Direction-of-Arrival Estimation for Uniform Rectangular Array: A Multilinear Projection Approach

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***Abstract*—In this paper, elevation and azimuth estimation with uniform rectangular array (URA) is addressed. Since the temporal samples received by the URA could be written into a tensorial form, we introduce the multilinear projection for developing a direction-of-arrival (DOA) estimator. In the noiseless condition, the multilinear projector is orthogonal to the steering matrix of the URA. Thus the proposed DOA estimator is designed to find minimal points of the inner product of the steering vector and the multilinear projector. Based on the multilinear algebraic framework, the proposed approach provides a better subspace estimate than that of the matrix-based subspace. Simulation results are provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Array signal processing, direction-of-arrival estimation, multilinear algebra, tensor decomposition, uniform rectangular array.**

I. Introduction

The topic of direction-of-arrival (DOA) estimation with uniform rectangular array (URA) has been widely investigated and applied in various fields such as communication, radar, sonar, etc. As one typical kind of planar arrays, URA could be deployed to estimate 2-D DOA, i.e., elevation and azimuth [1], [2]. Several DOA estimation methods, e.g., MUSIC [3] and ESPRIT, are based on the subspace estimation of the received samples. The subspace-based methods have shown their super-resolution ability and could achieve high DOA estimation accuracy. It is straightforward that the more accurate subspace we could obtain from the samples, the better DOA estimation performance we could expect.

Recently, a class of tensor-based methods have been introduced into the field of DOA estimation problem since the multidimensional array signal processing could be concluded in the multilinear algebraic framework. Several multidimensional array structures such as bistatic MIMO structure [4], [5] and URA have the potential to be constructed into a tensorial form. Each dimension of the tensor corresponds to a dimension of the multidimensional array. Then the higher-order singular value decomposition (HOSVD) which represents an extension of matrix-based SVD is used to calculate the tensor-based signal subspace [6]- [9]. The tensor-based signal subspace has shown to be a better estimate than that of the matrix-based ones under some mild conditions. A so-called tensor-based MUSIC method combines a set of noise subspaces which are orthogonal to the signal subspaces associated with each dimension [10], [11]. However, it may produces several false peaks which is caused by misparing of elevation and azimuth associated with different signals when searching over the whole spatial region.

In this paper, we propose a multilinear projection (MP) approach based DOA estimation method with URA. The MP approach is first proposed in [12] in the literature of computer vision and then extended to interference suppression [13]. Its main strategy aims to construct a multilinear projection tensor which is the orthogonal complement of tensor-based signal subspace. We follow this idea to establish a MUSIC-like DOA estimation approach which estimates all DOAs through searching over the spatial region to find the local minimal points of the inner product of the steering and the MP-based tensor. The proposed approach achieves auto-paring of the elevation and azimuth and has a superior DOA estimation performance over the matrix-based MUSIC method.

II. Signal Model

Consider an $M \times N$ URA equipped with half-wavelength spaced sensors placed on a X-Y plane. Assume there are Q narrowband uncorrelated signals impinging on this array. The sample of the (m, n)th sensor in the array at time t is expressed as

$$x_{mn}(t) = \sum_{q=1}^{Q} s_q(t) a_{mn}(\theta_q, \phi_q) + n_{mn}(t) \tag{1}$$

where $m = 1, 2, \ldots, M,\ n = 1, 2, \ldots, N$, (θ_q, ϕ_q) stands for the elevation and azimuth of the qth signal, $s_q(t)$ denotes the

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qth signal sampled at time t, $\mathbf{n}_{mn}(t)$ represents the complex white Gaussian noise, respectively. The a_{mn} stands for the (m,n)th sensor and its spatial phase factor for the qth signal is given by

$$a_{mn}(\theta_q,\phi_q) \triangleq e^{j(m-1)\pi\sin\theta_q\cos\phi_q}e^{j(n-1)\pi\sin\theta_q\sin\phi_q}. \quad (2)$$

Then the sample of the whole array at time t is written as

$$\mathbf{x}(t) \triangleq [x_{11}(t),\ldots,x_{1N}(t),\ldots,x_{M1}(t),\ldots,x_{MN}(t)]^T \quad (3)$$

$$= \sum_{q=1}^{Q} s_q(t)(\mathbf{a}_y(\theta_q,\phi_q)\otimes\mathbf{a}_x(\theta_q,\phi_q)) + \mathbf{n}(t) \quad (4)$$

$$= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (5)$$

where $(\cdot)^T, \otimes$ are used for denoting transpose, Kronecker product, and $\mathbf{A} \in \mathbb{C}^{MN\times Q}$ represents the steering matrix, given as

$$\mathbf{A} \triangleq [\mathbf{a}_y(\theta_1,\phi_1)\otimes\mathbf{a}_x(\theta_1,\phi_1),\ldots,\mathbf{a}_y(\theta_Q,\phi_Q)\otimes\mathbf{a}_x(\theta_Q,\phi_Q)] \quad (6)$$

and the steering vectors of Y- and X-axis are respectively defined as

$$\mathbf{a}_y(\theta_q,\phi_q) \triangleq [1,\ldots,e^{j(M-1)\pi\sin\theta_q\cos\phi_q}]^T \quad (7)$$

$$\mathbf{a}_x(\theta_q,\phi_q) \triangleq [1,\ldots,e^{j(M-1)\pi\sin\theta_q\cos\phi_q}]^T. \quad (8)$$

For a total of T samples, the received data of the URA is

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \quad (9)$$

Several subspace-based approaches which operate on (9) are proposed to solve the DOA estimation problem. Performing SVD to $\mathbf{X}$, we obtain the matrix-based subspace

$$\mathbf{X} = \begin{bmatrix}\hat{\mathbf{U}}_s & \hat{\mathbf{U}}_n\end{bmatrix}\begin{bmatrix}\hat{\mathbf{\Sigma}}_s & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{\Sigma}}_n\end{bmatrix}\begin{bmatrix}\hat{\mathbf{V}}_s & \hat{\mathbf{V}}_n\end{bmatrix}^H. \quad (10)$$

where $(\cdot)^H$ denotes the Hermitian transpose, $\hat{\mathbf{U}}_s$ and $\hat{\mathbf{U}}_n$ represent the estimated signal subspace and noise subspace, respectively. Also, $\hat{\mathbf{\Sigma}}_s$ is a diagonal matrix. If (9) is noiseless, the steering matrix (6) has the same range as the signal subspace and is orthogonal to the noise subspace. Therefore, the well-known matrix-based MUSIC method is expressed as

$$\arg\min_{\theta,\phi} ||\mathbf{a}(\theta,\phi)\hat{\mathbf{U}}_n||_2^2. \quad (11)$$

where $\|\cdot\|_2$ stands for the ℓ_2 norm. After searching over the spatial region, (11) yields a few minimal values. Thus we could obtain the estimates of elevation and azimuth.

III. Tensor Formulation

A. Tensor Notations

Following the notation in [6], we define a r-mode unfolding of a tensor $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_R}$ along as $\boldsymbol{\mathcal{A}}_{(r)}$ and the concatenation of two tensors along the r-mode as $\boldsymbol{\mathcal{B}} \sqcup_r \boldsymbol{\mathcal{C}}$. By setting $i_r = k$, we obtain a sub-tensor denoted by $\boldsymbol{\mathcal{A}}_{i_r=k}$.

Definition 1 (The r-mode tensor-matrix product): The r-mode product of a tensor $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_R}$ and a matrix $\mathbf{B} \in \mathbb{C}^{J_r\times I_r}$ along the r mode is given by

$$\boldsymbol{\mathcal{C}} \triangleq \boldsymbol{\mathcal{A}} \times_n \mathbf{B} \quad (12)$$

$$c_{i_1,i_2,\ldots,i_{r-1},j_r,i_{r+1},\ldots,i_R} = \sum_{i_r=1}^{I_r} a_{i_1,i_2,\ldots,i_R} b_{j_r,i_r}$$

where $\boldsymbol{\mathcal{C}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_{r-1}\times J_r\times I_{r+1}\times\cdots\times I_R}$.

Definition 2 (The generalized r-mode product): The generalized r-mode product between two R-order tensors $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_R}$ and $\boldsymbol{\mathcal{B}} \in \mathbb{C}^{J_1\times J_2\times\cdots\times J_R}$ is defined as

$$\boldsymbol{\mathcal{C}} \triangleq \boldsymbol{\mathcal{A}} \times_r \boldsymbol{\mathcal{B}} \quad (13)$$

where $\boldsymbol{\mathcal{C}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_{r-1}\times J_r\times I_{r+1}\times\cdots\times I_R}$ and $I_r = J_1\ldots J_{r-1}J_{r+1}\ldots J_R$. Its matrix form is expressed as $\boldsymbol{\mathcal{C}}_{(r)} = \boldsymbol{\mathcal{B}}_{(r)}\boldsymbol{\mathcal{A}}_{(r)}$.

B. Tensor Modeling

We could formulate a total of T received samples of the URA into a tensorial form, which enables the HOSVD to achieve a better signal subspace estimate of the tensor data. Note that the DOA estimation problem with URA could be regarded as an special type of two-dimensional harmonic retrieval problem, which its tensorial formulation has been well studied in [6]. Firstly, we fold the tth sample (5) into a $M\times N$ matrix, yielding

$$\mathbf{X}(t) = \sum_{q=1}^{Q} s_q(t)(\mathbf{a}_y(\theta_q,\phi_q)\circ\mathbf{a}_x(\theta_q,\phi_q)) + \mathbf{N}(t) \quad (14)$$

where $\circ$ denotes the outer product, and $\mathbf{N}(t)$ stands for the matrix form of the noise item in (5). Furthermore, (14) could be written into a form which shows a relationship with the tensor as

$$\mathbf{X}(t) = \boldsymbol{\mathcal{A}} \times_3 \mathbf{s}(t) + \mathbf{N}(t) \quad (15)$$

where $\mathcal{A} \in \mathbb{C}^{M\times N\times Q}$ denotes a three-way steering tensor. The q-th subtensor of $\mathcal{A}_{r_3=q}$ indicates the steering tensor of the qth signal and is expressed as

$$\boldsymbol{\mathcal{A}}_q(\theta_q,\phi_q) = \mathbf{a}_y(\theta_q,\phi_q)\circ\mathbf{a}_x(\theta_q,\phi_q) \quad (16)$$

Then, for a total of T samples, the received tensor as a concatenation of each received matrix at time $t = 1,2,\ldots,T$, $\boldsymbol{\mathcal{X}} \in \mathbb{C}^{M\times N\times T}$ is defined as

$$\boldsymbol{\mathcal{X}} = \mathbf{X}(1) \sqcup_3 \mathbf{X}(2) \sqcup_3 \cdots \sqcup_3 \mathbf{X}(T). \quad (17)$$

Then we construct a tensorial form of (9). Each dimension of $\boldsymbol{\mathcal{X}}$ stands for the Y-axis and X-axis of the URA and the temporal dimensions, respectively. Note that r-mode of a tensor has the same meaning with the r-dimension of an array. The HOSVD represents a lower rank approximation of a tensor. A straight forward but not optimal solution of HOSVD

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is achieved by alternatively performing SVD to the r-mode matrix unfolding of $\boldsymbol{\mathcal{X}}$ as

$$\boldsymbol{\mathcal{X}}_{(r)} = \begin{bmatrix}\hat{\mathbf{U}}_r^{[s]} & \hat{\mathbf{U}}_r^{[n]}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{\Sigma}}_r^{[s]} & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{\Sigma}}_r^{[n]}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{V}}_r^{[s]} & \hat{\mathbf{V}}_r^{[n]}\end{bmatrix}^H \quad (18)$$

where $\hat{\mathbf{U}}_1^{[s]} \in \mathbb{C}^{M\times Q}$, $\hat{\mathbf{U}}_2^{[s]} \in \mathbb{C}^{N\times Q}$ represent the r-mode signal subspaces and $\hat{\mathbf{U}}_1^{[n]} \in \mathbb{C}^{M\times(M-Q)}$, $\hat{\mathbf{U}}_2^{[n]} \in \mathbb{C}^{N\times(N-Q)}$ stands for the r-mode noise subspaces, respectively. Thus, the corresponding tensor-based signal subspace calculated through HOSVD is given as [6]

$$\hat{\boldsymbol{\mathcal{U}}}^{[s]} = \hat{\boldsymbol{\mathcal{G}}}^{[s]} \times_1 \hat{\mathbf{U}}_1^{[s]} \times_2 \hat{\mathbf{U}}_2^{[s]} \quad (19)$$

where $\hat{\boldsymbol{\mathcal{G}}}^{[s]} \in \mathbb{C}^{Q\times Q\times Q}$ denotes the core tensor and is taken from the upper-left of $\boldsymbol{\mathcal{G}}$. Similar to the matrix case, we have the following relationship

$$\boldsymbol{\mathcal{A}} \approx \hat{\boldsymbol{\mathcal{U}}}^{[s]} \times_3 \mathbf{Q} \quad (20)$$

where $\mathbf{Q} \in \mathbb{C}^{Q\times Q}$ denotes a full-rank matrix, which indicates $\boldsymbol{\mathcal{A}}$ and $\hat{\boldsymbol{\mathcal{U}}}^{[s]}$ share the same range. According to [6], the matrix-based and tensor-based signal subspace has the following relationship

$$[\hat{\boldsymbol{\mathcal{U}}}^{[s]}]_{(3)}^T = (\hat{\mathbf{T}}_1 \otimes \hat{\mathbf{T}}_2)\hat{\mathbf{U}}_s \quad (21)$$

where $\hat{\mathbf{T}}_i = \hat{\mathbf{U}}_i^{[s]}\hat{\mathbf{U}}_i^{[s],H}, i = 1, 2$. Since when $Q \leq \max\{M, N\}$, the tensor-based signal subspace is superior to the matrix-based one.

A tensor-based MUSIC DOA estimation method with URA combines a set of r-mode noise subspaces as [10]

$$\min ||\mathcal{A}(\theta,\phi) \times_1 \hat{\mathbf{U}}_1^{[n]^H} \times_2 \hat{\mathbf{U}}_2^{[n]^H}||_2^2. \quad (22)$$

This tensor-based MUSIC method may produces more DOA candidates than the actual number of signals since any combination of the elevation and azimuth associated with different signals will lead to a local minimal of (22).

C. MP Based DOA Estimation Approach

In this subsection, we propose a MP based MUSIC-like approach using the tensor-based signal subspace and multilinear algebra. First of all, we define the r-mode pseudo-inverse of a tensor as $\boldsymbol{\mathcal{A}}_{(r)}^{\dagger r} = (\boldsymbol{\mathcal{A}}_{(r)})^{\dagger}$, and $\dagger$ denotes the pseudo-inverse.

Property 1: The r-mode pseudo-inverse of a tensor satisfies [12]:

$$(\boldsymbol{\mathcal{A}} \times_r \boldsymbol{\mathcal{A}}^{\dagger n}) \times_r \boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}} \quad (23)$$

Property 2: the r-mode identity tensor has the following property :

$$\boldsymbol{\mathcal{I}}^r \times_r \boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}} \quad (24)$$

where $\boldsymbol{\mathcal{I}}^r$ is the r-mode identity tensor. Then the relationship between a r-mode identity tensor and an identity matrix is revealed as $\boldsymbol{\mathcal{I}}_{(n)}^r = \mathbf{I}_{I_r}$, $I_r = I_1 \ldots I_{r-1}I_{r+1} \ldots I_R$ where $\mathbf{I}$ is an identity matrix. Using properties 1 and 2, we could define the MP through orthogonal complement of the tensor-based signal subspace as

$$\hat{\boldsymbol{\mathcal{U}}}^{[n]} = \boldsymbol{\mathcal{I}}^3 - \hat{\boldsymbol{\mathcal{U}}}^{[s]} \times_3 (\hat{\boldsymbol{\mathcal{U}}}^{[s]})^{+3}. \quad (25)$$

Taking the generalized R-mode product between the MP and tensor-based signal subspace, we have

$$\hat{\boldsymbol{\mathcal{U}}}^{[n]} \times_3 \hat{\boldsymbol{\mathcal{U}}}^{[s]} = \hat{\boldsymbol{\mathcal{U}}}^{[s]} - \hat{\boldsymbol{\mathcal{U}}}^{[s]} \times_3 (\hat{\boldsymbol{\mathcal{U}}}^{[s]})^{+3} \times_3 \hat{\boldsymbol{\mathcal{U}}}^{[s]} = \boldsymbol{\mathcal{O}} \quad (26)$$

where $\boldsymbol{\mathcal{O}}$ is an all-zero tensor.

Thus the proposed MP approach is therefore given as

$$\arg\min_{\theta,\phi} ||\mathbf{a}(\theta,\phi)[\hat{\boldsymbol{\mathcal{U}}}^{[n]}]_{(3)}^T||_2^2. \quad (27)$$

Comparing with the matrix-based noise subspace, the proposed MP has the following relationship

$$[\hat{\boldsymbol{\mathcal{U}}}^{[n]}]_{(3)}^T = \mathbf{I}_{MN} - [\hat{\boldsymbol{\mathcal{U}}}^{[s]}]_{(3)}^T[\hat{\boldsymbol{\mathcal{U}}}^{[s]}]_{(3)}^{T,\dagger}. \quad (28)$$

The proposed DOA estimator (27) will have a better DOA estimation performance than that of the matrix-based MUSIC method since the tensor-based signal subspace are more accurate.

D. Computational complexity

The main computational complexity is mainly consist of two parts: the HOSVD and grid searching procedure. Because the calculation of the MP is needed only once during each time and demands much less computational complexity than the HOSVD, we neglect it for simplicity. If we respectively have G_e and G_a grids over elevation and azimuth, then the total computational complexity is $\mathcal{O}(4kQMNT) + \mathcal{O}(G_eG_aQMN)$ where k depends on the method we use to perform SVD.

IV. Simulations

In this part, two numerical examples are given to demonstrate the effectiveness of the proposed MP DOA estimation approach. In addition, the signal-to-noise ratio (SNR) is defined as $10\log_{10}(\|\mathbf{AS}\|_{\mathrm{F}}^2/\|\mathbf{N}\|_{\mathrm{F}}^2)$ where $\|\cdot\|_{\mathrm{F}}$ stands for the Frobenius norm. Throughout all the simulations, the size of URA is set as $M = 8$, $N = 7$. The root-mean-square-errors (RMSE) of elevation and azimuth are respectively defined as

$$\mathrm{RMSE}_\theta \triangleq \sqrt{\mathbb{E}\left\{\frac{1}{Q}\sum_{q=1}^{Q}(\hat{\theta}_q - \theta_q)^2\right\}} \quad (29)$$

$$\mathrm{RMSE}_\phi \triangleq \sqrt{\mathbb{E}\left\{\frac{1}{Q}\sum_{q=1}^{Q}(\hat{\phi}_q - \phi_q)^2\right\}}. \quad (30)$$

where $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation.

In the first example, we compare the spatial spectrums of the matrix-based MUSIC, tensor-based MUSIC and the proposed methods. The DOAs of two signals are set as $\theta = [43° \ 26°]$ and $\phi = [15° \ 30°]$. Moreover, we set $T = 6$ and SNR = 5dB. In Fig. 1, it is observed that all DOA estimation methods could distinguish these two signals. The spatial spectrum of MP approach has sharper peaks than that of the matrix-based method. However, there are several false peaks in Fig. 1(a), which degrades the DOA estimation performance of the tensor-based method. Also, the average CPU time of the proposed MP based method and matrix-based MUSIC method

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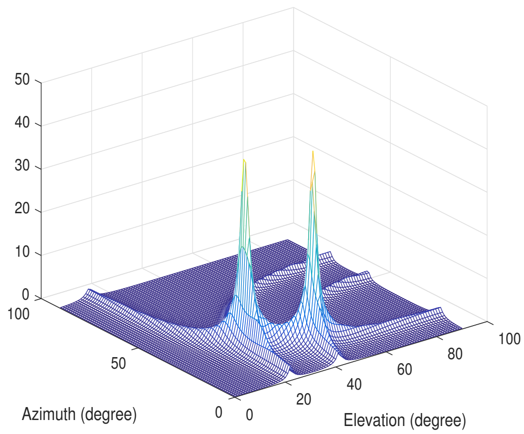
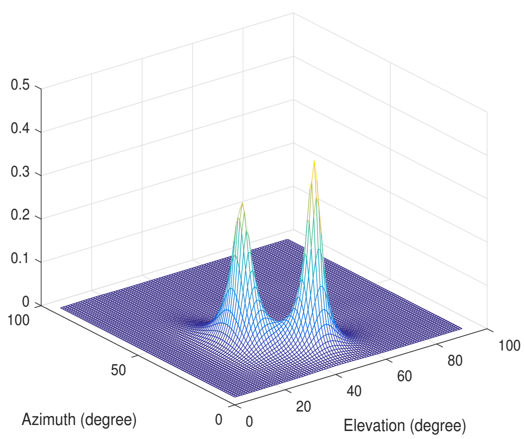
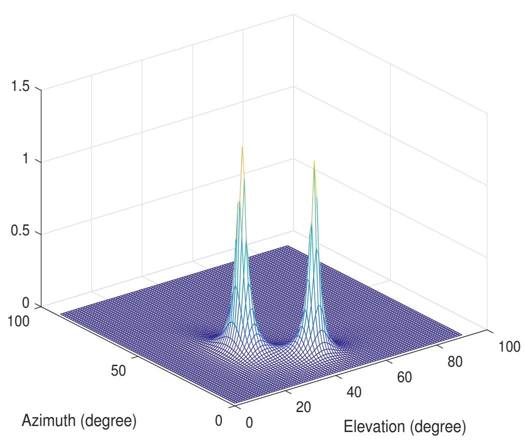


(a) Tensor-based MUSIC method

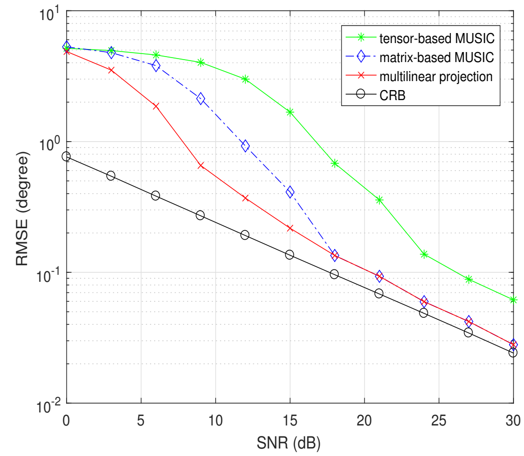


(b) Matrix-based MUSIC method

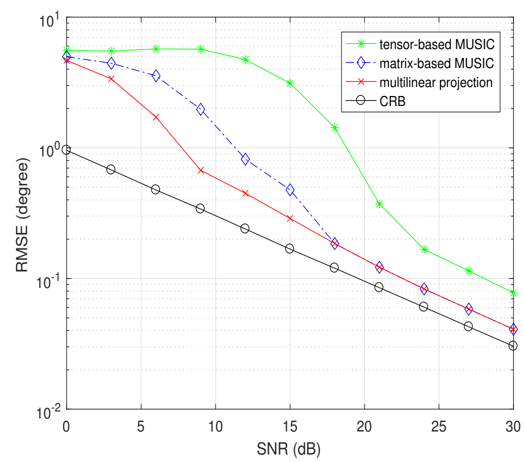


(c) MP method

Fig. 1. Comparison of spectral spectrums of three method. $M = 8$, $N = 7$, $Q = 10$, SNR $= 5$dB, $\theta = [43° \ 26°]$, $\phi = [15° \ 30°]$



(a) Elevation estimation



(b) Azimuth estimation

Fig. 2. RMSE performance of DOA estimation versus SNR, $M = 8$, $N = 7$, $Q = 6$, $\theta = [25° \ 35°]$, $\phi = [15° \ 25°]$.

are 0.065 s and 0.061 s with a search grid 0.01° on elevation and azimuth (on a laptop equipped with a Core i3 3.7 GHz processor, 8GB RAM and Matlab R2016b version).

In the second example, we evaluate the elevation and azimuth estimation performance of all these three methods versus SNR with a fixed number of samples $T = 6$. Note that the T is much smaller than the size of the URA. The DOAs of two signals are set as $\theta = [25° \ 35°]$, $\phi = [15° \ 25°]$, respectively. Also, the deterministic CRB is provided as a benchmark. In Fig. 2, the SNR is varied from 0dB to 30dB. We observe that the elevation and azimuth estimation performances of MP based approach outperform those of the matrix-based and tensor-based MUSIC methods, particularly when SNR$<$ 17dB. The MP method has as almost the same DOA estimation performance as that of the matrix-based MUSIC and both of them are close to the CRB when SNR$>$ 17dB.

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V. Conclusion

In this paper, we propose a DOA estimation approach with URA based on a multilinear projector. First, the received data is formulated into a tensor and obtain its tensor-based signal subspace. Then we utilizes multilinear projection to build a subspace is orthogonal to the steering tensor. A MP approach to DOA estimation is proposed following the strategy of the well-known MUSIC method. In addition, The computational complexity analysis of the MP method is also provided. Simulation results are given to show that the proposed DOA estimation method outperforms that of the matrix-based method.

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