Characterizing 3D Shapes: a complex network-based approach

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***Abstract*—In the past few years, 3D models have emerged as the focus of many new applications. This recent popularity of 3D models has stimulated researchers to investigate the problems of 3D shape retrieval and to develop more efficient search and retrieval methods. Aiming to contribute to the recent literature, this work proposes a novel 3D shape characterization method using the complex network theory. By modeling a 3D shape object as a complex network we are able to effectively represent, characterize and analyze the object is terms of the topological properties of the complex network. Comparison with two other known methods for 3D model description, shape histograms and shape distributions, on a 3D models data set shows that the proposed technique is a feasible approach to efficiently perform 3D shape characterization and discrimination.**

I. Introduction

Over the last years, there has been an increase in the use of three-dimensional (3D) models as a tool for describing and studying objects in many specific domains. In medicine, 3D models are a important tool, often used to better describe an structure or as an intermediate step in a procedure. For example, 3D bone models are an important issue in radiological and orthopedic environments. 3D bone models of the knee joint can be obtained from computed tomography (CT) and 3T magnetic resonance (MR) imaging [1]. 3D modelling is also a necessary step for reconstructive surgery with osseous free flaps [2]. Research in urban environment aims the generation of 3D city models with absence of data (such as elevation data) [3], [4]. In computer sciences, depth cameras can be used to create real time 3D face modeling systems and improve face recognition [5]. The impact of 3D digital models is so great that it is considered as the “4th wave of multimedia” [6] (audio, image and video are, respectively, the first, second and third wave).

From the use of 3D models emerges the need to create mechanisms to organize, research and recovery models in large repositories [7], [8], [9], [5]. However, 3D model recovering is a difficult and challenging task, due its greater complexity, diversity of information, data representation and number of dimensions. Literature provides an extensive amount of methods to describe and recognize a 3D object [10], [7], [8], [9].

In this paper, we propose to use complex networks to analyze and describe 3D models. Complex network is a special type of graph and it has particular properties not found in simple graphs. These properties are useful to analyze the topographic aspect of the network [11], [12] and, consequently, describe the object model as a network [13], [14], [15]. In fact, the study of complex networks can be described as an intersection between graph theory and statistical mechanics [11].

The remaining of the paper is organized as follows: Section II introduces the concept of Complex Network and describes how a point cloud of a 3D object can be modeled as a network. In Section III we propose to use the vertices’ degree distribution to compute a signature capable to describe the topological characteristics of the network representing the 3D object. We propose an experimental setup to evaluate our approach in Section IV. We discuss the results obtained by our approach and other compared methods in Section V, while Section VI concludes the paper.

II. Complex network approach for 3D shape analysis

The research in complex networks started with the studies of Flory [16], Rapoport [17], [18] and Erdos and Rényi [19], [20]. Literature shows an increase interest in complex networks in the last few years. Its recent popularity is due its great flexibility and generality. Complex networks are capable of representing virtually any natural structure, including those undergoing dynamic changes of topology [12]. As a result, many areas have focused on the the study of statistical properties of the complex networks [21], [22], [23], here included the many topics of computer vision [13], [14], [15], [11]. In the following sections we show how the complex network theory can be used to model and discriminate a 3D shape model.

A. Point cloud as a complex network

Usually, we can define a 3D object as a cloud of points where each point $p_i = (x_i, y_i, z_i)$, $i = i, \ldots, N$, occupies a given position in the Cartesian plane, $p_i \in R^3$. We can easily model this 3D object as a graph or network and use its topological properties for identification and comparison with other networks and, consequently, other 3D objects [9]. A graph $G = (V, E)$ is built by considering each point p_i of the 3D object as a vertice $v_i \in V$ of the graph G. The vertices

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associated to two points p_i and p_j are connected by a non-directed edge $e_{i,j} \in E$, $e_{i,j} = (p_i, p_j)$ and its weight, $w_{i,j}$, is defined as the Euclidean distance their respective points:

$$w_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \quad (1)$$

We used Euclidian distance as it is not affected by rotational and translational operations. To avoid influence of the scale of the 3D object, it is interesting to normalize the weight $w_{i,j}$ in the interval $[0, 1]$. This is performed by using the largest weight computed for an edge

$$w_{i,j} = \frac{w_{i,j}}{\max_{w_{i,j} \in W}}. \quad (2)$$

Since all vertices are connected to each other and, therefore, have the same number of connections, the initial network behaves as a complete and regular network. A regular network does not have any relevant property that can be used to describe the 3D object modeled. Thus, it is necessary to apply a transformation in order to convert this network into one that possesses relevant properties for 3D object analysis. A simple and straightforward approach is to simulate the dynamic evolution of the network through a set of thresholds T applied over the original set of edges E. Each threshold $t \in T$ enables us to select a subset of edges E_t, $E_t \subseteq E$, where each edge of $e_{i,j} \in E_t$ has weight $w_{i,j}$ equal to or smaller than t. By considering the original set of vertices V, this approach creates a new network $G_t = (V, E_t)$ representing an intermediary stage in the network evolution. Each network G_t has its own properties that change according to the value of the threshold t used as we discuss in the following sections. Figure 1 shows an example of the dynamic evolution of a network. Notice that the threshold acts as a visibility control in the network, limiting which vertices are reachable from a specific vertice.

B. Degree histogram

One of the most basic attributes of a graph or network is the *degree* (or *connectivity*) of a vertice v, $deg(v)$. This attribute represents the number of edges connected to v, i.e., the number of neighbors of a vertice v. The degree of a node v is defined as:

$$deg(v) = |\partial v| = |\{v' \in V | \{v, v'\} \in E\}|, \quad (3)$$

where ∂v is the set of neighbors of v, and $|A|$ denotes the number of elements of a set A [24].

The degree is a local attribute, i.e., it does not give us information about the whole structure of the network. A simple approach to obtain a concise description of the network topology is to compute the degree histogram of the network. The degree histogram provides a rough sense of the degree distribution and density, as also is a source of statistical information about the network topology. Mathematically, the degree histogram, h, is defined as

$$h(i) = \sum_{v \in V} \delta(deg(v), i) \quad (4)$$

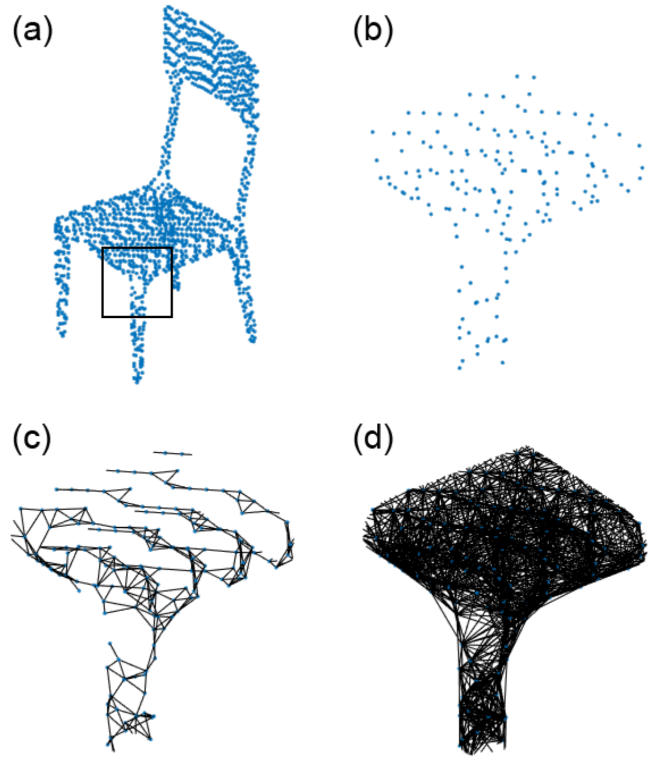

Fig. 1. Dynamic evolution of the network: (a) Point cloud of an object; (b) Region selected from (a); (c) Complex network computed using $t = 0.025$; (d) Complex network computed using $t = 0.050$.

$$\delta(j, i) = \begin{cases} 1, & j = i \\ 0, & j \neq i \end{cases} \quad (5)$$

where $\delta(j, i)$ is the Kronecker's delta. From the probability density function $p(i)$ of the degree histogram $h(i)$ it is possible to compute a set of first-order statistics [25], [14]. These statistics are a feasible approach to characterize the network topology. In this paper, the following set of 9 features were evaluated: mean, variance, contrast, energy, entropy, kurtosis, skewness, smoothness and inverse difference moment (IDM)

III. Complex network signature

In order to characterize a 3D shape object it is necessary to build a feature vector capable of describing it. To accomplish that we propose to create a feature vector representing the variation of the properties of the complex network as it goes through dynamic evolution.

From the initial network modeled from the original 3D object we are able to compute different complex network variations. Each variation is the result of applying a threshold $t \in T$ over the original one, thus simulating its dynamic evolution. The threshold affects the topology of the network and changes its degree distribution, represented by the degree histogram, and, consequently, it affects the values of the 9 features compute from the histogram. Thus, by considering different threshold values t, we are able to compute a feature vector φ containing temporary characteristics of the network:

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$$\varphi = [F_k(t_1), \ldots, F_k(t_M)], t_i \in T, \quad (6)$$

where $F_k(t)$ represents a subset of features selected from the 9 features available. Since we have 9 features, there are $2^9 - 1$ combinations of features, i.e. $k = 1, \ldots, 511$. We define a feature combination k as the selection of the features associated with the bits 1's in the binary representation of k. Table I shows this scheme for selecting a subset of features. In this paper, we considered the following order of features during the selection: mean, variance, contrast, energy, entropy, kurtosis, skewness, smoothness and inverse difference moment (IDM).

TABLE I
SCHEME FOR SELECTING A SUBSET OF FEATURES

k	Binary (k)	Features (F_k)
3	000000011	smoothness and IDM
18	000010010	entropy and smoothness
278	100010110	mean, entropy, skewness and smoothness

IV. Experimental evaluation

To evaluate the discrimination ability of our method we designed an experiment using a dataset of artificial 3D models (available at http://segeval.cs.princeton.edu) [26]. This dataset represents a set of 19 different types of 3D models (classes) with 20 samples each, totalising 380 3D objects. Each sample represents a variation (e.g., different orientation, articulation etc) of the 3D model representing the class.

For each model, we computed our approach using different threshold values, t. For each threshold, we computed the degree histogram of the respective complex network and its 9 features, as previously proposed. We evaluated the resulting feature vectors using k-Nearest Neighbor (k-NN) with $k = 1$. This is a very simple classification technique, where each sample is classified according to the k closest training samples in the feature space [27].

To improve the evaluation of our 3D shape description approach, we also implemented and compared ours with other two known approaches: (i) 3D shape histograms and (ii) Shape distributions. A brief description of these approaches is given as follows:

3D shape histograms [8]: this approach uses 3D shape histograms to compute a discrete representation of a 3D object. To accomplish that, the method decomposes the space where the object is using one of the three suggested techniques: (i) shell model, (ii) sector model and (iii) the combination of shell and sector models. For each technique the method obtains a different histogram, where each bin stores the amount of vertices in the correspondent partition of the decomposed space. Since the orientation of the object may affect the bin counting, the methods uses a normalization as a pre-processing step. This normalization moves the center of mass of the object onto the origin and, in the sequence, applies the Principal Axes Transform [8] over the object to ensure that the variance of the objects' points are aligned with the axes. In this experiment, we computed a histogram using 10 shells and 8 sectors instead of the configuration proposed in the original paper (20 sectors and 6 or 12 shells) as it leads to a higher success in this set of artificial 3D models.

Shape distributions [7]: this method uses a sampled probability distribution for describing 3D objects. The method uses a shape function computed from randomly select points on the surface of the 3D object. This results in a histogram (probability distribution) which reflects geometric properties of the 3D object that can be used for its characterization. One drawback of this approach lies in the choice of the shape function, which must be carefully selected to result in a probability distribution that provides a good shape signature. According to authors [7], although many and different shape functions can be used to describe the object, D2 shape function is the one achieving the best results and was used in ours experiments. D2 shape function measures the distribution of Euclidean distances between pairs of randomly selected points on the surface of a 3D model. We used L1 norm of the probability density function of the D2 shape function as the dissimilarity measure. We also performed a normalization step to align the mean sample values of two compared probability density functions.

V. Results and discussion

Initially, our approach computes the euclidean distance among each pair of points of a 3D shape object and, in the sequence, normalizes all values according to the largest one. This is performed to avoid scales variations. As a result, we obtain a complete and regular network as a representation of the original point cloud. Since a regular graph does not present any relevant feature that could be used for 3D model matching, we apply the concept of dynamic evolution to transform the original network into one with desirable properties. Then, for each network, we obtain its degree histogram and a set of 9 features can be computed to be used to discriminate the original 3D object. However, to effectively apply our proposed approach for 3D model description it is necessary to define the set of thresholds T used to perform the dynamic evolution of the network, as also which combination of the 9 features considered yields the best discrimination of the 3D model.

First, we evaluated the impact of different sets of threshold in the classification performance of the method. For this experiment, we considered one single feature from each histogram: the energy of the histogram. Table II shows the results achieved for each set of thresholds. In order to establish each threshold set, we defined the arithmetic progression (AP) $t_n = t_0 + (n-1)t_{inc}$ to mathematically compute them. Results show that the method yields best success rates when the threshold set ranges from 0.020 to 0.500 (Set 1), i.e., the vertices are allowed to connect with other vertices whose positions ranging from close to an intermediate distance. We also notice that the closest vertices are the more significant when building the network. Even if we choose the same number of thresholds (Sets 2 and 3), there is a substantial

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TABLE III
BEST RESULTS AND THEIR RESPECTIVE CONFIGURATIONS

Histogram features		# of descriptors			Success rate (%)		
k	F_k	Set 1	Set 4	Set 7	Set 1	Set 4	Set 7
290	mean, energy and smoothness	75	39	27	66.32	68.16	**70.79**
306	mean, energy, entropy and smoothness	100	52	36	69.21	68.95	70.53
418	mean, variance, energy and smoothness	100	52	36	66.32	67.63	70.53
432	mean, variance, energy and entropy	100	52	36	69.47	70.26	70.00
464	mean, variance, contrast and entropy	100	52	36	68.16	70.26	69.21
434	mean, variance, energy, entropy and smoothness	125	65	45	68.95	69.21	70.79
496	mean, variance, contrast, energy and entropy	125	65	45	**69.74**	**70.79**	69.74
498	mean, variance, contrast, energy, entropy and smoothness	150	78	54	69.21	70.00	70.53

TABLE II
RESULTS ACHIEVED FOR DIFFERENT SETS OF THRESHOLDS WHEN USING ONLY THE ENERGY OF THE HISTOGRAM.

Set #	Initial term (t_0)	Increment (t_{inc})	Final term (t_n)	No of thresholds (n)	Success rate (%)
1	0.020	0.020	0.500	25	60.26
2	0.260	0.020	0.740	25	55.26
3	0.500	0.020	0.980	25	48.42
4	0.020	0.040	0.500	13	59.74
5	0.260	0.040	0.740	13	53.68
6	0.500	0.040	0.980	13	49.47
7	0.020	0.060	0.500	9	58.95
8	0.260	0.060	0.740	9	54.21
9	0.500	0.060	0.980	9	50.79

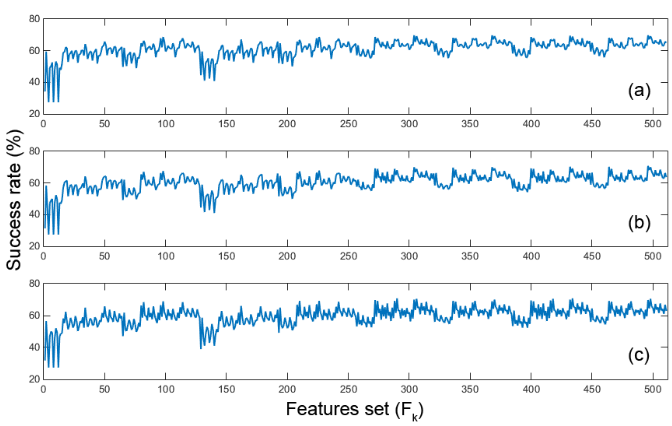

Fig. 2. Success rate yielded for different sets of histogram features for different thresholds sets from Table II: (a) Set 1, $t_{inc} = 0.020$; (b) Set 4, $t_{inc} = 0.040$; (c) Set 7, $t_{inc} = 0.060$.

drop in the success rate if the smaller threshold values are unconsidered.

We also evaluated the impact of the number of thresholds by controlling the increment between them (Table II). As we increase the increment between thresholds, there is a slight decrease in the success rate. Set 4 (59.74%) and Set 7 (58.95%) present, respectively, success rates 0.52% and 1.31% inferior in comparison to Set 1. However, while Set 1 uses 25 thresholds, Set 4 and Set 7 uses, respectively, 13 and 9 threshold values. As one can see, the decrease in the success rate is tolerable if we consider the amount of thresholds used.

TABLE IV
COMPARISON OF THE SUCCESS RATES FOR DIFFERENT SHAPE DESCRIPTORS.

Method	# of descriptors	Objects correctly classified	Success rate(%)
3D shape histogram [8]	640	165	43.42
Shape distribution [7]	99	256	67.37
Proposed approach	27	269	70.79

One could argument that this small variation in the success rate is just perceived when using the energy of the histogram as a descriptor, i.e., for other sets of histogram features F_k the variation in the success rate would be much larger as we increase the increment between thresholds. In order to address this issue, we evaluated all possible sets of histogram features F_k, $k = 1, \ldots, 511$, for the three values of increment used, as shown in Figure 2. In fact, we notice there are variations among the results of the three thresholds sets. However, these variations in results are small as the resulting curves present roughly the same aspect.

Moreover, we also notice that for some combinations of descriptors the success rate increases as we decrease the number of thresholds. To illustrate that, we selected and compared some of the best results and their respective configurations (threshold set and histogram features), as shown in Table III. In this table, the number of descriptors is given as the "number of histogram features" times "the number of thresholds" in the respective set. In general, the use of fewer thresholds achieves a better success rate, which occurs when we increase the increment between them. This indicates that threshold values close to each other may be generating complex networks too similar in terms of properties. The addition of properties too similar to the feature vector acts negatively in the performance, slightly diminishing the success rate.

In Table IV we show success rates yielded for the proposed and the compared approaches. For our approach, we considered the following set of parameters: threshold Set 7 ($t_0 = 0.020$, $t_{inc} = 0.060$ and $t_n = 0.500$) and three histogram features (mean, energy and smoothness). In this

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experiment, results of our approach surpasses the compared methods. Moreover, ours use a smaller set of descriptors, thus proving to be more robust in the discrimination and classification of the 3D shape objects evaluated. In addition to this, we must emphasize that due to the use of Euclidean distance in the computation of the complex network, our approach is invariant to rotation.

On the compared approaches, the good performance of the shape distribution method [7] is explained for two reasons: is also uses Euclidean distance to compute the shape function, and the distances are computed between pairs of randomly selected points, which makes it insensitive to small perturbations (e.g., articulation). The inability of 3D shape histogram method [8] to accurately discriminate objects is explained mostly by the presence of articulation. Even though this method normalizes the 3D shape to achieve rotation invariance, articulation and other small variations disturb how the histogram's bins are mapped, making two similar models different in their histogram representation.

VI. Conclusion

In this paper, we proposed a novel approach to discriminate 3D models based on complex network theory. We investigated how a 3D shape object can be effectively represented as a complex network and how the degree based topological properties provide a feasible set of attributes to characterize and analyze the original object.

We performed an experimental evaluation of our approach using a 3D model data set [26]. We also compared ours with two other methods found in literature: 3D Shape Histograms and Shape Distributions. Results show that our method is able to discriminate different 3D shape objects, significantly surpassing the compared approaches.

For future work, we plan to improve our experimental setup by using a larger data set (e.g., The Princeton Shape Benchmark [28]) and include other 3D shape analysis methods.

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