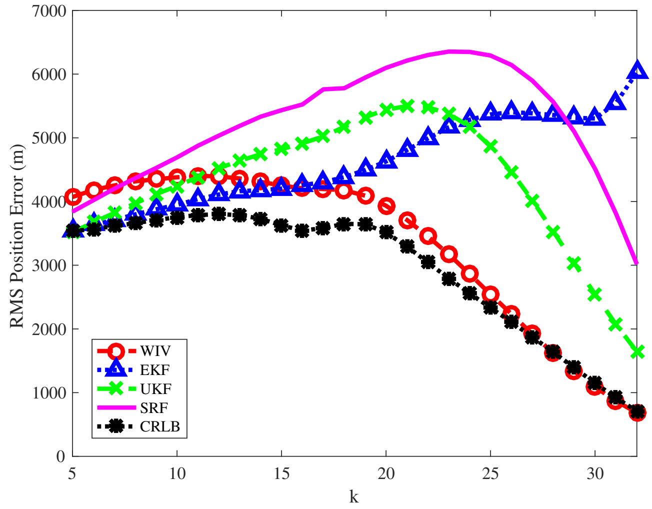

Fig. 2. RMS position error results for the scenario in Fig. 1.

TABLE I
TRACK DIVERGENCE RESULTS AND RELATIVE COMPUTATION TIME OF THE EKF, UKF, SRF AND FOR THE SCENARIO GIVEN IN FIG. 1.

Algorithm	EKF	UKF	SRF	WIV
Number of diverged tracks	243	3	59	0
Relative computation time to the EKF	1	1.6	1.46	1.07

After comparing the estimation performance, we also compared the computation time of the UKF, SRF, and the proposed approach relative to that of the EKF in Table I. This table shows that the WIV is faster than the UKF and SRF and is comparable to that of the EKF.

V. CONCLUSION

This paper develops a recursive Bayesian estimator that is computationally cheaper and more suitable for practical applications compared to its batch counterpart developed in our earlier work [17]. Simulation results indicate that the proposed algorithm outperforms the EKF, UKF, and SRF in the simulated poor observable geometry, while requiring comparable computational resources.

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