Frequency Domain Multipath Fading Channel Simulator Integrated with OFDM Transmitter for E-UTRAN Baseband Traffic Generator

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***Abstract*—The purpose of the study is to develop an efficient 3GPP compliant method to simulate multiple independent fading radio channels in software defined Evolved Universal Terrestrial Radio Access Network (E-EUTRAN) traffic generator. In this paper, frequency domain representation of commonly accepted Tapped Delay Line (TDL) model is discussed and three transformation algorithms are evaluated. The effects of multipath fading channel are applied to the signal at the level of Orthogonal Frequency Division Multiplexing[1] (OFDM) transmitter prior to IFFT stage. Models 0 and 1 are based on Digital Fourier Transform (DFT) of TDL with and without consideration of Intercarrier Interference (ICI) phenomenon. Model 2 is the novel method that extends quasi-stationary model with low-cost linear approximation of ICI applied directly in frequency domain in order to gain overall accuracy with small computational effort. When limiting the ICI term to 16 neighboring subcarriers, Model 2 exhibits 12 dB SNR improvement comparing to stationary model and offers execution time advantage comparing to TDL model when the number of terminals sharing radio resources is high.**

***Index Terms*—OFDM, Multipath channels, Simulation**

I. Introduction

Long Term Evolution (LTE) is the mobile wireless communication standard published by 3GPP in 2010 as an evolution of UMTS third generation networks. LTE offers a significant increase in system capacity, scalability, spectral efficiency and latency comparing to predecessors like WCDMA or HSPA+. Moreover, new generation networks tend to increase their effectiveness in term of mobility. The end user travelling a high speed train expects to achieve the same network performance as in a stationary environment. This fact requires the new generation base stations to use advanced channel estimation and radio resource scheduling algorithms that can adapt to variable conditions of wireless multi-path fading environment.

Testing and verification are important aspects in a process of modern wireless communication system development. Significant number of different test scenarios characterized by topography, amount of terminals, their positions and velocities makes a stochastic simulation an effective and commonly accepted approach to model a wireless fading channel for link level dynamic capability testing. Many providers on the market of RF test equipment offer different solutions for wireless channel emulation, but all exploit Tapped Delay Line (TDL) channel model. Most of them make use of a standalone hardware module, which includes ADC/DAC chains and FPGA or DSP processing, connected using RF cables between tested LTE base station and a group of User Equipment (UE) devices. In order to reduce complexity of test setup during performance tests when hundreds or thousands of UEs are required, a single traffic generator hardware may be used to emulate the behaviour of E-UTRAN cell from base station's perspective.

Examples of commercial channel simulators are Aeroflex TM500 [1] or Keysight Propsim F8 [2]. However, using such implementation of channel simulator may be ineffective during high-capacity and multi-user performance testing scenarios, when providing uncorrelated fading channels to analog signals coming from multiple users is necessary. The lack of correlation is mandatory to appropriately simulate the real E-UTRAN cell, where multiple attached devices move with different speed in different positions. Emulation of such environment may be required to appropriately test the performance of base station's algorithms. It allows to verify if the scheduler can track and adapt to variations of the wireless channel, as radio resource grants for multiple UE should be optimized for momentary conditions across the frequency band.

Frequency domain representation of fading channel has been well characterized in the literature for the sake of equalization algorithms [3]–[6]. Some of the previous studies have already focused on realization of fading channel simulator in frequency domain. Hardware channel simulator for LTE MIMO systems using Fourier Transform to obtain algebraic product of the FIR filtering has been evaluated [7]. The simulator operates on analog time domain input/output signal and internally applies ADC/DAC conversion and FFT/IFFT operations. It was later shown by the authors that this approach has no real advantage over time domain FIR filtering due to high resource consumption [8]. In [9], the authors proposed a frequency domain autocorrelation function of the fading channel. This method still requires time domain transformation and severely limits the variety of supported frequency correlation profiles. The other approach based on AR modelling proposed in [10] is entirely realized in frequency domain, however, amount of maintained Jakes fading processes depends on the number

[1]In this paper, term OFDM refers both to OFDMA scheme used in LTE downlink and SC-FDMA scheme used in LTE uplink. Equations do not consider D.C. subcarrier for better clarity.

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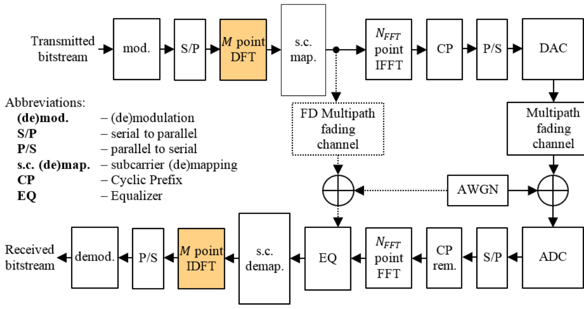

Fig. 1. Block diagram of SC-FDMA scheme. Blocks marked in orange are additions comparing to OFDMA. Dotted lines show an alternative way to simulate multipath fading channel in frequency domain.

of subcarriers, not the number of channel taps in selected Power Delay Profile (PDP) and all coefficients of channel frequency response must be generated regardless of whether they are applied for subcarriers used for data transmission during particular OFDM symbol. A non-unitary DFT based approach was used in [11] to implement frequency-domain channel simulator in GPU. Authors proposed several methods to efficiently generate channel frequency response vector from given PDP. The assumption of channel stationarity was taken during the research without consideration of mobile terminal movement. The lack of comparison with time-domain Tapped Delay Line (TDL) model causes the methods to be incomplete in term of compliance with 3GPP link level simulation requirements.

In this paper, we would like to discuss and compare three DFT based possibilities of efficient applying Rayleigh fading channel model to LTE signal in frequency domain prior to OFDM modulation and Cyclic Prefix insertion. Performance comparison with commonly accepted time domain TDL model [12] will be held. The methods are designated for E-UTRAN simulators, including multi-user LTE uplink emulators where multiple UE may be combined at the stage of Resource Element mapping and share the same OFDM modulator. The main advantage of frequency domain approach is that subcarriers not used for transmission during a given symbol, especially guardbands, can be omitted in a process of frequency response generation, as their energies are set to zero [13]. Moreover, this approach provides a capability to completely bypass time domain transformation, reducing simulation's complexity. The first of discussed methods – Model 0 – ideally preserves channel variances during OFDM symbol period, whereas Model 1 assumes quasi-stationarity of the channel impulse response, gaining speed at the cost of accuracy due to neglect of ICI phenomenon. Model 2 is the novel method which extends Model 1 with linear approximation of ICI, increasing overall accuracy.

Discussed models do not consider a case when Intersymbol Interference (ISI) occurs.

II. LTE Requirements for Fading Models

In [13], 3GPP specifies a set of requirements for LTE base stations in term of uplink signal reception under multipath-fading. The overall multipath propagation conditions are modelled by three components composed together [14]:

- Power Delay Profile in a form of TDL in order to express frequency-selective nature of the channel.
- A Doppler spectrum characterized by maximum Doppler frequency. It is used to simulate time-varying nature of the channel due to motion of terminal.
- Correlation between transmit and receive antennas in case of MIMO systems.

Standard defines three Power Delay Profiles to model different type of propagation environments. The names of the profiles are: Extended Pedestrian A (EPA), Extended Vehicular A (EVA) and Extended Typical Urban (ETU) [13]. The profiles are defined by 7 to 9 taps represented by a pair of excess delay d_l and static attenuation factor a_l, each corresponding to a single delayed copy of transmitted signal. The number of channel taps will be further denoted by L and enumerated by $l = 0, ..., L-1$. Time Domain TDL model, implemented as a FIR filter, is presented in Fig. 2.

Additionally, all taps have to be modulated by a complex random Gaussian processes to simulate the time-variant nature of Rayleigh fading channel. The random processes should be modelled as wide sense stationary and exhibit appropriate time-domain correlation. In frequency domain, their envelope should be distributed according to classical Doppler spectrum [13], [15], also known as Jakes Power Density Function (PDF), which depends on Doppler frequency f_d:

$$S(f) = \begin{cases} \frac{1}{\sqrt{1-(\frac{f}{f_d})^2}} & \text{if } f \in < -f_d, f_d > \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Generation of Rayleigh fading channel coefficients has been extensively studied in the literature and many different methods has been proposed. The appropriate method should be chosen depending on target implementation platform, desired accuracy and computational complexity budget. The most common methods for generating Rayleigh faded complex samples with PDF expressed by (1) are:

- Sum-of-Sinusoids (SOS) method proposed by Jakes [15] and then extensively enhanced [16], [17],
- IDFT method [18],
- noise coloring method [19].

For the sake of the research presented in this paper, the SOS method modification proposed by Zheng and Xiao was chosen [17]. The independent Rayleigh fading channel coefficient $c_l(n)$, modulating the signal in l-th TDL branch, is generated according to the formula given below:

$$\begin{aligned} c_l(n) = \sqrt{\frac{2}{K}}\Big(& \big(\sum_{k=1}^{K} \cos(2\pi f_d T_s n \cos(\frac{2\pi k - \pi + \theta_k}{4\pi}) + \phi_{k,l})\big) + \\ & j\big(\sum_{k=1}^{K} \cos(2\pi f_d T_s n \sin(\frac{2\pi k - \pi + \theta_k}{4\pi}) + \varphi_{k,l})\big)\Big) \end{aligned} \tag{2}$$

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where K is the number of sinusoids, T_s is sample period, $\phi_{k,l}$, $\varphi_{k,l}$ and θ_k are independent random variables uniformly distributed over $[-\pi;\pi)$.

III. Time Domain TDL model

A. Discrete-Time Description

Multipath frequency-selective fading channel with specified PDP applied to digital baseband signal $x(n)$ can be implemented as a FIR filter with time variant impulse response [12]:

$$y(n) = \sum_{m=-\infty}^{\infty} h(n,m)\, x(n-m) + w(n) \tag{3}$$

where $w \sim \mathbb{CN}(\mu,\sigma^2)$ is AWGN and $h(n,m)$ is given by:

$$h(n,m) = \sum_{l=0}^{L-1} a_l\, c_l(n)\, \delta(m-d_l) \tag{4}$$

Scalars a_l and d_l are static attenuation factor and excess delay of tap l in selected PDP. Function $\delta(.)$ is the Kronecker delta.

Presented architecture does not track sample position within the data stream, thus generation of $L \cdot (N_{FFT} + N_u^{CP})$ channel coefficients $h(n,m)$ per single OFDM symbol is necessary as the location of cyclic prefix is not known. In the reported research, the TDL model serves only as a reference.

B. OFDM Block-Based Time Domain Interpretation

In systems based on OFDM techniques like SC-FDMA or OFDMA, cyclic prefix is added at the beginning of each data block. It serves as a guard interval used to eliminate ISI from the previous symbol and is detached at the receiver. Because the data stream is transmitted in blocks, not continuously, (3) can be rewritten to express the data vector y_u received in u-th OFDM symbol after cyclic prefix removal [3].

Let N_{FFT} denotes the FFT size of OFDM system and $\tau = \max(d_l)$ the delay spread of multipath channel. Let x_u and w_u be time domain vectors of transmit data and AWGN vector in symbol u, respectively. The time-varying channel impulse response can be expressed in a form of $N_{FFT} \times N_{FFT}$ matrix, considering that the convolution must be treated as circular due to presence of cyclic prefix [6]:

$$H_u = \begin{bmatrix} h(0,0) & 0 & \cdots & h(0,\tau) & \cdots & h(0,1) \\ \vdots & \ddots & 0 & \cdots & \ddots & \vdots \\ h(\tau,\tau) & \cdots & h(\tau,0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h(N_{FFT}-1,\tau) & \cdots & h(N_{FFT}-1,0) \end{bmatrix} \tag{5}$$

The vector of received data for symbol u in time domain after passing through the multipath fading channel is given by:

$$y_u = H_u\, x_u + w_u \tag{6}$$

Equations (6) and (5) are valid only under assumption that the length of the cyclic prefix N_u^{CP} is longer than the delay spread τ, thus no ISI occurs.

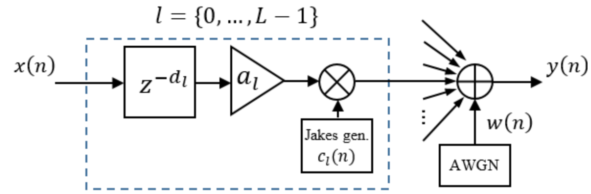

Fig. 2. Time domain TDL model implemented as FIR filter.

IV. Frequency domain representations of TDL

A. Model 0 - Accurate

Let vectors r_u, s_u and η_u be frequency domain representations of y_u, x_u and w_u. The data vector after FFT stage at the OFDM receiver is given by:

$$r_u = G_u\, s_u + \eta_u \tag{7}$$

Channel frequency response matrix G_u is given by [3]:

$$G_u = D\, H_u\, D^{\dagger} \tag{8}$$

where D denotes the DFT matrix and superscript $\dagger$ denotes Hermitian transpose.

In order to preserve high compatibility with time domain model in term of channel variance and ICI, Jakes generators must produce LN_{FFT} channel coefficients per single OFDM symbol. Calculation of matrix G_u and further multiplication by vector s_u requires high amount of operations. Complexness can be reduced considering the fact that H_u is a sparse $N_{FFT} \times N_{FFT}$ matrix with LN_{FFT} non-zero elements and the exact positions of M non-zero values of vector s_u are known. For example, in case of $N_{FFT} = 2048$ configuration, $M \leq 1200$ due to presence of guard subcarriers [13].

Each tap of the channel impulse response is slowly changing over time, thus the frequency spectrum will be mainly concentrated around D.C. DFT bin and negligible energy will be scattered around the remaining part of the spectrum, i.e. the effect of ICI will have the greatest impact on the closest neighbouring subcarriers [4]. This fact may be used to reduce complexity of matrix G_u through reduction to a band matrix $\overline{G}_u$ by rounding elements outside diagonal band of width b to zero:

$$[\overline{G}_u]_{i,j} = \begin{cases} [G_u]_{i,j} & \text{if } j \geq i-b \text{ and } j \leq i+b \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

B. Model 1 - Quasi-stationary Impulse Response

Model 0 can be simplified by assuming channel impulse response invariance during OFDM symbol period. This will introduce an error depending on velocity caused by the fact that the effects of ICI are completely neglected, but complexity will be significantly decreased. As a consequence of channel quasi-stationarity, matrix H_u from (5) becomes a Toeplitz matrix, thus its Fourier Transform pair (10) is diagonal matrix. Equation (8) may be reduced to 1-D non-uniform DFT to compute the vector of diagonal elements [11]:

$$G_u^{(1)}(k) = \sum_{l=0}^{L-1} a_l\, c_l(u)\, e^{-j2\pi k d_l \frac{1}{N_{FFT}}} \tag{10}$$

where superscript (1) denotes Model 1.

Vector of frequency domain received data $r_u^{(1)}$ is given by:

$$r_u^{(1)}(k) = \begin{cases} G_u^{(1)}(k)\, s_u(k) + \eta_u(k) & \text{if } k\text{-th s.c. is alloc.} \\ \eta_u(k) & \text{otherwise} \end{cases} \tag{11}$$

This method requires generation of only L channel coefficients per symbol and $\mathcal{O}(M \cdot (L+1))$ complex add and multiply operation pairs, assuming M non-zero subcarriers in vector s_u.

C. Model 2 - Linear ICI Approximation

Assumption of channel quasi-stationarity can be acceptable for low Doppler frequencies, however, some use cases may require utilization of algorithms that consider interferences between subcarriers, simultaneously being more computationally efficient than Model 0. Model 1 can be extended by low-cost channel variance approximation applied directly in frequency domain without a need for additional on-line DFT computation during simulation.

Because the coherence time of the channel is much lower than OFDM symbol duration, variations of each tap in channel impulse response (5) during a single symbol can be approximated as a straight line with low slope [4]. Frequency response matrix $G_u^{(2)}$ can be expressed as a sum of diagonal matrix $G_u^{(1)}$, with elements given by (10), and matrices Q_l multiplied by scalars, each representing ICI introduced by l-th tap:

$$G_u^{(2)} = G_u^{(1)} + \sum_{l=0}^{L-1} \Delta c_l(u)\, a_l\, Q_l \tag{12}$$

where $\Delta c_l(u)$ is a slope factor of linear approximation for symbol u and tap l. It is to express an average variation of tap coefficient between consecutive OFDM symbols per sample:

$$\Delta c_l(u) = \frac{c_l(u-1) - c_l(u)}{N_{FFT} + N_u^{CP}} \tag{13}$$

Matrix Q_l, computed as DFT of matrix P_l incorporating single tap with unitary slope factor variation, is given by:

$$Q_l = D P_l D^{\dagger} \tag{14}$$

$$P_l = shift\left(\begin{bmatrix} \frac{N_{FFT}}{2}-1 & 0 & \cdots & 0 \\ 0 & \frac{N_{FFT}}{2}-2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & -\frac{N_{FFT}}{2} \end{bmatrix}, d_l \right) \tag{15}$$

where $shift(\cdot)$ is a circular right shift operator defined as:

$$shift\left(\begin{bmatrix} v_{0,0} & \cdots & v_{0,n} \\ \vdots & & \vdots \\ v_{n,0} & \cdots & v_{n,n} \end{bmatrix}, a \right) = \begin{bmatrix} v_{0,a} & \cdots & v_{0,n-a} \\ \vdots & & \vdots \\ v_{n,a} & \cdots & v_{n,n-a} \end{bmatrix} \tag{16}$$

Elements of frequency response matrices Q_l depends only on selected PDP and N_{FFT} which are static simulation parameters. Hence, the matrices can be precomputed and stored in RAM prior to simulation. Matrix Q_l may be reduced to a band matrix $\overline{Q}_l$ analogously as in (9).

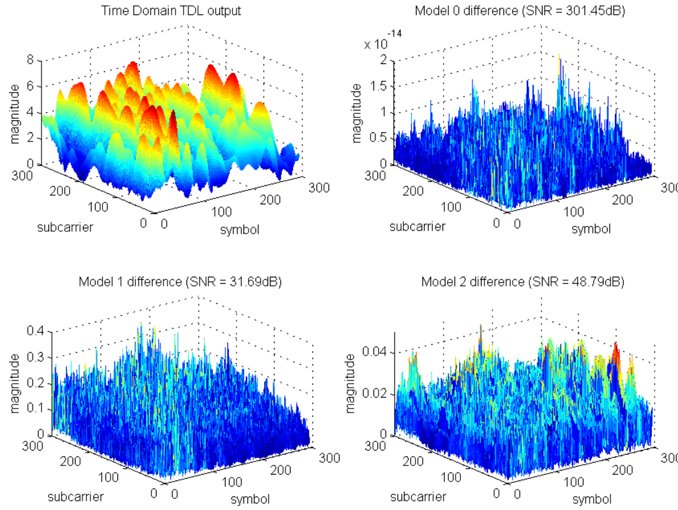

Fig. 3. Difference between frequency domain and time domain models for random QPSK input data. Reduction to band matrix is not used. Configuration: normal cyclic prefix, carrier bandwidth 5MHz, EVA300 profile.

V. Simulation

Frequency domain models are compared against the time domain TDL model in term of accuracy and speed. The algorithms are implemented both in C++ (x86 platform) and MATLAB. AWGN and path loss are not incorporated during simulation. Accuracy is evaluated in a sense of response difference to a randomized stimulus on subcarrier data level (see Fig. 1), measured using Signal to Noise Ratio (SNR) defined as:

$$SNR = 20\log_{10}\frac{\|r_f\|}{\|r_f - r_t\|} \tag{17}$$

where r_f and r_t are received vectors at subcarrier level for frequency and time domain models respectively. Operator $\|v\| = \sqrt{\frac{\sum_{i=1}^{N} v_i^2}{N}}$ denotes Root Mean Square (RMS) of vector $v = [v_1, ..., v_N]$.

Fig. 3 shows time-frequency response comparison per subcarrier without reduction to a band matrix as given by (9). The error of Model 0 is at the level of quantization noise. The SNR of other models is much lower, but Model 2 offers about 17 dB improvement in respect to Model 1. Fig. 4 exposes relationship between Doppler frequency f_d and SNR of Models 0-2 with various numbers of considered neighbouring ICI channels b. The second criterion is a speedup rate depending on number of subcarriers allocated for transmission. Speedup is calculated by dividing frequency domain by time domain model execution period. Measured times include generation of channel coefficients c_l according to (2), taking the number of sinusoids $K = 8$ and Look-Up Table method to resolve trigonometric functions. Fig. 5 shows results for 5 MHz LTE.

Models 2 exhibits similar SNR characteristics as Model 0 for small b, but higher values does not bring noticeable increase of SNR. In considered case of 5 MHz LTE, Model 2 with $b = 16$ exhibits 12 dB improvement comparing to Model 1 on the expense of increase in the computation time, but

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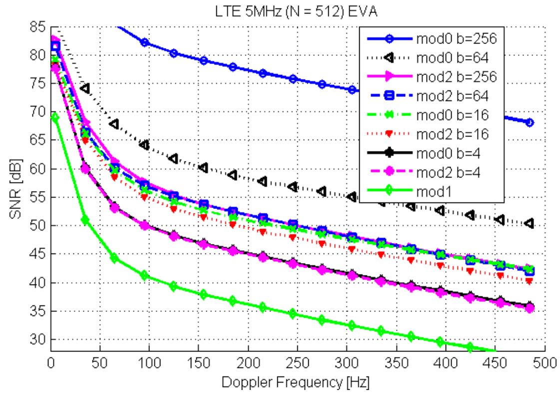

Fig. 4. SNR of frequency domain models for selected values of *b*.

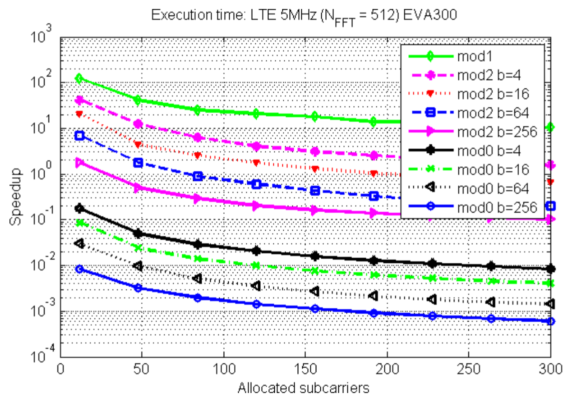

Fig. 5. Execution time improvement of C++ implementation relative to non-stationary TDL.

still offering advantage comparing to TDL model when the allocation size is small, i.e. number of transmitting UE is high.

VI. Conclusion

To cover the minimum 3GPP high-speed scenario requirements for Wide Area Base Station, generation of Doppler spectrum with $f_d = 300\,\text{Hz}$ is mandatory [13]. This corresponds to UE velocity of 120 $\frac{\text{km}}{\text{h}}$ at E-UTRA frequency band 7 [14]. However, Base Station manufacturers may follow more strict internal requirements to improve system performance from the perspective of mobile end user.

Quasi-stationary Model 1 provides good reduction of complexity comparing to time domain TDL model at the cost of accuracy decreasing with Doppler frequency growing. It still can be acceptable for the most practical Doppler frequencies, because the ICI term may be treated as an additional Gaussian noise during equalization [6]. However, this creates a risk of test model oversimplification. Implementation costs of TDL and Model 0 are extremely high when hundreds or thousands of independent terminals are to be simulated. According to simulation results presented in this paper, Model 2 can be used as a good trade-off between accuracy and speed in such scenarios. Additionally, computational complexity of frequency domain models depends on number of subcarriers allocated for transmission during a given symbol, especially no calculations have to be done unless the transmission occur. This fact may be used to increase the performance and capabilities of software based multi-UE traffic generators, as the maximum number of terminals transmitting during the same OFDM symbol is strictly limited.

Further study will be focused on increasing universality and application flexibility of the proposed Model 2 of frequency-domain channel emulation. Then, all discussed algorithms will be implemented on DSP based E-UTRAN traffic generator and optimised for selected hardware platform.

Acknowledgement

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