Robust Precoding Scheme for Multi-user MIMO Visible Light Communication System

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***Abstract*—This paper considers a multi-user multiple-input multiple-output (MU-MIMO) visible light communication (VLC) interference channel. The multi-user interference (MUI) can be successfully eliminated with the perfect knowledge of channel state information (CSI). However, the perfect information may not be available at transmitter, which will lead to severe interference and consequently degrade the system performance. Robust precoding design with the assist of block diagonalization (BD) scheme is proposed to minimize the mean square error (MMSE), which not only completely suppresses the MUI but also maximizes the sum rate of the MU VLC system. Simulation results are presented to validate the effectiveness of the proposed algorithm.**

I. Introduction

In recent years, visible light communication (VLC) has been emerged as research interests to be an alternative technology which can satisfy the growing needs in wireless data and surmount the crowed radio spectrum for wireless communication systems [1]. It can serve room illumination and wireless data transmission simultaneously where the light-emitting diodes (LEDs) had become the suitable lighting equipment because of its characteristic of extreme high switching rate. Due to the limited modulation bandwidth of the LEDs, the VLC spectral efficiency improvement has become a challenging topic. Multiple-input multiple-output (MIMO) techniques can be considered a tremendous technology to achieve high data rate in indoor VLC systems without increasing system bandwidth [2]–[4]. The precoder and equalizer joint design of optical wireless MIMO system is studied with only one user considered [5]. Nevertheless, as the VLC broadcast nature, multiple users should be investigated, in which the system performance could be degraded resulting from the multi-user interference (MUI). Therefore, the MUI elimination is of critical importance for MU VLC system.

Precoding is a promising approach to suppress the MU interference from cellular to VLC systems [6]–[10], [12]. The special case that the one user is equipped with only one photodetectors (PD) has been investigated over MISO VLC systems where the zero-forcing (ZF) beamforming was designed to maximize the achievable sum rate for downlink MISO VLC system [6]. Without perfectly eliminating the interference, the optimal transmit beamforming is designed to maximize the sum rate directly [7]. The optimal transceiver relying on the objective function of minimizing the maximum mean square error (MMSE) between the legitimate transmitted and received signals of the users are designed in [8]. The MU-MIMO optical orthogonal frequency division multiplexing (OOFDM) aided indoor VLC scheme is proposed where the MUI is mitigated by the MIMO precoder [9]. Moreover, the block diagonalization (BD) precoding technique is developed which maximizes the sum rate performance [10]. All above mentioned works are under the assumption at perfect channel state information (CSI). However, the channel uncertainty need to be considered in practical systems. Dirty paper coding (DPC), channel inversion (CI) and BD scheme are implemented in [12] and three different transmission precoding schemes' performance was studied under the imperfect CSI assumption where the considerable bit error rate (BER) can be achieved via BD technique. But the achievable sum rate has not been investigated which is an important metric for evaluating communication efficiency.

In this paper, we study the robust beamforming for MU-MIMO VLC downlink broadcast system where the MUI is the primary challenge resulting in system sum rate degradation. Under the assumption of perfect CSI, the BD technique combining with water-filling algorithm are exploited to entirely eliminate the MUI and the maximum sum rate is achieved. However, in practical system, the channel uncertainty may cause that the MUI cannot perfectly eliminated, and the underlying problem becomes non-convex. As result, a robust precoding design is proposed to minimize the MSE, which is equivalent to the sum-rate optimization problem. Instead of MU-MISO transceiver design by minimize MSE under the assumption of the perfect channel information [8], our approach involves MU-MIMO interference channel and the channel uncertainty. Simulation validates that the proposed robust algorithm can achieve the improvement of sum rate performance in contrast to the case in which channel uncertainty has not been taken into considered.

The rest of the paper is organized as follows. Section II presents the MU-MIMO VLC model. Section III investigates the beamforming design with perfect CSI for MU-MIMO VLC systems. Section IV discusses the robust approach with imperfect channel information while simulation results are demonstrated in Section V. Finally, Section VI concludes this work.

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II. System and Channel Model

We consider an indoor MU-MIMO VLC system that consists of N transmitting LED arrays and K user terminals, each equipped with L LEDs and M_k receiver photodetectors (PDs) respectively. The total number of PDs in this indoor system is $M = \sum_{k=1}^{K} M_k$. The vector $\mathbf{u}_k \in \mathbb{R}^{N\times 1}$ represent the information signal of k-th user, which is zero mean and belongs to $[-1, 1]$. The precoding matrix for the k-th user is $\mathbf{W}_k = \left[\mathbf{w}_{1k}^T, \mathbf{w}_{2k}^T, \ldots, \mathbf{w}_{Nk}^T\right]^T \in \mathbb{R}^{N\times N}$ and each row of $\mathbf{W}_k$ represent a precoding matrix at each LED array for k-th user. The precoded data signal at n-th LED array is $x_n = \sum_{k=1}^{K} \mathbf{w}_{nk}\mathbf{u}_k$. The transmitted optical signal at n-th LED array can be written as

$$s_n = x_n + p_n, \tag{1}$$

where p_n is the direct current (DC) bias and also the average optical intensity of the n-th LED. The constraint to ensure the non-negativity and limited linear range requirement of the LED array input signal can be expressed as

$$0 < s_n \leqslant p_{\max}. \tag{2}$$

Since we have $\mathbf{u}_k \in [-1, 1]$, the range of s_n becomes

$$-\sum_{k=1}^{K} |w_{nk}| + p_n \leqslant s_n \leqslant \sum_{k=1}^{K} |w_{nk}| + p_n, \quad \forall n. \tag{3}$$

Therefore, the constraint for the precoding matrix to ensure (2) can be presented as

$$\sum_{k=1}^{K} \|\mathbf{w}_{nk}\|_1 \leqslant p_n', \quad \forall n, \tag{4}$$

where $p_n' = \min\{p_n, p_{\max} - p_n\}$, and $||\cdot||_1$ is the L_1 norm operator.

There are two types of links in the VLC system which cause the received signals constituted by two components. One is the direct line-of-sight (LOS) component propagated from LED to the receiver and the other is diffuse component caused by reflections from the walls. It is sensible that the volume of our research is just an internal part of a larger indoor normal environment. Therefore, the diffuse component can be neglected [11].

With only the LOS link considered, each element $h_{ij}^{(k)}$ of channel matrix $\mathbf{H}_k \in \mathbb{R}^{M_k\times N}$ denotes the path loss between i-th PD of the k-th user and j-th LED array, where the path loss can be written as [11], [12]

$$h_{ij}^{(k)} = \sum_{l=1}^{L} \frac{AR_o(\phi_{ijl})}{d_{ijl}^2} g(\psi_{ijl}) \cos(\psi_{ijl}), 0 \leqslant \psi_{ijl} \leqslant \Psi_{FOV}, \tag{5}$$

while $h_{ij}^{(k)} = 0$ for $\psi_{ijl} \geqslant \Psi_{FOV}$. In (5), d_{ijl} denotes the distance between the l-th LED in the j-th LED array and i-th PD, ϕ_{ijl} is the angle of emission relative to the optical axis of the LED array, ψ_{ijl} is the incidence angle relative to PD, Ψ_{FOV} denotes the optical field of view (FOV) of the PD and $g(\psi_{ijl})$ is the gain of optical concentrator which given by

$$g(\psi_{ijl}) = \begin{cases} \frac{\kappa^2}{\sin^2(\Psi_{FOV})} & , 0 \leqslant \psi_{ijl} \leqslant \Psi_{FOV}, \\ 0 & , \psi_{ijl} \geqslant \Psi_{FOV}, \end{cases} \tag{6}$$

where κ is the refractive index of concentrator. $R_o(\phi_{ijl})$ is the Lambertain radiant intensity of each LED,

$$R_o(\phi_{ijl}) = \frac{(m+1)}{2\pi} \cos^m(\phi_{ijl}), \tag{7}$$

with m being the order of Lambertain emission determined by the semi-angle for half illuminance of the LED $\Phi_{1/2}$, and $m = \ln 2 / \ln\left(\cos\Phi_{1/2}\right)$.

Hence, the received optical signal after removing the DC components at the k-th user can be expressed as

$$\begin{aligned} \mathbf{r}_k &= \gamma_k \mathbf{H}_k \begin{bmatrix} x_1 & \ldots & x_N \end{bmatrix}^T + \mathbf{z}_k \\ &= \gamma_k \mathbf{H}_k \sum_{j=1}^{K} \mathbf{W}_j \mathbf{u}_j + \mathbf{z}_k \\ &= \gamma_k \mathbf{H}_k \left(\mathbf{W}_k \mathbf{u}_k + \sum_{j\neq k}^{K} \mathbf{W}_j \mathbf{u}_j \right) + \mathbf{z}_k, \end{aligned} \tag{8}$$

where γ_k is the detector responsivity of the k-th PD, $\mathbf{z}_k \in \mathbb{R}^{M_k\times 1}$ is additive white Gaussian noise with zero mean and variance σ_k^2, i.e., $\mathbf{z}_k \sim CN\left(0, \sigma_k^2\right)$ and $\mathbf{W}_k \in \mathbb{R}^{N\times N}$ is the transmit beamformer.

III. Sum rate maximization Problem Formulation

With assumption that the channel state information is perfectly know at transmitter side, the transmitted data can be efficiently precoded and the MUI $\mathbf{H}_k\mathbf{W}_j$ will be diminished by utilizing BD algorithm that is

$$\mathbf{H}_k \mathbf{W}_j = 0, \quad (j \neq k). \tag{9}$$

Supposing

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_1^T & \ldots & \mathbf{H}_{k-1}^T & \mathbf{H}_{k+1}^T & \ldots & \mathbf{H}_K^T \end{bmatrix}, \tag{10}$$

which combines all channel matrices expect the k-th user's matrix $\mathbf{H}_k$, the singular value decomposition (SVD) of $\tilde{\mathbf{H}}_k$ can be obtained as

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k \mathbf{\Sigma}_k \begin{bmatrix} \mathbf{V}_k^{r_k} & \mathbf{V}_k^{N-r_k} \end{bmatrix}^T, \tag{11}$$

where $r_k = \text{rank}(\tilde{\mathbf{H}}_k)$, $\mathbf{V}_k^{r_k}$ holds the first r_k right singular vectors corresponding to non-zero eigenvalues of $\tilde{\mathbf{H}}_k$, and $\mathbf{V}_k^{N-r_k}$ contains the last $(N - r_k)$ right singular vectors corresponding to zero eigenvalues of $\tilde{\mathbf{H}}_k$. Since $\mathbf{V}_k^{N-r_k}$ forms an orthogonal basic for the null space of $\tilde{\mathbf{H}}_k$, $\mathbf{W}_k$ can be any linear combination of $\mathbf{V}_k^{N-r_k}$, i.e. $\mathbf{W}_k = \mathbf{V}_k^{N-r_k}\mathbf{\Lambda}_k$, where $\mathbf{\Lambda}_k \in \mathbb{R}^{(N-r_k)\times N}$ can be any arbitrary matrix subject to the inequality constraint (4). For simplicity, $\mathbf{V}_k$ denotes $\mathbf{V}_k^{N-r_k}$ in the following contents.

The maximum sum rate of MU-MIMO VLC can be

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achieved by solving the following problem, that is,

$$\max_{\mathbf{\Lambda}_k} \quad \frac{1}{2}\sum_{k=1}^{K}\log_2\left|\mathbf{I}+\frac{\gamma_k^2}{\sigma_k^2}\mathbf{H}_k\mathbf{V}_k\mathbf{\Lambda}_k\mathbf{\Lambda}_k^T\mathbf{V}_k^T\mathbf{H}_k^T\right| \tag{12}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\|\mathbf{\Lambda}_k^T\mathbf{V}_k^T\mathbf{e}_n\|_1 \leqslant p_n', \quad n=1,2...N,$$

where $\mathbf{e}_n$ denotes a zero vector with n-th element being one and $\mathbf{I}$ represents the identity matrix. We can treat $\mathbf{H}_k\mathbf{V}_k$ as the effective channel matrix $\mathbf{H}_e$ for k-th user, and it has a singular value decomposition represented as

$$\mathbf{H}_e=\mathbf{H}_k\mathbf{V}_k=\mathbf{U}_e\mathbf{\Sigma}_e\mathbf{V}_e^T. \tag{13}$$

By defining

$$\mathbf{\Lambda}_k=\mathbf{V}_e\text{diag}\left(\sqrt{\lambda_1},\ldots,\sqrt{\lambda_{r_k}}\right), \tag{14}$$

the underlying problem (12) can be represented as

$$\max_{\{\lambda_1,\ldots,\lambda_{r_k}\}} \quad \frac{1}{2}\sum_{k=1}^{K}\log_2\left|\mathbf{I}+\frac{\gamma_k^2}{\sigma_k^2}\mathbf{\Sigma}_k\mathbf{\Sigma}_k^T\text{diag}\left(\lambda_1,\ldots,\lambda_{r_k}\right)\right| \tag{15}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\|\text{diag}\left(\sqrt{\lambda_1},\ldots,\sqrt{\lambda_{r_k}}\right)\mathbf{V}_e^T\mathbf{V}_k^T\mathbf{e}_n\|_1 \leqslant p_n', \forall n,$$

where the optimal solution $\{\lambda_1^{opt},\ldots,\lambda_{r_k}^{opt}\}$ can be obtained via water-filling algorithm [13].

IV. Sum Rate Maximization with Imperfect Channel Information

Due to the imperfection of the CSI in practical system, we develop a robust design to provide the robustness to the uncertainty. We assume that $\hat{\mathbf{H}}$ represents the estimated CSI for MU-MIMO VLC system, and channel estimation errors are taken into account, therefore the real channel in LOS link can be represented as

$$\mathbf{H}=\hat{\mathbf{H}}+\Delta\mathbf{H}, \tag{16}$$

where $\Delta\mathbf{H}$ is the corresponding channel estimation errors modeled as zero-mean Gaussian random model with variance σ_e^2 [5]. The corresponding received signal with imperfect CSI at output of the k-th receiver becomes

$$\mathbf{r}_k=\gamma_k\left(\hat{\mathbf{H}}_k+\Delta\mathbf{H}_k\right)\left(\mathbf{W}_k\mathbf{u}_k+\sum_{j\neq k}^{K}\mathbf{W}_j\mathbf{u}_j\right)+\mathbf{z}_k, \tag{17}$$

where $\Delta\mathbf{H}_k$ can be denoted as $\Delta\mathbf{H}_k=\sigma_e^2\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2}$. The elements of the $M_k\times N$ matrix $\mathbf{H}_w$ are independent and identically distributed (i.i.d) Gaussian random variables with zero mean and unit variance. $\mathbf{R}_r$ is the $M_k\times M_k$ correlation matrix reflecting the correlations between receive antennas, and $\mathbf{R}_t$ is the $N\times N$ correlation matrix reflecting the correlations between transmit antennas. Furthermore, the (i,j)-th entry of the corresponding covariance matrices can be chosen as $\alpha^{|i-j|}$ and $\beta^{|i-j|}$ and the factors α and β represent transmit correlation and receive correlation respectively.

According to [14], we have the mutual information of k-th user,

$$\begin{aligned}\mathcal{I}\left(x_k;y_k|\hat{\mathbf{H}}_k\right)=&h\left(x_k|\hat{\mathbf{H}}_k\right)-h\left(x_k|y_k,\hat{\mathbf{H}}_k\right)\\ \geqslant&\frac{1}{2}\log_2\left|\mathbf{I}+\gamma_k^2\hat{\mathbf{H}}_k\mathbf{W}_k\mathbf{W}_k^T\hat{\mathbf{H}}_k^T\hat{\mathbf{Q}}_k^{-1}\right|,\end{aligned} \tag{18}$$

where $\hat{\mathbf{Q}}_k=\sigma_k^2\mathbf{I}+\gamma_k^2(\sum_{j=1}^{K}\sigma_e^2\,\text{tr}(\mathbf{W}_j\mathbf{W}_j^T\mathbf{R}_t)\mathbf{R}_r+\sum_{j\neq k}^{K}\hat{\mathbf{H}}_k\mathbf{W}_j\mathbf{W}_j^T\hat{\mathbf{H}}_k^T)$. $\text{tr}(\cdot)$ represents the trace of a matrix.

By maximizing the sum of the lower bound of the mutual information over all k, the robust sum rate maximization formulation can be written as

$$\max_{\mathbf{W}_k} \quad \frac{1}{2}\sum_{k=1}^{K}\log_2\left|\mathbf{I}+\gamma_k^2\hat{\mathbf{H}}_k\mathbf{W}_k\mathbf{W}_k^T\hat{\mathbf{H}}_k^T\hat{\mathbf{Q}}_k^{-1}\right| \tag{19}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\|\mathbf{W}_k^T\mathbf{e}_n\|_1 \leqslant p_n', \quad \forall n,$$

which is neither convex or concave with respect to $\mathbf{W}_k$, though the estimated interference can be eliminated via BD technique, that is, $\hat{\mathbf{H}}_k\mathbf{W}_j\mathbf{W}_j^T\hat{\mathbf{H}}_k^T=\mathbf{0}, j\neq k$. In order to overcome the difficulty in solving the non-convex problem, the objective function (19) needs further reformulation.

Base on (9) and (17), the corresponding estimated signal at the k-th receiver can be presented as

$$\hat{\mathbf{u}}_k=\mathbf{F}_k^T\left(\gamma_k\hat{\mathbf{H}}_k\mathbf{W}_k\mathbf{u}_k+\gamma_k\sum_{j=1}^{K}\Delta\mathbf{H}_k\mathbf{W}_j\mathbf{u}_j+\mathbf{z}_k\right), \tag{20}$$

where $\mathbf{F}_k\in\mathbb{R}^{M_k\times N}$ is the receive beamformer. Therefore, the MSE matrix can be expressed as

$$\begin{aligned}\mathbf{E}_k\triangleq&\,\mathbb{E}_{\mathbf{u},\mathbf{z}}\left(\left(\hat{\mathbf{u}}_k-\mathbf{u}_k\right)\left(\hat{\mathbf{u}}_k-\mathbf{u}_k\right)^T\right)\\ =&\,\mathbf{I}-\gamma_k\mathbf{F}_k^T\hat{\mathbf{H}}_k\mathbf{W}_k-\gamma_k\mathbf{W}_k^T\hat{\mathbf{H}}_k^T\mathbf{F}_k\\ &+\gamma_k^2\sum_{j=1}^{K}\sigma_e^2\,\text{tr}\left(\mathbf{W}_j\mathbf{W}_j^T\mathbf{R}_t\right)\mathbf{F}_k^T\mathbf{R}_r\mathbf{F}_k+\sigma_k^2\mathbf{F}_k^T\mathbf{F}_k\\ &+\gamma_k^2\mathbf{F}_k^T\hat{\mathbf{H}}_k\mathbf{W}_k\mathbf{W}_k^T\hat{\mathbf{H}}_k^T\mathbf{F}_k.\end{aligned} \tag{21}$$

By introducing a weight matrix $\mathbf{B}_k$ for each $\mathbf{E}_k$, we constructs the following problem

$$\min_{\mathbf{W}_k,\mathbf{B}_k,\mathbf{F}_k} \quad \frac{1}{2}\sum_{k=1}^{K}\left(\text{tr}\left(\mathbf{B}_k\mathbf{E}_k\right)-\text{logdet}\left(\mathbf{B}_k\right)\right) \tag{22}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\|\mathbf{W}_k^T\mathbf{e}_n\|_1 \leqslant p_n', \quad \forall n,$$

which is convex with respect to $\mathbf{B}_k$, $\mathbf{F}_k$ and $\mathbf{W}_k$, respectively. The optimum solutions can be obtained via alternating minimization approach over $\mathbf{F}_k$, $\mathbf{B}_k$ and $\mathbf{W}_k$ sequently. The algorithm is briefly presented as follows.

a) Given $\mathbf{B}_k$ and $\mathbf{W}_k$, by taking the differential of the objective function with respect to $\mathbf{F}_k$, the optimal solution

$\mathbf{F}_k$ can be obtained by $\frac{\partial \mathbf{E}_k}{\partial \mathbf{F}_k} = 0$, that is,

$$\mathbf{F}_k^* = \gamma_k \left(\gamma_k^2 \hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T + \mathbf{Q}_k \right)^{-1} \hat{\mathbf{H}}_k \mathbf{W}_k, \tag{23}$$

where $\mathbf{Q}_k = \gamma_k^2 \sum_{j=1}^{K} \sigma_e^2 \operatorname{tr}\left(\mathbf{W}_j \mathbf{W}_j^T \mathbf{R}_t\right) \mathbf{R}_r + \sigma_k^2 \mathbf{I}$. By substituting (23) into (21), the minimum MSE matrix is given by

$$\mathbf{E}_k^* = \mathbf{I} - \gamma_k^2 \mathbf{W}_k^T \hat{\mathbf{H}}_k^T \left(\gamma_k^2 \hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T + \mathbf{Q}_k \right)^{-1} \hat{\mathbf{H}}_k \mathbf{W}_k. \tag{24}$$

b) The optimal $\mathbf{B}_k$ can be obtained from the first order optimality condition of the objective function in (22), that is,

$$\mathbf{B}_k^* = \left(\mathbf{E}_k^*\right)^{-1}. \tag{25}$$

Substituting the optimal $\mathbf{B}_k$ into the objective function (22), we have the equivalent problem of the form

$$\begin{aligned} \max_{\mathbf{W}_k} \quad & \frac{1}{2} \sum_{k=1}^{K} \log\det\left(\left(\mathbf{E}_k^*\right)^{-1} \right) \\ \text{s.t.} \quad & \sum_{k=1}^{K} \|\mathbf{W}_k^T \mathbf{e}_n\|_1 \leqslant p_n', \quad \forall n, \end{aligned} \tag{26}$$

where the objective function is equivalent to the one in the robust sum rate maximization formulation (19) with BD algorithm, that is,

$$\begin{aligned} & \frac{1}{2} \sum_{k=1}^{K} \log\det\left(\left(\mathbf{E}_k^*\right)^{-1} \right) \\ =& \frac{1}{2} \sum_{k=1}^{K} \log\det\left(\left(\frac{\mathbf{Q}_k}{\gamma_k^2 \hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T + \mathbf{Q}_k} \right)^{-1} \right) \\ =& \frac{1}{2} \sum_{k=1}^{K} \log\det\left(\mathbf{I}_k + \gamma_k^2 \hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T \mathbf{Q}_k^{-1} \right). \end{aligned} \tag{27}$$

c) Given $\mathbf{F}_k$ and $\mathbf{B}_k$, the optimum $\mathbf{W}_k$ can be achieved by solving the following optimization problem, such as,

$$\begin{aligned} \min_{\mathbf{W}_k} \quad & \operatorname{tr}\left(\mathbf{B}_k \left(\mathbf{I}_k - \gamma_k \mathbf{F}_k^T \hat{\mathbf{H}}_k \mathbf{W}_k - \gamma_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T \mathbf{F}_k \right) \right) \\ & + \operatorname{tr}\left(\mathbf{B}_k \left(\gamma_k^2 \mathbf{F}_k^T \hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^T \hat{\mathbf{H}}_k^T \mathbf{F}_k \right) \right) \\ & + \sum_{j=1}^{K} \gamma_k^2 \sigma_e^2 \operatorname{tr}\left(\mathbf{B}_k \mathbf{F}_k^T R_r \mathbf{F}_k \right) \operatorname{tr}\left(\mathbf{W}_j \mathbf{W}_j^T R_t \right) \\ \text{s.t.} \quad & \sum_{k=1}^{K} \|\mathbf{W}_k^T \mathbf{e}_n\|_1 \leqslant p_n', \quad \forall n, \end{aligned} \tag{28}$$

which is derived from plugging (21) into (22). The optical power constraint here is different from traditional electronic constraints in RF what are often based on the L_2 norm [15]. With the help of equation $\operatorname{tr}\left(\mathbf{A}^H \mathbf{B} \mathbf{A} \mathbf{C}\right) = \operatorname{vec}\left(\mathbf{A}\right)^H \left(\mathbf{C}^T \otimes \mathbf{B}\right) \operatorname{vec}\left(\mathbf{A}\right)$, the last two terms of the objective function in (28) are transformed into quadratic forms. Since the other terms are linear or constant and the constraint is linear, the original optimization problem becomes a convex linearly constrained quadratic program (LCQP) problem which can be solved by CVX to figure out the optimal $\mathbf{W}_k$.

V. Simulation Results and Discussion

In this section, the performance of the proposed sum rate maximization schemes are investigated via simulations in practical VLC scenarios. Four LED TX arrays are located in the ceiling in a $5 \times 5 \times 3$ m^3 space of an indoor environment, with the position [1.5, 1.5, 3], [3.5, 1.5, 3], [1.5, 3.5, 3], [3.5, 2.5, 3], respectively. Two user terminals, each with two PDs, are placed at a height of 0.8 m with 0.02 m separation between PDs of the same user terminal. This configuration constitutes a $4 \times [2, 2]$ MU-MIMO VLC system based on the model presented in Section II. The parameters used for the simulations are listed in Table I unless otherwise specified.

TABLE I
Simulation Parameters

Description	Value
Number of LEDs per Array	60×60
Transmitter Semi-angle	60 deg
Field of View	60 deg
Physical Area of PD	1.0 cm^2
PD Responsively	0.53 A/W

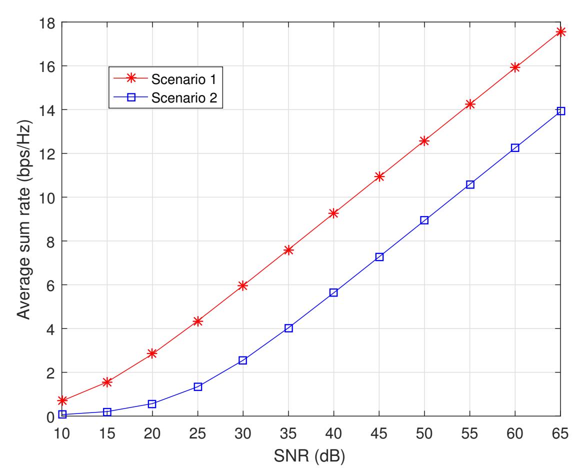

Fig. 1. Sum rate performance of the water-filling algorithm with different user's locations

In the following, we investigate the performance of the water-filling algorithm with the same transmitting semi-angles in all transmitting LEDs. Two scenarios are considered with respect of the distance between two users. In scenario 1, the distance between the two users are relatively far away, which is set as [1, 1, 0.8], [4, 4, 0.8]. While scenario 2 is assumed that the two users are close to each other, where the locations of two users are [2, 2, 0.8] and [2.5, 2.5, 0.8] respectively. Fig. 1 shows the achievable sum rate via the water-filling algorithm with different scenarios, in which a higher sum rate can be achieved when the two users are not near with each other. Moreover, the sum rate performance is degraded by the high channel correlation due to the short distance between the two users, which demonstrates the advantage of spatial multiplexing.

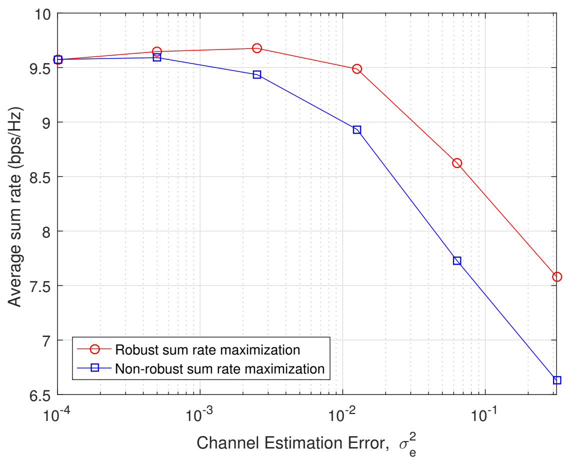

Fig. 2. Impact of the channel estimation error and the robustness of the BD-WMMSE method.

Fig. 2 illustrates the robustness of the proposed design against channel imperfection, where the achievable sum rate is presented. The coordinates of two users are fixed at [2.5, 2.5, 0.8] and [4, 4, 0.8] respectively, $p'_n = 20$ and $\alpha = \beta = 0.5$. As we expected the average sum rate performance improves with the reduction of the channel estimation errors and the proposed robust BD-WMMSE design outperforms the non-robust scheme. When the channel estimation errors becomes small, two algorithms have the similar performance because that the channel uncertainty no longer dominate the achievable SNR .

VI. Conclusion

In this paper, we maximized the average sum rate for indoor MU-MIMO VLC downlink broadcast channel where BD beamforming with the assist of WMMSE algorithm is exploited to suppress the MUI with the channel estimation errors. Robust design is considered to maximize the sum rate with the imperfect CSI, and the optimal solution is obtained by solving the sum-rate maximization problem which is further reformulated as WMMSE problem. Simulation result shows that the robust design can provide robustness against the channel uncertainty.

Acknowledgement

The work of Zhirong Zeng and Huiqin Du are supported by the National Natural Science Foundation of China (No.61401178) and the Open Research Fund of National Mobile Communications Research Laboratory Southeast University(No.2017D12).

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