A Kernel Density-Based Particle Filter for State and Time-Varying Parameter Estimation in Nonlinear State-Space Models

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***Abstract*—In linear/nonlinear dynamical systems, there are many situations where model parameters cannot be obtained a priori or vary with time. As a consequence, the estimation algorithms that are based on the exact knowledge of these model parameters cannot be accurate in this context. In this work, a kernel density-based particle filter is investigated to jointly estimate the states and unknown time-varying parameters of a dynamical system described by nonlinear state and measurement equations. The approach combines an auxiliary particle filter with the kernel smoothing method so as to obtain a stationary kernel density for the unknown parameters. The performance of the proposed approach is investigated for positioning using measurements from a global navigation satellite system that are possibly contaminated by multipath interferences.**

I. Introduction

For state estimation problems in linear/nonlinear dynamical systems, the state and measurement models described in state-space forms need to be known, i.e., the parameters of these models have to be exactly specified a priori [1]. However, there are situations in which the values of these model parameters cannot be obtained a priori or vary with time. As a consequence, the estimation algorithms, which are based on the exact knowledge of the model parameters, can be no longer accurate in this context. Thus the joint state and parameter estimation (i.e., state estimation in the presence of model uncertainty) for linear/nonlinear dynamical systems is a challenging problem in many practical areas, such as target tracking [2], satellite positioning [3] and communication systems [4].

There are three main classes of joint state and parameter estimators for a dynamical system. The first class of methods consists of augmenting the state vector by including the unknown model parameters. Then the parameters can be estimated by using estimation algorithms based on the frame of the Kalman filter, such as the ensemble Kalman filter [5]. In the second class of methods, the problem of joint state and parameter estimation is considered as a special case of maximum-likelihood estimation with incomplete data. The estimation problem is then solved in the frame of the expectation-maximization (EM) algorithm, i.e., the state estimation is performed by using the model parameters in the expectation step and then the model parameters are updated using the estimated state and the corresponding measurements in the maximization step. These two expectation and maximization steps are generally implemented until a convergence condition is satisfied [6]. A third idea is to represent the joint posterior distribution of the state and parameters by using a set of weighted random samples (also known as particles) generated according to an on-line Bayesian approach which is based on sequential Monte Carlo (SMC) techniques [7]. When a new observation becomes available, the particles are updated in order to approximate the joint posterior distribution sequentially. When the unknown parameter vector is static in an on-line Bayesian approach, successive time propagations can lead to particle degeneracy. This problem can be solved by introducing diversity in the set of particles by adding artificial random noise to the particles, i.e., by approximating the static parameters by some slowly changing time-varying ones [8]. The alternative solution, also known as particle learning, is to sample new parameter values at each iteration by constructing sufficient statistics associated with unknown parameters [9]. Regarding applications with time-varying unknown parameters, it is interesting to mention a recent on-line Bayesian approach exploiting a changepoint model for the unknown parameters [10].

This paper considers the case where the unknown parameters in the measurement model is time-varying and is assigned a non-informative prior distribution. A kernel density-based particle filter resulting from a kernel smoothing method embedded into an auxiliary particle filter is investigated to jointly estimate the states and unknown time-varying parameters of a nonlinear dynamical system. The performance of the proposed approach is evaluated by using measurements associated with a global navigation satellite system (GNSS) possibly corrupted by multipath (MP) interferences.

The rest of this paper is organized as follows. Section II introduces a Bayesian formulation of the joint state and

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parameter estimation problem. Section III describes the auxiliary particle filter considered in this paper for handling time-varying parameters. A kernel density smoother is presented in Section IV and the resulting kernel density-based particle filter is investigated in Section V. In Section VI, the proposed approach is used to solve a GNSS-based positioning problem in the presence of MP interferences. Conclusions are finally reported in Section VII.

II. Problem Formulation

The discrete-time state model which describes the propagation of the state vector $\boldsymbol{x}_k$ in a nonlinear dynamical system can be formulated as

$$\boldsymbol{x}_k = \boldsymbol{f}\left(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}\right) \tag{1}$$

where $k = 1, ..., K$ denotes the kth sampling time instant, $\boldsymbol{u}_k$ is an independent and identically distributed (i.i.d.) system noise process whose probability density function (pdf) is assumed to be known and possibly non-Gaussian. The nonlinear measurement model including an unknown time-varying parameter vector $\boldsymbol{\theta}_k$ can be defined as

$$\boldsymbol{z}_k = \boldsymbol{h}\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k, \boldsymbol{v}_k\right) \tag{2}$$

where $\boldsymbol{z}_k$ is a measurement vector at time instant k and $\boldsymbol{v}_k$ is an i.i.d. non-Gaussian measurement noise process. In this work, we assume that the function $\boldsymbol{f}(\cdot)$ is known, whereas the structure of the function $\boldsymbol{h}(\cdot)$ is available and its uncertainty can be expressed as an unknown time-varying parameter vector $\boldsymbol{\theta}_k$ having a non-informative prior distribution $p(\boldsymbol{\theta})$, i.e., some parameters in $\boldsymbol{h}(\cdot)$ randomly change over a finite interval at each time instant. Accordingly, the general state-space model in (1) and (2) can be described by using the following conditional pdfs

$$\boldsymbol{x}_k \sim p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}\right) \tag{3a}$$

$$\boldsymbol{z}_k \sim p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right) \tag{3b}$$

where the state $\boldsymbol{x}_k$ is defined as a first-order Markov process, i.e., its conditional pdf given the past states $\boldsymbol{x}_{0:k-1} = \{\boldsymbol{x}_0, ..., \boldsymbol{x}_{k-1}\}$ only depends on $\boldsymbol{x}_{k-1}$ through the transition pdf $p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}\right)$, the conditional pdf of $\boldsymbol{z}_k$ given the states $\boldsymbol{x}_{0:k}$ and the past measurements $\boldsymbol{z}_{1:k-1} = \{\boldsymbol{z}_1, ..., \boldsymbol{z}_{k-1}\}$ only depends on the state $\boldsymbol{x}_k$ and the unknown parameter vector $\boldsymbol{\theta}_k$ through the measurement pdf $p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right)$. The problem addressed in this paper is to evaluate the posterior pdf $p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k}\right)$ when the unknown time-varying parameter vector $\boldsymbol{\theta}$ in the measurement model has a non-informative prior distribution. According to the Bayesian estimation principle, the posterior pdf $p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k}\right)$ can be recursively updated as follows

$$\begin{aligned} p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k}\right) &\propto p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right) p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k-1}\right) \\ &= p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right) p\left(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}\right) p\left(\boldsymbol{\theta}_k\right) \end{aligned} \tag{4}$$

where

$$p\left(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}\right) = \int p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}\right) p\left(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1}\right) d\boldsymbol{x}_{k-1}. \tag{5}$$

Note that (5) represents a prediction step resulting in the prior pdf of the state at time k. In most practical applications, it is difficult to obtain an analytic solution of the posterior pdf in (4), In these applications, it is quite classical to consider particle filter approximating the posterior distribution of interest by using a set of weighted particles leading to

$$p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k}\right) \approx \sum_{i=1}^{N_\mathrm{s}} \omega_k^i \delta\left(\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k\right) - \left(\boldsymbol{x}_k^i, \boldsymbol{\theta}_k^i\right)\right) \tag{6}$$

where N_s is the number of particles, $\delta(\cdot)$ is the Dirac delta function, $\left(\boldsymbol{x}_k^i, \boldsymbol{\theta}_k^i\right)$ is the ith particle and ω_k^i is an appropriate weight at time k.

III. Auxiliary Particle Filter Including Time-varying Parameters

The choice of the importance distribution in SMC techniques directly impacts the estimation performance. In the general case, it is difficult to determine the optimal importance distribution, which requires the ability to evaluate the integral of the current state [11]. By considering the current measurement $\boldsymbol{z}_k$ before the particles are propagated, the auxiliary particle filter (APF) proposed in [12] generates particles from the sample at time $k-1$ that are most likely to be close to the true state at time k. According to Bayes theorem, the posterior pdf $p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k, i|\boldsymbol{z}_{1:k}\right)$ depending on the state $\boldsymbol{x}_k$, the unknown parameter vector $\boldsymbol{\theta}_k$ and the auxiliary variables i can be derived as

$$p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k, i|\boldsymbol{z}_{1:k}\right) \propto p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right) p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i\right) p\left(\boldsymbol{\theta}_k\right) \omega_{k-1}^i \tag{7}$$

where $i = 1, ..., N_\mathrm{s}$ is the index of the particle at time $k-1$. Accordingly, the importance distribution is defined as follows

$$q\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k, i|\boldsymbol{z}_{1:k}\right) \propto p\left(\boldsymbol{z}_k|\boldsymbol{\mu}_k^i, \boldsymbol{\theta}_k\right) p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i\right) p\left(\boldsymbol{\theta}_k\right) \omega_{k-1}^i \tag{8}$$

where $\boldsymbol{\mu}_k^i$ characterizes $\boldsymbol{x}_k$ given $\boldsymbol{x}_{k-1}^i$ and is usually specified as the expectation, i.e., $\boldsymbol{\mu}_k^i = \mathrm{E}\left[\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i\right]$. The posterior pdf $p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k|\boldsymbol{z}_{1:k}\right)$ can be obtained by marginalizing $p\left(\boldsymbol{x}_k, \boldsymbol{\theta}_k, i|\boldsymbol{z}_{1:k}\right)$ with respect to the auxiliary variable. Thus the sampling weight at time k which is proportional to the ratio of the right-hand side of (7) and (8) is

$$\omega_k \propto \frac{p\left(\boldsymbol{z}_k|\boldsymbol{x}_k, \boldsymbol{\theta}_k\right)}{p\left(\boldsymbol{z}_k|\mu_k^i, \boldsymbol{\theta}_k\right)}. \tag{9}$$

IV. A Kernel Density Smoother

As mentioned above, the prior distribution $p(\boldsymbol{\theta})$ of the unknown time-varying parameter vector $\boldsymbol{\theta}$ is assumed to be non-informative [13], i.e., it carries no information about this parameter vector. A kernel smoothing method was proposed in [8] in order to approximate the prior distribution of $\boldsymbol{\theta}$ by using a mixture of multivariate Gaussian distributions. Using the results of [8], the distribution of the unknown parameter vector $\boldsymbol{\theta}$ at time k can be represented as follows

$$p\left(\boldsymbol{\theta}_k\right) \approx \sum_{i=1}^{N_\mathrm{s}} \omega_k^i \mathcal{N}\left(\boldsymbol{\theta}|\boldsymbol{m}_k^i, h^2 \boldsymbol{V}_k\right) \tag{10}$$

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where $\mathcal{N}(\cdot)$ represents a Gaussian pdf with mean $\boldsymbol{m}_k^i$ and variance $h^2\boldsymbol{V}_k$, h is the kernel smoothing parameter. More precisely, we propose to define the mean value $\boldsymbol{m}_k^i$ and covariance matrix $\boldsymbol{V}_k$ as in [14]

$$\boldsymbol{m}_k^i = a\boldsymbol{\theta}_k^i + (1-a)\,\bar{\boldsymbol{\theta}}_k \tag{11}$$

$$\boldsymbol{V}_k = \sum_{i=1}^{N_\mathrm{s}} \omega_k^i \left(\boldsymbol{\theta}_k^i - \bar{\boldsymbol{\theta}}_k\right)\left(\boldsymbol{\theta}_k^i - \bar{\boldsymbol{\theta}}_k\right)^\mathrm{T} \tag{12}$$

where $\bar{\boldsymbol{\theta}}_k = \sum_{i=1}^{N_\mathrm{s}} \omega_k^i \boldsymbol{\theta}_k^i$, $a = \sqrt{1-h^2}$ is the so-called shrinkage parameter of the kernel mean, which pushes particles $\boldsymbol{\theta}^i$ towards their overall mean $\bar{\boldsymbol{\theta}}$.

V. Kernel Density-Based Particle Filter

The proposed approach, which is referred as to kernel density-based particle filter, can be obtained by embedding the kernel density approximation into the frame of the APF, where the kernel smoothing approximation is computed iteratively until a termination condition is satisfied. The proposed approach mainly consists of three steps that are summarized in Algorithm 1. In the initialization step, the auxiliary particles of the state and the initial samples of the parameters are generated from their corresponding prior distributions, i.e., $\boldsymbol{\mu}_k^i \sim p\left(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i\right)$ and $\boldsymbol{\theta}_k^i(0) \sim p(\cdot)$ defined in (10) with $i = 1, ..., N_\mathrm{s}$. In the iteration step, the parameter samples are iteratively updated by using the current measurement and auxiliary particles of the state. The basic idea behind these iterations is to eliminate parameter samples with small weights and to concentrate on samples with large weights by using the resampling procedure of the APF. Thus parameter samples will converge around a single point after a few iterations at each time instant, i.e., $\bar{\boldsymbol{\theta}}_k$. Moreover, the covariance matrix $\boldsymbol{V}_k$ in the smooth kernel density will converge to stationary values since parameter samples associated with high weights have been statistically selected many times. In the propagation step, the state particles can be propagated by using the APF including parameter samples.

Algorithm 1: Kernel Density-Based Particle Filter.

```
Step 1: Initialization.
    Calculate μ_k^i ~ p(x_k | x_{k-1}^i) and sample parameter particles
    θ_k^i(0) ~ p(·) where i = 1, ..., N_s
Step 2: Iteration.
    FOR r = 1, 2, ... do
      Calculate weights ω_k^i(r) = p(z_k | μ_k^i, θ_k^i(r))
      FOR j = 1, ..., N_s do
        Resample parameter samples with indices i^j(r) from
        probabilities {ω_k^i(r)}_{i=1}^{N_s}
      END FOR
      FOR j = 1, ..., N_s do
        Set parameters θ_k^j(r) = θ_k^{i^j(r)}
        Update parameter samples from the smooth kernel density
        θ_k^j(r+1) ~ N(θ | m_k^j(r), h^2 V_k(r))
        where m_k^j and V_k are computed according to (11) and (12)
      END FOR
      If |θ̄_k(r+1) − θ̄_k(r)| / |θ̄_k(r+1)| < δ or r ≥ r_max where 0 < δ ≪ 1,
      then the iteration terminates, else set r = r + 1
    END FOR
Step 3: Propagation.
    FOR i = 1, ..., N_s do
      Regenerate parameter samples according to probability density
      function θ_k^i ~ N(θ | θ̄_k, V_k)
      Calculate weights ω_k^i = p(z_k | μ_k^i, θ_k^i)
    END FOR
    FOR j = 1, ..., N_s do
      Resample state particles with indices i^j from probabilities
      {ω_k^i}_{i=1}^{N_s}
      Assign weights ω_k^j ∝ p(z_k | x_k^j, θ_k^j) / p(z_k | μ_k^{i^j}, θ_k^j)
    END FOR
```

VI. Experimental Results

A. Simulation Scenario

We have applied the kernel density-based particle filter to a GNSS-based vehicle positioning problem in the presence of MP interferences. In this application, the discrete-time state model describing the propagation of the vehicle state can be formulated as

$$\boldsymbol{x}_k = \boldsymbol{\varPhi}_{k|k-1}\boldsymbol{x}_{k-1} + \boldsymbol{e}_{k-1} \tag{13}$$

with

$$\boldsymbol{x}_k = (x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, b_k, d_k)^\mathrm{T}$$

where $k = 1, ..., K$ denotes the kth time instant, $\boldsymbol{x}_k$ is the state vector containing the vehicle position (x_k, y_k, z_k) and velocity $(\dot{x}_k, \dot{y}_k, \dot{z}_k)$ in the earth-centered earth-fixed (ECEF) frame (Cartesian coordinates), and the GNSS receiver clock offset b_k and drift d_k, $\boldsymbol{e}_k = (e_x, e_y, e_z, e_b, e_d)^\mathrm{T}$ is a zero mean Gaussian noise with covariance matrix $\boldsymbol{Q}_k$. The definitions of the matrices $\boldsymbol{\varPhi}_{k|k-1}$ and $\boldsymbol{Q}_k$ can be found in [15].

As the GNSS receiver tracking loop filters MP interferences whose relative delays vary with time, the remaining MP interferences can be modelled as a time-varying bias affecting the pseudo-range (PR) measurements. In practice, MP interferences not only depend on the relative position between the receiver and GNSS satellites, but also on the environment where the receiver is located, especially in urban canyons. Thus the MP bias affecting the PR measurements rapidly changes when the receiver is moving. Since it is difficult to use a specific propagation model to accurately capture the dynamics of the MP bias, the bias resulting from the MP interferences can be considered as an unknown time-varying parameter vector (that can be assigned a non-informative prior distribution). Accordingly, the mth in-view satellite PR measurement model includes an unknown MP bias at time k and can thus be defined as

$$Z_{m,k} = \sqrt{(x_{m,k}-x_k)^2 + (y_{m,k}-y_k)^2 + (z_{m,k}-z_k)^2} + b_k + \theta_{m,k} + e_{m,k} \tag{14}$$

where $Z_{m,k}$ $(m = 1, ..., M)$ is the PR measurement associated with the mth in-view satellite, M is the number

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TABLE I
SIMULATION PARAMETERS.

Process noise (velocity)	$\sigma_a = 1$ m/s^2
Clock offset noise	$\sigma_b = 3c \times 10^{-10}$ m
Clock drift noise	$\sigma_d = 2\pi c \times 10^{-10}$ m/s
GNSS measurement noise	$\sigma_r = 5$ m

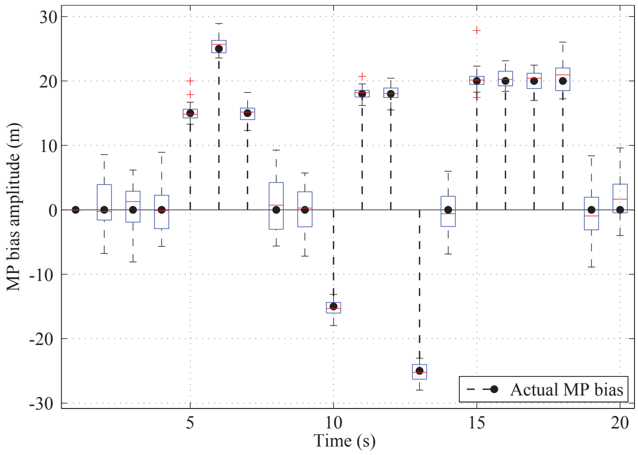
$c = 3 \times 10^8$ m/s denotes the velocity of light.

of in-view satellites, $(x_{m,k}, y_{m,k}, z_{m,k})$ and (x_k, y_k, z_k) are the mth satellite and vehicle positions in the ECEF frame, b_k is the GNSS receiver clock offset, $\theta_{m,k}$ is the MP bias associated with the mth PR measurement, and $e_{m,k}$ is the mth satellite PR measurement noise with a Gaussian distribution $e_{m,k} \sim \mathcal{N}\left(0, \sigma_r^2\right)$.

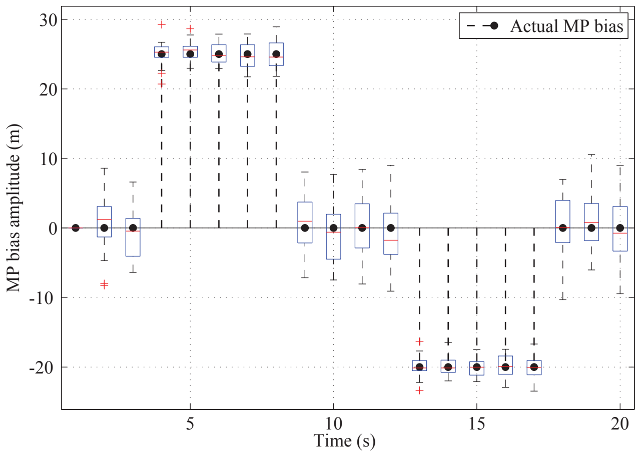
B. Simulation Results

Several simulations have been conducted to study the performance of the proposed approach. The state space model has been first simulated with the parameters given in Table I. In theory, the MP error can reach magnitudes close to 0.5 of a code chip, i.e., 150m in the C/A case, depending on the receiver correlation technology [16]. We have assumed in this study that the prior density of the MP bias θ_m has a non-informative uniform distribution on the interval $(-75\text{m},\ 75\text{m})$ at each time instant, i.e., $\theta_{m,k} \sim \mathrm{U}\left(-75,\ 75\right)$ where $m = 1, ..., M$. We have also assumed that there are 4 in-view satellite pseudo-range measurements during the whole simulation. The simulation time is set to 20s and the estimation period equals 1Hz. Different MP scenarios have been tested by randomly adding MP biases of various amplitudes to pseudo-range measurements of satellites #1 and #2 at specified time instants. The kernel density-based particle filter was implemented with 5000 particles and the smoothing parameter and maximum number of iterations for the kernel density estimator were set to $h = 0.1$ and $r_{\max} = 10$, respectively. $N_{\mathrm{m}} = 50$ Monte Carlo simulations have been run for any scenario to compute the root mean square errors (RMSEs) of the estimates defined by $\sqrt{N_{\mathrm{m}}^{-1} \sum_{i=1}^{N_{\mathrm{m}}} \left(\hat{\boldsymbol{x}}_k\left(i\right) - \boldsymbol{x}_k\right)^2}$, where $\hat{\boldsymbol{x}}_k\left(i\right)$ is the ith state estimate, and $k = 1, \ldots, K$ where K is the number of time instants.

The box plots and RMSEs of 50 MP bias estimates are depicted in Figs. 1 and 2, respectively. The results reported in these figures indicate that the proposed approach provides good results for MP bias estimation. However, due to the effect of the measurement noise, the variations of the estimated biases in the absence of MP interferences are larger than those obtained in the presence of interferences (as shown in Fig. 1), i.e., the RMSE of bias estimates in absence of the MP interferences is relatively larger (as shown in Fig. 2). Considering that the parameter estimation procedure is performed at each time instant in this work, a threshold depending on the covariance of the measurement noise can be specified in order to eliminate the effect of unnecessary MP bias estimates on the state estimate, i.e., the estimate of MP bias can be no longer taken into account in the propagation step of the proposed approach when $\bar{\theta}_{m,k} < 2\sigma_r$.

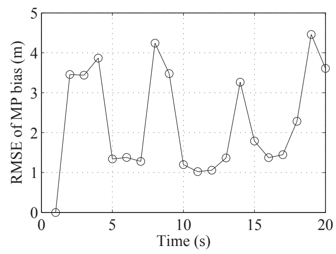


(a) PR measurement #1

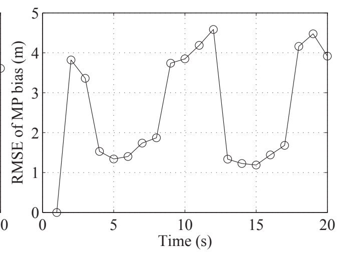


(b) PR measurement #2

Fig. 1. Box plots of 50 MP bias estimates.



(a) PR measurement #1



(b) PR measurement #2

Fig. 2. RMSEs of 50 MP bias estimates.

The successive iterations of $\bar{\theta}_{1,k}$ and $V_{1,k}$ for the MP bias θ affecting the PR measurement #1 at the time instant $k = 16$ are depicted in Fig. 3. It is clear that $\bar{\theta}_{1,k}$ and $V_{1,k}$ converge quickly to a stationary value with iteration times increasing. Thus the smooth kernel density associated with the unknown parameter can be considered as a stationary distribution after

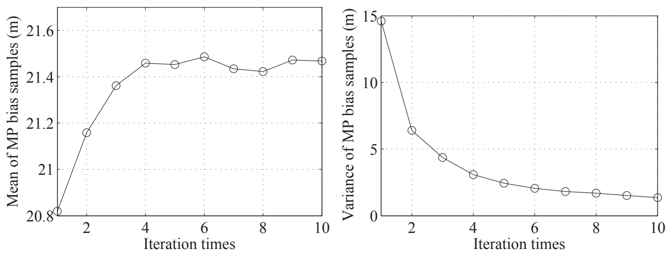
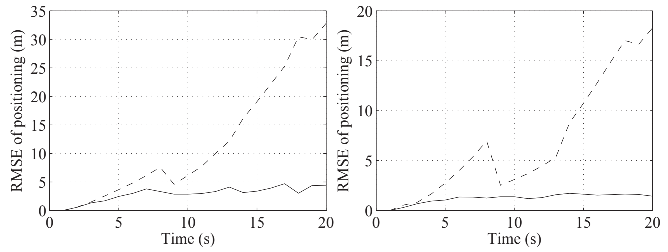


Fig. 3. Changes of mean $\bar{\theta}_{1,k}$ and variance $V_{1,k}$ versus iteration times (Actual MP bias $\theta_{1,k} = 20\mathrm{m}$ where $k = 16$).



(a) Horizontal direction (b) Vertical direction

Fig. 4. RMSEs of state estimates obtained with the proposed approach and the standard APF. Proposed approach: solid line; standard APF: dashed line.

few iterations. Finally, the RMSEs of the estimated positions (the horizontal and vertical directions versus time) with the proposed approach and the standard APF are depicted in Fig. 4. Positioning results can be improved in the presence of MP interferences due to the fact that the uncertainty in the measurement model has been reduced by using the proposed approach. Since the propagation model of the unknown time-varying parameter is not necessary to take into accounted, the proposed approach is very appropriate to the case where parameters of the measurement model are subjected to abrupt changes.

VII. Conclusion

This paper studied a kernel density-based particle filter to jointly estimate the state vector and the unknown time-varying parameters in linear/nonlinear dynamical systems. The proposed approach was obtained by embedding the kernel smoothing method into the frame of the auxiliary particle filter. Its performance was evaluated for a localization problem aiming at estimating a state vector from non-linear state and measurement equations, in the presence of a possible additive bias due to multipath. The proposed approach proved its efficiency for time-varying parameter estimation, resulting in an improved state estimation accuracy. In this work, the parameter estimation procedure was performed at each time instant. Future work includes the consideration of an adaptive detection rule for determining whether the model parameters are subjected to abrupt changes or not. This rule would avoid to estimate the time varying parameter at each time instant, and would thus allow the computational complexity of the algorithm to be reduced.

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