Sparsity-based Direction of Arrival Estimation in the Presence of Gain/Phase Uncertainty

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***Abstract*— Estimating the direction of arrival (DOA) in sensor arrays is a crucial task in array signal processing systems. This task becomes more difficult when the sensors have gain/phase uncertainty. We have addressed this issue by modeling the problem as a combination of two sparse components, the DOA vector and the gain/phase uncertainty vector. Therefore, a sparse decomposition technique is suggested to jointly recover the DOAs and the sensors with gain/phase uncertainty. The simulation results confirm that the suggested method offers very good performance in different scenarios and is superior to its counterparts.**

***Index Terms*—sparsity, DOA estimation, gain/phase uncertainty**

I. Introduction

Direction Of Arrival (DOA) estimation is an important task in array signal processing and it has been used in different industries such as sonar [1], radar, wireless communications, seismic sensing [2], and radio astronomy [3]. The aim of DOA estimation is to estimate the angle of arrival of signals on an antenna array to increase the sensitivity of the system or enable adaptive beamforming of the antenna pattern. The MUSIC and ESPRIT methods are the conventional decomposition-based DOA estimation techniques [4], [5], [6]. These methods need a considerable number of snapshots to estimate the covariance matrix correctly. The main issue with the conventional methods is that they need a prior knowledge of the number of sources. By the development of sparse signal processing [7] and compressed sensing [8]–[10], advanced DOA estimation techniques have been suggested in the literature. The advantages of these techniques include their robustness against noise and computational efficiency. The L_1-SVD and SPICE are common sparsity-based methods. The L_1-SVD method applies L_1-norm minimization to reconstruct the sparse DOA signal [11]. The sparse iterative covariance-based estimation (SPICE) method is acquired by the minimization of a covariance matrix fitting criterion [12]. The sparse spatial spectral estimation (SpSF) technique estimates the DOA of multiple sources using the sparsity of the spatial covariance matrix [13]. In [14], a sparse reconstruction method is proposed for DOA estimation with the aid of active nonuniform array. These methods have good performance in ideal conditions but in actual arrays there are some errors in the array that degrade the performance of the DOA estimation methods. One of such issues is the gain/phase uncertainty of the antenna array. In [15], a method has been suggested for DOA estimation in the presence of gain/phase uncertainty. In this paper, we address the same issue by another modeling. Using the fact that only a few of the array sensor elements may have gain/phase uncertainty, and also the point that we have small number of targets, we consider the sparsity property for the gain/phase uncertainty as well as the DOA signal. A jointly sparse recovery method is suggested to estimate the DOAs and the gain/phase uncertainty values of the array elements. The performance of the proposed method has been evaluated in different scenarios. The simulation results confirm the superiority of the suggested scheme over its counterparts.

The rest of the paper is organized as follows: In section II, we present the modeling of the DOA estimation problem. The proposed DOA estimation method is illustrated in section III. Section IV includes the simulation results and Section V concludes the paper.

II. System Model

In this section, we describe the DOA estimation problem. We consider an M-element linear array with uniform inter-element space of $d = \lambda/2$ where λ is the wavelength. P signals emitted from P sources impinge on a sensor array. The distinct arrival angles are represented as θ_p . The received signal at time t can be expressed as follows:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\tilde{\mathbf{s}}(t) + \mathbf{n}(t) \tag{1}$$

where $\tilde{\mathbf{s}}(t) = [\tilde{\mathbf{s}}_1(t), \cdots, \tilde{\mathbf{s}}_P(t)]^T$ denotes the signal vector generated by P sources and $\mathbf{n}(t) = [n_1(t), \cdots, n_M(t)]^T$ is the noise vector and $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$ is the recieved signal vector in the sensor array. $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_P)]_{M\times P}$ is the steering matrix where $\mathbf{a}(\theta_p) = [1, e^{-j\frac{2\pi}{\lambda}d\sin(\theta_k)}, \cdots, e^{-j\frac{2\pi}{\lambda}(M-1)\sin(\theta_p)}]^T$. To express the DOA estimation problem in the sparse domain, the angular range is discretized with J uniform samples such that $J >> P$. Therefore, the extended steering matrix $\mathbf{\Psi}$ denotes a dictionary matrix that contains all the possible directions corresponding to the discretized angles: $\mathbf{\Psi} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_J)]_{M\times J}$ Where $\theta_j = 180j/J, j = 1, \cdots, J$. The DOA problem (1) can be represented as:

$$\mathbf{x}(t) = \mathbf{\Psi}\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{s}(t)$ is a $J \times 1$ vector where its q^{th} element is non-zero if $\theta_q = \theta_p$. This problem formulation is appropriate for the ideal case where there is no error in the sensor array. In the case that we have some gain/phase uncertainties in the sensor, we need to have a more correct modeling which would be presented in the next section.

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III. The proposed method

In this section, we illustrate the proposed method. To have a clear modeling of the DOA estimation problem in the presence of gain/phase uncertainty, we consider a matrix for describing the gain/phase uncertainty effect. The gain/phase uncertainty matrix is an $M \times M$ diagonal Matrix $\mathbf{G}' = diag(\mathbf{g})$ where $\mathbf{g}_i = 1 + \mathbf{\Delta g}_i$ and $\mathbf{\Delta g}_i$ indicates for the gain/phase uncertainties of the sensors. Therefore, the array received signal will be described as:

$$\mathbf{x}(t) = \mathbf{G}'\mathbf{\Psi}\mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

For a perfect sensor i, the gain/phase uncertainty is zero, $\mathbf{\Delta g}_i = 0$. The matrix $\mathbf{G}'$ can be written as:

$$\mathbf{G}' = \mathbf{I} + \mathbf{G} \tag{4}$$

Similar to (2), we can rewrite (3) in the form of the dictionary $\mathbf{\Psi}$ (the extended steering matrix) as:

$$\mathbf{x} = \mathbf{\Psi}\mathbf{s} + \mathbf{v} + \mathbf{n} \tag{5}$$

We have omitted the time variable, t, in the above relation where

$$\mathbf{v} = \mathbf{G}\mathbf{\Psi}\mathbf{s} \tag{6}$$

We use the fact that only a few of the antenna elements may have gain/phase uncertainty. Therefore, the matrix $\mathbf{G}$ would have a few non-zero diagonal entries. Considering this property and (6), the vector $\mathbf{v}$ would be sparse such that:

$$support(\mathbf{v}) \subset support(diag(\mathbf{G})) \tag{7}$$

Moreover, since only P entries of s corresponding to the target angles are non-zero, the vector s would be sparse. Utilizing the sparsity of $\mathbf{v}$ and s, we formulate a jointly sparse DOA estimation problem as:

$$\min \|\mathbf{x} - \mathbf{\Psi}\mathbf{s} - \mathbf{v}\|_2^2 + \lambda_1\|\mathbf{s}\|_1 + \lambda_2\|\mathbf{v}\|_1 \tag{8}$$

In order to solve the above optimization problem, we introduce an auxiliary variable, $\mathbf{z}$, and apply the ADMM technique:

$$\begin{aligned} &\min \|\mathbf{x} - \mathbf{\Psi}\mathbf{s} - \mathbf{v}\|_2^2 + \lambda_1\|\mathbf{z}\|_1 + \lambda_2\|\mathbf{v}\|_1 \\ &\quad \text{subject to} \quad \mathbf{z} = \mathbf{s} \end{aligned} \tag{9}$$

The auxiliary variable enables us to decompose the problem. The augmented lagrangian function is obtained as:

$$\begin{aligned} &L(\mathbf{z}, \mathbf{s}, \mathbf{v}, \mathbf{\Lambda}) = \\ &\|\mathbf{x} - \mathbf{\Psi}\mathbf{s} - \mathbf{v}\|_2^2 + \lambda_1\|\mathbf{z}\|_1 + \lambda_2\|\mathbf{v}\|_1 + \rho/2\|\mathbf{z} - \mathbf{s} + \mathbf{\Lambda}/\rho\|_2^2 \end{aligned} \tag{10}$$

where $\mathbf{\Lambda}$ is the dual variable. The first step of the algorithm is obtained by minimizing the lagrangian function with respect to s. Therefore, we have:

$$\frac{\partial L}{\partial \mathbf{s}} = 0 \Rightarrow \mathbf{s}^{k+1} = (2\mathbf{\Psi}^T\mathbf{\Psi} + \rho\mathbf{I})^{-1}(\rho\mathbf{z}^k + 2\mathbf{\Psi}^T(\mathbf{x} - \mathbf{v}^k) + \mathbf{\Lambda}^k) \tag{11}$$

The second step would be to minimize the lagrangian function with respect to the auxiliary variable $\mathbf{z}$ which yields:

$$\frac{\partial L}{\partial \mathbf{z}} = 0 \Rightarrow \mathbf{z}^{k+1} = shrink(\mathbf{s}^{k+1} - \frac{\mathbf{\Lambda}^k}{\rho}, \frac{\lambda_1}{\rho}) \tag{12}$$

where the shrinkage function is defined as:

$$shrink(w, \tau) = \begin{cases} w - \tau & w > \tau \\ w + \tau & w < -\tau \\ 0 & otherwise \end{cases} \tag{13}$$

The third step of the suggested scheme is derived by the minimization with respect to $\mathbf{v}$:

$$\frac{\partial L}{\partial \mathbf{v}} = 0 \Rightarrow \mathbf{v}^{k+1} = shrink(\mathbf{x} - \mathbf{\Psi}\mathbf{s}^{k+1}, \frac{\lambda_2}{2}) \tag{14}$$

The last step would be updating the dual variable:

$$\mathbf{\Lambda}^{k+1} = \mathbf{\Lambda}^k + \rho(\mathbf{z}^{k+1} - \mathbf{s}^{k+1}) \tag{15}$$

The details of the proposed iterative method are given in Algorithm 1.

Algorithm 1 The proposed algorithm

input:
The steering matrix $\mathbf{\Psi} \in \mathbb{C}^{M \times J}$.
The received signal vector $\mathbf{x} \in \mathbb{R}^{M \times 1}$.
The maximum number of iterations $iter_{max}$.
output:
The estimated DOA signal $\widehat{\mathbf{s}} \in \mathbb{R}^J$.
procedure THE PROPOSED METHOD($\widehat{\mathbf{s}}, \mathbf{x}$)
    $\mathbf{\Lambda}^0 \leftarrow 0$
    $\mathbf{s}^0 \leftarrow 0$
    for $k = 1 \ldots iter_{max}$ **do**
        $\mathbf{s}^{k+1} \leftarrow (2\mathbf{\Psi}^T\mathbf{\Psi} + \rho\mathbf{I})^{-1}(\rho\mathbf{z}^k + 2\mathbf{\Psi}^T(\mathbf{x} - \mathbf{v}^k) + \mathbf{\Lambda}^k)$
        $\mathbf{z}^{k+1} \leftarrow shrink(\mathbf{s}^{k+1} - \frac{\mathbf{\Lambda}^k}{\rho}, \frac{\lambda_1}{\rho})$
        $\mathbf{v}^{k+1} = shrink(\mathbf{x} - \mathbf{\Psi}\mathbf{s}^{k+1}, \frac{\lambda_2}{2})$
        $\mathbf{\Lambda}^{k+1} \leftarrow \mathbf{\Lambda}^k + \rho(\mathbf{z}^{k+1} - \mathbf{s}^{k+1})$
    end for
    $\widehat{\mathbf{s}} \leftarrow \mathbf{s}^{iter_{max}}$
    return $\widehat{\mathbf{s}}$
end procedure

IV. Simulation Results

In this section, the simulation results are illustrated. We consider a uniform linear array with 50 sensors and half-wavelength interelement spacing. In our proposed method we set $\lambda_1 = 0.5$, $\lambda_2 = 0.5$ and $\rho = 1.1$ and the maximum number of iteration is considered 100. The time snapshot is $N = 50$ and the method in [15] has been selected as the benchmark algorithm. In the first scenario, we assume that there are signal sources in 6 locations: $\theta = [22°, 30°, 40°, 50°, 63°, 75°]$ and 10% of the array sensors are imperfect (they have gain/phase uncertainty). The MSE of the DOA vector, s, versus SNR is evaluated and depicted in Figure 1.

The MSE defined as:

$$\mathbf{MSE} = \frac{1}{NH}\sum_{i=1}^{N}\sum_{h=1}^{H}(\widehat{\mathbf{S}}_{h,i} - \mathbf{S}_i)^2 \tag{16}$$

where N is number of snapshots and H is number of iterations. With increasing SNR, the MSE decreases for both

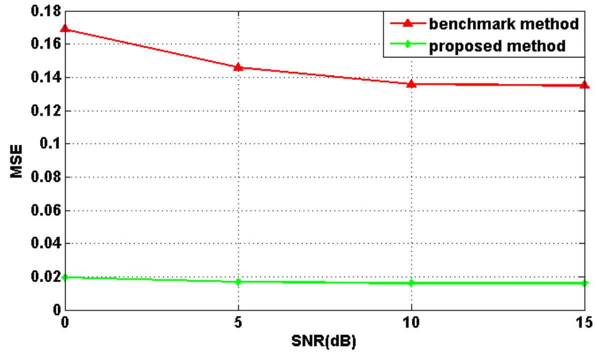

Fig. 1. The MSE of the estimated DOA versus SNR when 10% of the array sensors are imperfect.

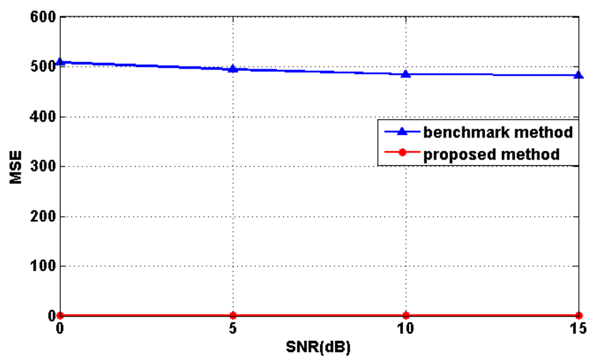

Fig. 2. The MSE of the recovered vector $\mathbf{v}$ versus SNR when 10% of the array sensors are imperfect.

of the methods, however our method achieves much lower DOA estimation error compared to the benchmark. As another comparison, we consider the the MSE of the vector, $\mathbf{v}$, defined in (6) as a measure of the reconstruction of the gain/phase uncertainty values. We plot the MSE of the vector, $\mathbf{v}$, with respect to the SNR value in Figure 2.

We observe that the vector, $\mathbf{v}$, is not estimated so properly using the benchmark algorithm, while our suggested method is capable of resolving the gain/phase uncertainty issue. The main reason that we can mention for this poor performance of the method in [15] is that their problem modeling does not seem to be correct. The authors assume that the gain/phase uncertainty matrix $\mathbf{G}'$ is a low-rank one so the nuclear norm minimization is used to solve the problem. However, this assumption is not true since according to (4), the gain/phase uncertainty matrix is a diagonal matrix with non-zero diagonal entries which yields a full-rank matrix. We believe that this incorrect modeling results in poor performance for the method in [15].

In the second scenario, similar to the pervious case, we consider 10% of array sensors to have gain/phase uncertainty. In contrast to the first scenario, we select 6 source locations uniformly at random which is changed at each snapshot. The average MSE of DOAs over all the snapshots versus SNR has been shown in Figure 3.

According to this figure, we see that the suggested method offers lower estimation error for different SNR values. The

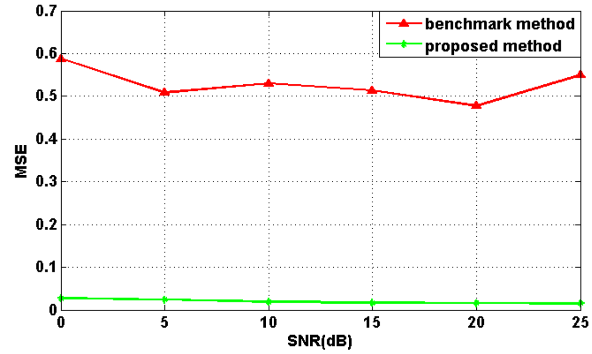

Fig. 3. The average MSE of the estimated DOA versus SNR when 10% of the array sensors are imperfect and source locations change randomly at each snapshot.

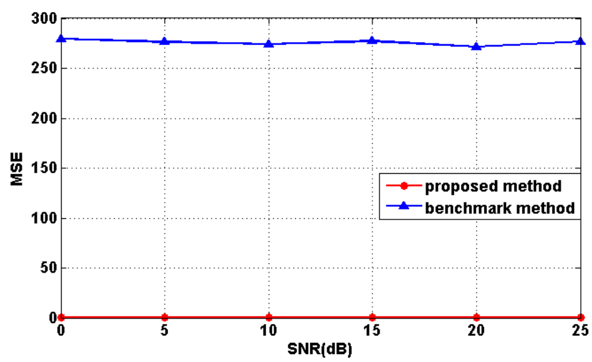

Fig. 4. The average MSE of the recovered vector $\mathbf{v}$ versus SNR when 10% of the array sensors are imperfect and source locations change randomly at each snapshot.

average MSE of the recovered vector $\mathbf{v}$ versus SNR has been plotted in Figure 4.

This figure also indicates that the proposed method performs significantly better than its counterpart.

In the third case, the location of targets are fixed as the first scenario, but the location of imperfect sensors are changed at each snapshot. The average MSE of DOA vector as well as the average MSE of the recovered vector $\mathbf{v}$ versus SNR have been depicted in Figures 5 and 6, respectively.

We observe that both of the methods achieve lower average

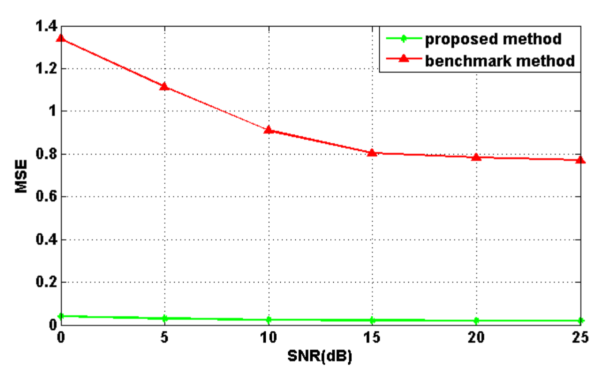

Fig. 5. The average MSE of the estimated DOA versus SNR when the location of the imperfect sensors change randomly at each snapshot.

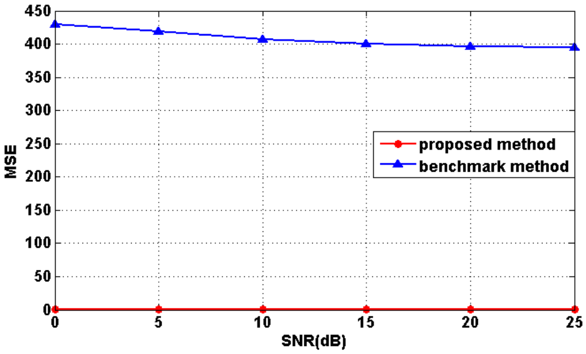

Fig. 6. The average MSE of the recovered vector, $\mathbf{v}$, versus SNR when the location of the imperfect sensors change randomly at each snapshot.

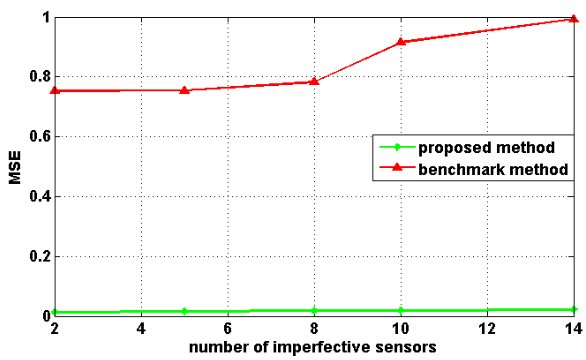

Fig. 7. The MSE of the estimated DOA versus the number of imperfect sensors.

MSE for higher SNR values, however our proposed method outperforms the method in [15], significantly.

In the fourth scenario, we investigate the efficiency of the methods in the case of increasing the number of corrupted sensors. The 6 sources are fixed as the first scenario, and the number of imperfect sensors are changed. Figure 7 indicates the MSE of the estimated DOAs with respect to the number of imperfect sensors.

According to this figure, we see that the estimation error increases with the number of imperfect sensors which is expected. Moreover, similar to the pervious scenarios, the proposed method offers supperior performance compared to its counterpart.

As the last experiment, we consider three incoherent sources with $\theta_1 = 20°$, $\theta_2 = 40°$ and $\theta_3 = 70°$ impinging on a uniform linear array with 50 elements. The SNR value is set to $20dB$. To have a subjective comparison, we depict the estimated DOAs in Figure 8.

According to this figure, the proposed method estimates the DOA of all the three sources properly, however the method in [15] is incapable of finding the correct DOAs.

V. Conclusion

In this paper, we considered the DOA estimation problem in the presence of gain/phase uncertainty of the array sensors. We offer a modeling for the DOAs as well as the gain/phase uncertainties. With the aid of the joint sparsity property of

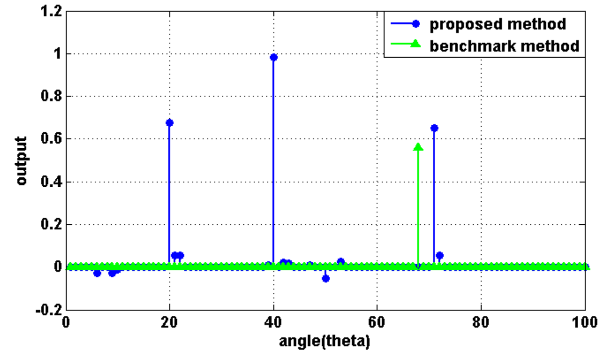

Fig. 8. The estimated DOA in 1 snapshot for SNR=$20dB$.

the DOAs and a vector obtained from the uncertainties, we suggest an iterative algorithm to approximate the DOAs. The simulation results in different scenarios validate the superiority of the suggested method.

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