A Robust Signal Quantization System Based on Error Correcting Codes

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***Abstract*—In this paper we propose a robust representation of a digital signal based on error correction codes. For each frame of the signal (N successive samples) a binary decomposition, as a (successive power 2) weighted sum of binary vectors, is first considered. Then, each binary vector is projected into the set of codewords of a corresponding block code.**

The codes are designed so that their correction powers increases inversely to the weight of the binary vectors since the binary vectors with high weight are less sensitive to disturbance. The corresponding representation (decoding) thus appears as a form of signal quantization that can provide an interesting protection against noise and/or channel distorsion. Some applications showing the utility of the proposed representation are given.

I. Introduction

Channel coding or error correcting coding [1] is one of the basic building blocks of a digital communication system. Since any transmission link, be it wire or wireless, is bound to undergo distorsion and noise corruption, the capacity of codes [2] to correct a part of the resulting errors is necessary for a correct communication. In view of its efficiency, the principle of channel coding has been used beyond the context of robustness in signal transmission, as for instance in cooperative networks [3], [4] and in secrecy coding (see [5] and references therein) to name a few.

In this paper, we exploit the error correction coding for signal representation.

Signal representation is one of the main chapters of signal processing, with a very long and rich history. A basic approach to this is to design a set of appropriate atoms allowing to capture the useful information of the signal with a reduced number of parameters. The purpose of this paper is not to propose such sophisticated and efficient methods as the well established time-scale, time-frequency analysis or the more recent ones exploiting sparsity [6]. The fidelity of the representation, with respect to some objective criterion or with respect to the final end-receiver, is a common key point to all these well elaborated approaches. Now, observe that when the purpose of the representation is, for instance, signal detection and identification or recognition, this fidelity requirement becomes less stringent. If we consider such application context (see [7] and [8] for the identification of audio signal) then we allow the representation to deviate from the original signal. However, the proposed representation is built upon the basic principle of projection into a set of atoms. The atoms are derived here, from the codewords of some error correction codes, following the idea of [9]. More precisely, given a vector of successive samples of the signal, we first consider its binary decomposition as a (successive power 2) weighted sum of binary vectors.

Fig. 1: General idea of our proposition.

Then, each binary vector is projected into the set of codewords of a corresponding block code. The codes are designed so that their correction powers increases inversely to the weight of the binary vectors since the binary vectors with high weight are less sensitive to disturbance. The corresponding representation thus appears as a form of signal quantization. Hence, if the capacity of correction of the codes is high enough, then the quantized signal can be recovered after noise corruption by a simple decoding. The proposed method is described in section II. The applications presented in section III illustrate the interest of the method.

II. Applying channel coding principles to signal

A. Overview of our proposition

We consider a signal transmitted across a noisy channel. The signal undergoes a channel coding at the transmitting part, and the corresponding channel decoding at the receiving part, as illustrated by Fig. 1.

If the signal is an audio signal, coding may be performed in the frequency domain, where perceptual constraints are easier to express. For this purpose, we will use a Modified Discrete Cosine Transform (MDCT [10]), as will be detailled later.

Whatever the coding domain (time or frequency), the coding scheme will differ from a classical channel coding by the fact that the coded signal must be in the same space of representation as the original signal. The principle is the following. A quantized vector X of n time- or frequency-coefficients of the input signal can be expressed as:

$$X = (-1)^{B_{m-1}} \sum_{i=0}^{m-2} B_i 2^i, \tag{1}$$

where each B_i is a binary vector of length n and m is the number of quantization bits. Using block coders, the codewords of which are of length n, each vector B_i will be coded by the binary codeword D_i that is the closest to

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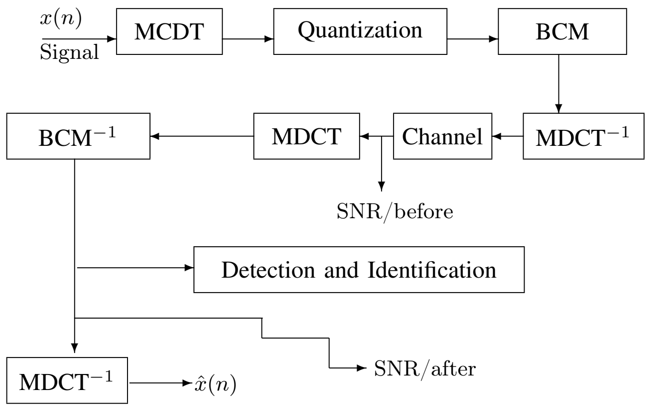

Fig. 2: Complete overview of the channel-coded signal transmission chain.

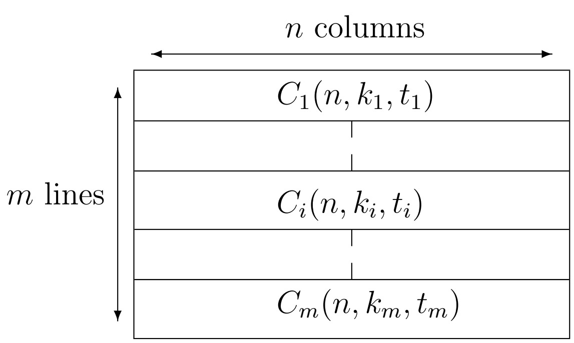

Fig. 3: Principle of a block coded modulation.

itself. Hence, the coded version of X is given by the vector of length n:

$$X_c = (-1)^{D_{m-1}} \sum_{i=0}^{m-2} D_i 2^i \quad (2)$$

The coders will be chosen according to the principles of block coded modulations [9], as explained later.

Finally, we consider the transmission chain represented by Fig 2, which we will detail hereafter.

B. Block coded modulation (BCM)

Introduced in 1977 by Imai and Hirikawa [9], the BCM allows to optimize jointly the coding and the modulation.

The principle of BCM is illustrated by Fig. 3. Considering a M-ary modulation with $M = 2^m$, this coding exploits the fact that the most significant bits are less vulnerable to the noise of the transmission channel. To transmit $k_1 + k_2 + \ldots + k_m$ bits, one codes each word of k_i bits by a block code of length $n \geq k_i$. The resulting $m \times n$ binary matrix is then transmitted as n M-ary symbols. The higher the bit weight, the lower the error rate. Therefore several codes of decreasing error-correction capabilities are used. Codes with higher error-correction capabilities encode the lines of least significant bits, while the lines of most significant bits are encoded by codes with low error-correction capabilities. In other terms, $k_1 \leq k_2 \leq \ldots \leq k_m$.

C. Frequency domain representation

For a time-frequency representation of the signals to be coded, we will use the Modified Discrete Cosine Transform (MDCT [10]). Considering a signal analysis by frames of length N with 50% overlap, each frame is represented by $N/2$ MDCT coefficients. The signal can be perfectly retrieved through applying the inverse MDCT to each vector of $N/2$ MDCT coefficients and recombining the resulting time-domain vectors of length N with 50% overlap, if the analysis and synthesis windows are adequately chosen [11].

This choice of frequency transform is motivated by two main reasons, related to modification of each MDCT vector [12] through channel coding. The first one is that it is preferable to use a transform with an overlap in the synthesis, to avoid discontinuities in the reconstructed signal, which are audible as "clicks" in the case of audio signals. These discontinuities can appear if successive MDCT vectors undergo different modifications. From this point of view, any transform could have been used, provided the perfect reconstruction be ensured. But the other advantage of the MDCT is the following: after an analyzis-modification-synthesis process, if one analyzes again the signal in the MDCT domain, each MDCT vector appears exactly as it was after the modification. In other terms, it is not modified by the frame-overlap of the reconstruction. This property is not verified for analysis-synthesis schemes with other transforms.

D. How to apply BCM to signal coefficients ?

Let $\left(C_i(n, k_i, t_i)\right)_{1 \leq i \leq m}$ be a family of codes of length n, where k_i and t_i denote the dimension of the i^{th} code and its error-correction capacity, respectively.

A matrix of m codes from this family can be used to encode n coefficients quantized with m bits per coefficient. Since the coded signal must be in the same space of representation as the original signal, the classical channel coding scheme consisting in transforming a k-dimensional vector in an n-dimensional one cannot be used here. Instead, each vector X of n coefficients is transformed into another vector X_c of length n as described in subsection II-A, such that for each bit weight i, the codeword D_i is chosen in the codebook of $C_i(n, k_i, t_i)$. Hence, coding here is similar to a decoding process.

As indicated by Fig. 4, the probability of error on each bit depends on the signal to noise ratio (SNR) and on the bit weight. Due to their low error rates, some bits levels (depending on the SNR) do not require any coding, while the lowest levels require a high correction capacity, otherwise the decoding will increase the error rate.

For a given SNR and a given bit weight, let P_e^{before} be the probability of error before decoding. The probability of having k errors on n bits is given by:

$$P^{before}(k/n) = \binom{n}{k} (P_e^{before})^k (1 - P_e^{before})^{(n-k)} \quad (3)$$

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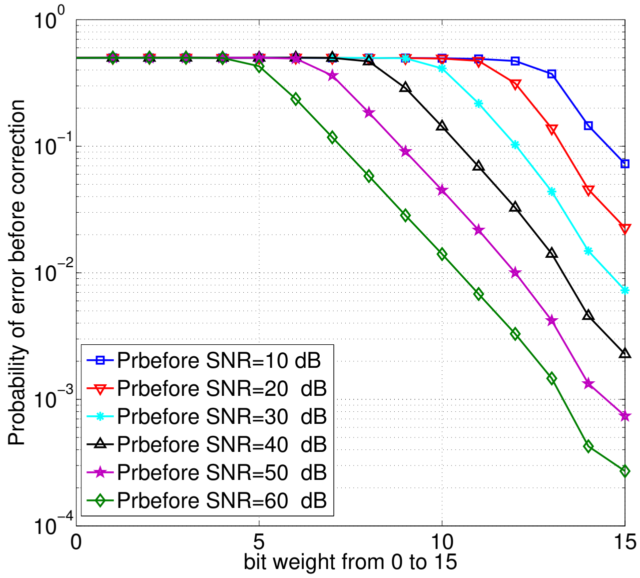

Fig. 4: Probability of error according to the bit weight, for various SNRs. Results from simulations on 310000 samples quantized on 16 bits corrupted by an additive white Gaussian noise.

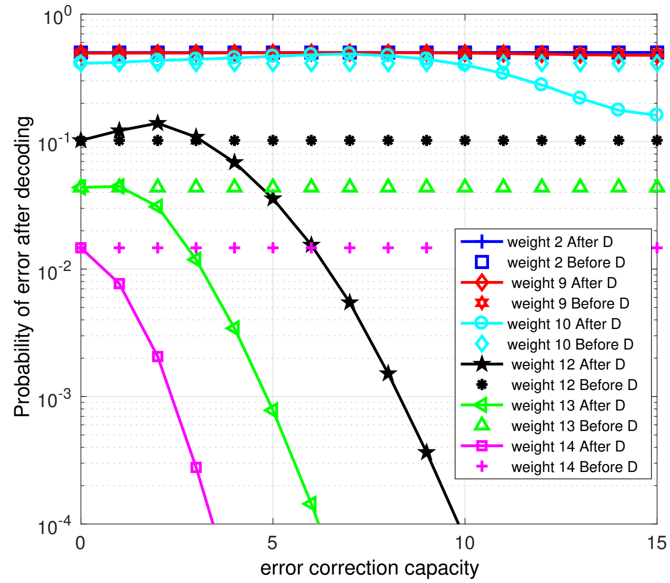

Fig. 5: Probability of error on various bit levels, before and after decoding, versus error-correction capacity of the code, for SNR of 30 dB.

If the code has a correcting capacity $t > 0$, the probability of error after decoding is given by:

$$P_e^{decod} = \frac{1}{n} \sum_{k=t+1}^{n} P^{before}(k/n) N_{err}^{decod}(k/n), t > 0 \quad (4)$$

where $N_{err}^{decod}(k/n)$ denotes the mean number of errors per n bits after decoding when there was k errors on n before decoding. Fig. 5 displays P_e^{decod} versus the correcting capacity t , for various bit weights and a SNR of 30 dB, for $n = 31$.

From these results, one can choose for each SNR and each bit weight a code of which correcting capacity ensures that the binary probability of error after decoding will be lower than before decoding, leading to an increase of the SNR_{dB}.

III. Application to SNR reduction

We simulated the transmission of signals through the chain represented in Fig. 2, where the channel adds a stationary white Gaussian noise.

For each SNR and each bit weight, Eq. (2) provides the probability of error after decoding for any correction capacity t. Hence, drawing curves like those of Fig. 5 allows to choose the appropriate correction capacity to reduce the error rate.

In the following experiments, we used BCH encoders [13], [14] of length $n = 31$, where the error-correction capacities are in the set $\{1, 2, 3, 5, 7\}$. When the required minimum correction capacity exceeds 7, we consider a zero-dimension coding, consisting in replacing the n bits by n zeros for even weights or n ones for odd weights, which zeroes the errors on those bit weights. This alternance of zeros and ones was found to reduce the effect of errors of lower bit levels (with zero-dimension code) on higher bit levels.

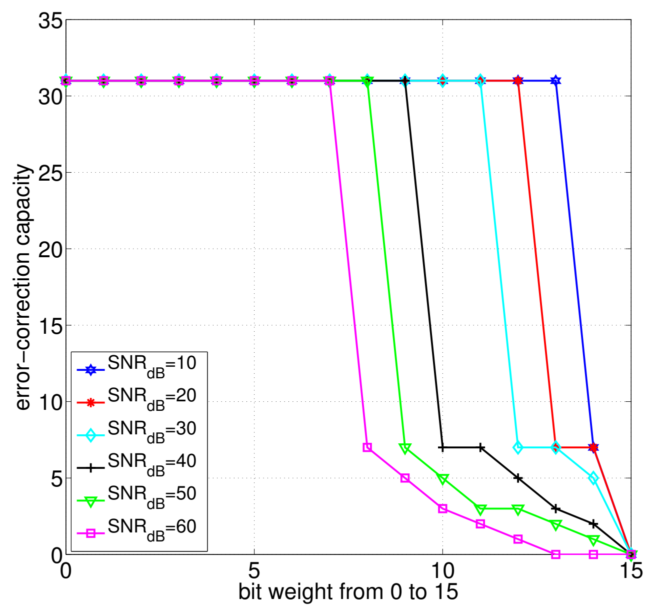

Fig. 6: Error-correction capacity required for each SNR and bit weight.

Fig 6 and Table I indicate for each SNR and each bit weight the minimum required error-correction capacity and the dimension of the code we used, respectively.

A. Results for uniformly distributed samples

We randomly generated a signal of length $n = 31$, according to the uniform law on the integer interval $[-2^{15}; 2^{15} - 1]$, and used it as input of the transmission chain represented in Fig 2, without the MDCT and MDCT^{-1} boxes. This operation was repeated one million times for various SNRs on the channel.

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TABLE I: Code dimension chosen for each SNR_{dB} and each bit weight, to fulfill the minimal error correction capacities indicated by Fig. 6.

Bit weight	15	14	13	12	11	10	9	8	7	6...0
10 dB	31	6	0	0	0	0	0	0	0	0
20 dB	31	6	6	0	0	0	0	0	0	0
30 dB	31	11	6	6	0	0	0	0	0	0
40 dB	31	21	16	11	6	6	0	0	0	0
50 dB	31	26	21	16	16	11	6	0	0	0
60 dB	31	31	31	26	21	16	11	6	0	0

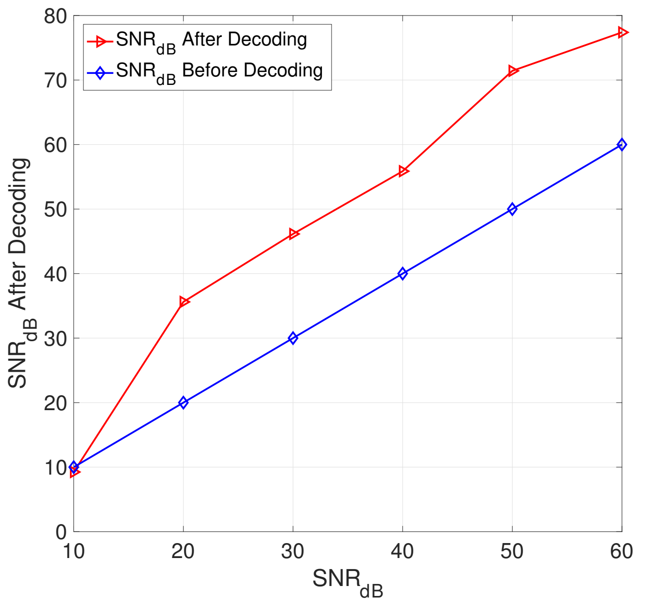

Fig. 7: Signal-to-noise ratio (SNR) after decoding, versus SNR before decoding. Results from 10^6 simulations of a signal of length $n = 31$ generated according to the uniform law on the integer interval $[-2^{15}; 2^{15} - 1]$

For each channel SNR, we computed the SNR of the decoded signal and compared it to the channel SNR (SNR without correction).

The results are represented in Fig. 7. In most cases, as expected, choosing a set of coders ensuring the error-rate reduction for each bit level leads to a clear enhancement of the SNR. For lower SNRs (*e.g.* 10 dB) however, coding reduces the SNR. The reason is that we chose not to code the 15$^{\text{th}}$ bit level, which corresponds to the sign bit, although it is prone to errors.

B. Application to an alarm signal

We applied the proposed coding scheme to an alarm of a priority car. The signal has a duration of 10 s and is sampled at 44100 Hz. Refering to Fig. 2, we considered two ways of coding: (i) in the time domain, by blocks of 31 samples (without the the MDCT and MDCT^{-1} boxes); (ii) in the frequency domain. In the latter case, the MDCT was computed on 1024 coefficients, which were coded by blocks of 31, letting the last coefficient uncoded.

The SNRs after decoding are represented for both cases in Fig 8. The SNR after decoding is greater than the channel

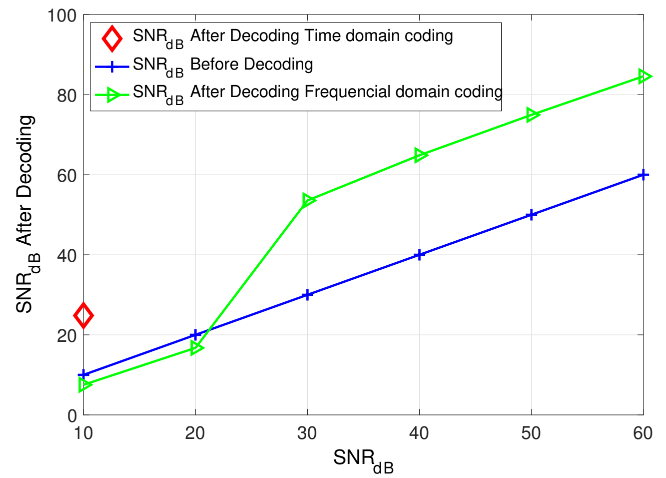

Fig. 8: Signal-to-noise ratio after decoding, versus SNR_{dB} before decoding. Results from simulation on a signal of priority car alarm

SNR in both cases (except for SNR=10 dB and 20 dB before decoding, in the case of frequency domain coding). The SNR for the time-domain coding is infinite for SNR of 20 dB or greater.

For the frequency domain coding, the first and the last windows of the MDCT are not coded, because coding these windows affects the perfect reconstitution of the signal. The non-corrected errors in the first and last windows of MDCT explain why the SNR after decoding for time-domain coding is greater than the SNR after decoding for frequency-domain coding.

C. Distortion of the coded signal

For the previous example of a priority car alarm, and for a channel SNR of 30 dB, Fig. 9 and 10 represent the spectrogram of the original signal and the spectrogram of the signal coded in the time domain, respectively.

The spectrogram structure of the time-domain quantized signal of the priority car alarm is the same as the original one, though more energetical in high frequencies. Coding in time domain does not change much the perception of the alarm, unlike coding in frequential domain, which introduces a perceptible noise in the signal and modifies completely the structure of the spectrogram. The audio files can be heard at http://www.mi.parisdescartes.fr/%7Emahe/Recherche/RobustAudio

IV. Conclusion

We have proposed a quantization based on error correcting codes that makes signals robust to the noise of a transmission channel. This quantization exploits the principles of block coded modulations, through applying codes of different correction capacities to each bit level of a vector of time-domain or frequency-domain coefficients.

We have shown that choosing for each channel SNR and for each bit weight a correction capacity ensuring an error rate lower with than without channel decoding enhances the SNR of the received signal. The counterpart of the proposed method is the distortion caused by the coding process to the signal. However, for signals that do not require a high fidelity to the original one, like an alarm signal, it is possible to robustly code the signal while preserving its time-frequency structure.

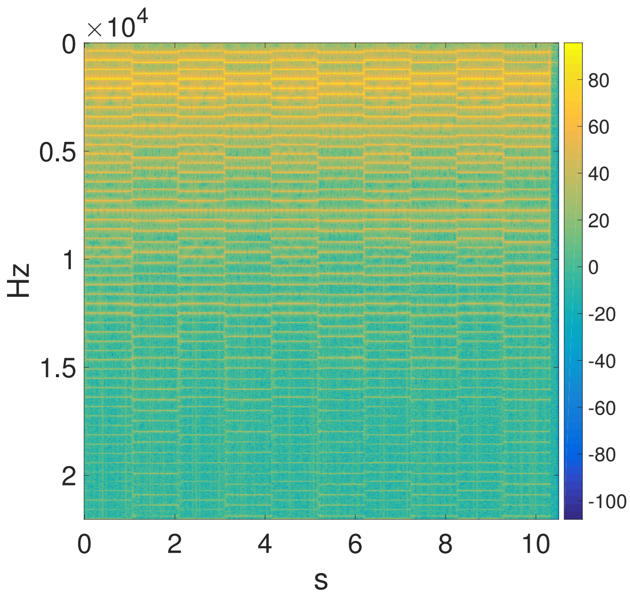

Fig. 9: Spectrogram of a priority car alarm signal

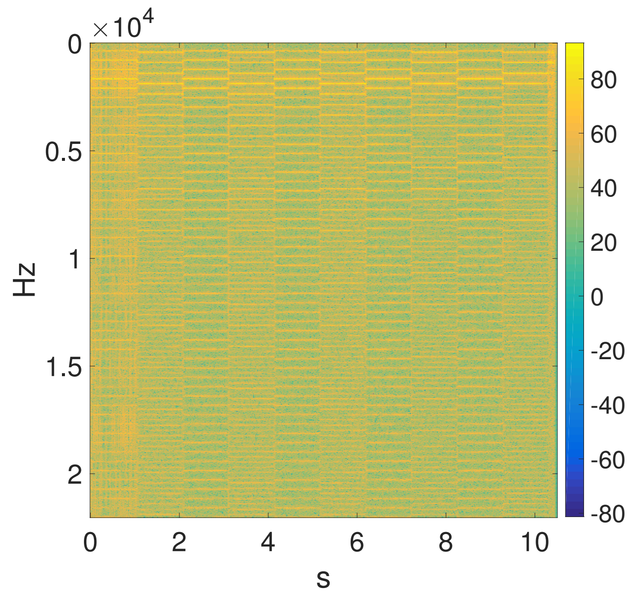

Fig. 10: Spectrogram of the priority car alarm quantized signal, with time-domain coding

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