H_∞ Estimator for Gearbox Diagnosis in Variable Speed Conditions

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***Abstract*—It is well known that a faulty gearbox vibration signal exhibits an amplitude modulation (AM) as well as a phase modulation (PM). These modulation carry out a lot of useful information about health condition. This paper presents two approaches for modeling amplitude and phase modulation in gearbox vibration signal. These last are used to describe the vibration signal by a state space model. Then, the H_∞ estimator is designed to estimate the modulation appearing in the vibration signal. This estimator is obtained by minimizing the worst possible amplification effects of disturbances (measurement and modeling noises) on the estimation error. Such an estimator does not require any assumption on the statistic properties of the noises. Since additive noises in gearbox vibration signal are non Gaussian and non white, this estimator is more suitable in practical gearbox diagnosis. To evaluate the performance of the two approaches, we use a synthetic and an experimental gearbox vibration signal.**

***Index Terms*—Gearbox diagnosis, variable speed conditions, H_∞ estimator, Legendre polynomials**

I. Introduction

The analysis of gearbox fault is well established in the literature. It is known that when a fault occurs on a gear tooth, the vibration signal exhibits an amplitude modulation (AM) and a phase modulation (PM). McFadden has proposed a highly effective technique to estimate the modulations using a time domain synchronous average and Hilbert transform [1]. He proved that the analysis of these modulations is powerful for detecting the presence of a fault. However, the effectiveness of this technique is subject to the selection of band width for the bandpass filtering [2]. This technique fails when it comes to the non-stationary case. The non-stationary phenomena exist in rotating machine due to the change of the load (in crusher machine) and to the external conditions for e.g in wind turbines. One of the methods to process a non-stationary signal is order tracking. Two classes of method were developed for this task: the non-parametric and parametric estimation. The first class uses the joint time-frequency methods to the analysis of non-stationary signal since they are able to provide an overall view of the signal structure. During these three last decades, many authors have investigated this subject. And most of the non-parametric techniques use the windowed Fourier Transform (WFT), the Wigner-Ville distribution (WVD) and the wavelet transform. The application of such a methodology concerns the analysis of highly transient phenomena in machines. Their ability to track these transient events is challenging. These methods are limited by the well-known compromise between the time and frequency resolution due to 'uncertainty principle'. In addition, the resolution of the WFT depends on the type of the window applied. Also, the performance of the WVD is seriously influenced by the so-called 'cross-terms' and 'negative energy' that affect the interpretation of the time-frequency distribution [3]. And the use of the wavelet transform is interesting to analyze the relatively strong frequency component. But it gives a significant error for dealing with higher frequency components [4].

In the other hand, Kalman estimator is a tool among the parametric estimation method for the diagnosis of gear fault under non-stationary conditions. Zhan and Jardine [5], [6] have presented an interesting modified Kalman estimator using an adaptive autoregressive modeling for gear fault diagnosis. In the 1997, Vold developed the so-called Vold-Kalman tracking filter to estimate multiple frequency components [7]. Besides, Pan et al. proposed some enhanced derivatives of the Vold-Kalman filter [8], [9]. All these approaches based on the Kalman estimator is designed with the hypothesis of a white-Gaussian noise. This unrealistic assumption on the noises naturally limits its application in several experimental situations. And when the noises are colored, the Kalman based estimation may becomes suboptimal [10].

In the 1990 years, a new approach to design an optimal estimator from a linear state space model appeared. This estimator called H_∞ estimator consists in minimizing the worst possible amplification effects of disturbances (measurement and modeling noises) on the estimation error. The main interest of the H_∞ estimator is that it doesn't require any assumption on the noise source statistics. The noise signals must only be of bounded energy. In practical vibration signal analysis, where there is significant uncertainty in the noise statistics and the vibration signal model, the H_∞ estimator is more suitable. The effectiveness of this estimator has already been demonstrated in these previous works [11], [12]. In this work, we use this estimator to estimate the modulation signal.

The H_∞ estimator is used with a state space model. And one of the most used method to model these modulation is the stochastic smoothness constraint [8]. It consists to assume that the second derivative of the modulation is a white Gaussian noise. In fact, this approach is equivalent to model the modulation by a polynomial. In this paper, we introduce a new approach using an orthogonal polynomial, "the discrete

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Legendre polynomials", for modeling the modulation. Then, we compare the performances obtained by the two modeling approach combined with the H_∞ estimator.
This paper is organized as follows. In Section II, the H_∞ estimator is introduced. In the same section the two approaches for modeling the AM and the PM are presented. Based on the output signal-to-noise ratio, the performance of the H_∞ estimator using the two modeling approaches is analyzed in Section III. In Section IV, we present the results obtained by applying the proposed method to experimental gearbox vibration signal. Our conclusions are given in Section V.

II. H_∞ ESTIMATION ALGORITHM

A. From vibration signal to state model

The gearbox vibration signal can be modeled by AM-PM process where the carrier frequency is the gear meshing frequency and its harmonics [1]. If a localized fault occurs on gear tooth, the modulation functions will be affected and will change periodically at the rotating frequency of the faulty gear. Thus, under non-stationary conditions, the time-varying modulated signal in discrete time can be described by the following equation

$$y_k = \sum_{i=1}^{M} A_{i,k} \cos\left(\theta_{i,k} + \phi_{i,k}\right) + v_k \tag{1}$$

where:

- M is the *a priori* significant number of components of the observed signal,
- $\theta_{i,k} = 2\pi i \sum_{j=1}^{k} \frac{f_{mesh,j}}{f_s}$ for $i = 1...M$ is the instantaneous angular displacement in which $f_{mesh,j}$ is the instantaneous gear meshing frequency at jth instant and f_s is the sampling frequency,
- v_k is the unwanted part of the signal at kth instant,
- $A_{i,k}$ and $\phi_{i,k}$ are respectively AM and the PM functions of the ith component.

The amplitude and phase modulation include components having the periodicity of both the driving and driven gear.
Define the carrier frequency matrix by $\boldsymbol{h_{i,k}} = \begin{bmatrix} \cos(\theta_{i,k}) & -\sin(\theta_{i,k}) \end{bmatrix}^T$ and the instantaneous modulating envelope by $\boldsymbol{a_{i,k}} = \begin{bmatrix} A_{i,k}\cos(\phi_{i,k}) \\ A_{i,k}\sin(\phi_{i,k}) \end{bmatrix} = \begin{bmatrix} a^c_{i,k} \\ a^s_{i,k} \end{bmatrix}$, the signal is then written as

$$y_k = \sum_{i=1}^{M} \boldsymbol{h^T_{i,k}} \boldsymbol{a_{i,k}} + v_k \tag{2}$$

where $[\bullet]^T$ stands for the transpose symbol. The AM and PM of interest are now embedded in the envelope $\boldsymbol{a_{i,k}}$. These last are unknown and time-varying. Two approaches of modeling these modulations are presented in this paper.

1) The stochastic smoothness constraint (SSC): This approach is presented by Pan et al [8] where

$$a_{i,k+1} - 2a_{i,k} + a_{i,k-1} = w_k \tag{3}$$

The state model related to this approach is developed in [8]. This model is equivalent to use a polynomial of degree two which may be non orthogonal.

2) Orthogonal Legendre polynomial approximation (OLPA): This approach consist to model $a^c_{i,k}$ and $a^s_{i,k}$ by a polynomial function of power of time. For this, we use the discrete orthogonal Legendre polynomial. Jabloun et al. [**?**] mention that using orthogonal polynomial base improves the estimation accuracy. Our modeling is as follows:

$$a^c_{i,k} = \sum_{d=0}^{D} \alpha_{i,d,k} L_{d,k} \tag{4}$$

and

$$a^s_{i,k} = \sum_{d=0}^{D} \beta_{i,d,k}(t) L_{d,k} \tag{5}$$

where $\alpha_{i,d,k}$ and $\beta_{i,d,k}$ are the model parameters and $L_{d,k}$ is the Legendre polynomial of degree d for $d = 0, \ldots, D$. The discrete Legendre polynomial is given by this following recursive relation [17]:

$$L_{0,k} = 1 \tag{6}$$

$$L_{1,k} = 1 - \frac{2k}{N} \tag{7}$$

$$\begin{aligned} L_{d,k} =& \left(2d-1\right)\left(N-2k\right) L_{d-1,k} \\ &- \frac{\left(d-1\right)\left(N+d\right) L_{d-2,k}}{d\left(N-d+1\right)}, d \geq 2 \end{aligned} \tag{8}$$

where N is the length of the data. The set $\{L_{d,k}\}_{0 \leq d \leq D}$ are the discrete polynomials orthogonal over the range $0 \leq k \leq N$. Define the vector

$$\boldsymbol{x_{i,k}} = \begin{bmatrix} \alpha_{i,0,k} & \ldots & \alpha_{i,d,k} & \beta_{i,0,k} & \ldots & \beta_{i,d,k} \end{bmatrix}^T \tag{9}$$

and let all the model parameters be assembled in the vector $\boldsymbol{x}$

$$\boldsymbol{x_k} = \begin{bmatrix} \boldsymbol{x_{1,k}}^T & \boldsymbol{x_{2,k}}^T & \ldots & \boldsymbol{x_{M,k}}^T \end{bmatrix}^T \tag{10}$$

Similary, let

$$\boldsymbol{b_k} = \begin{bmatrix} L_{0,k} & L_{1,k} & \ldots & L_{D,k} \end{bmatrix}^T \tag{11}$$

be the Legendre polynomial base and

$$\boldsymbol{\bar{h}_{i,k}} = \begin{bmatrix} \boldsymbol{b_k}^T \cos(\theta_{i,k}) & -\boldsymbol{b_k}^T \sin(\theta_{i,k}) \end{bmatrix}^T \tag{12}$$

be the ith measurement vector. Hence, we can write the vibration signal as

$$y_k = \boldsymbol{h_k}^T \boldsymbol{x_k} + v_k \tag{13}$$

where $\boldsymbol{h_k} = \begin{bmatrix} \boldsymbol{\bar{h}_{1,k}}^T & \boldsymbol{\bar{h}_{2,k}}^T & \ldots & \boldsymbol{\bar{h}_{M,k}}^T \end{bmatrix}^T$ is a known measurement vector. Since the parameter vector are time-varying, we impose a first order stochastic smoothness constraint. This can be written as

$$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \boldsymbol{w_k} \tag{14}$$

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where $\boldsymbol{w_k}$ is the unknown drift term. Equations (13) and (14) represent the state model of the vibration signal embedded in the following system

$$\begin{cases} \boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \boldsymbol{w_k} \\ y_k = \boldsymbol{h_k}^T \boldsymbol{x_k} + v_k \end{cases} \quad (15)$$

Then, the fault detection problem is stated as follows. Given a measurement y_k, we have to estimate the model parameters $\alpha_{i,d,k}$ and $\beta_{i,d,k}$ for $i = 1, \ldots, M$, $d = 0, \ldots, D$ and $k = 1, \ldots, N$.

B. H_∞ estimator

Consider the state space model (15). We make no assumptions on the nature of the unknown quantities $\boldsymbol{w_k}$ and v_k. They must only have bounded energy. Let $\boldsymbol{e_k} = \boldsymbol{x_k} - \boldsymbol{\hat{x}_k}$ be the estimation error when $\boldsymbol{\hat{x}_k}$ is the estimate of $\boldsymbol{x_k}$. Different from the Kalman estimator which minimizes the variance of the estimation error, the H_∞ estimator is designed to provide a uniformly small estimation error for any $\boldsymbol{w_k}$, v_k and $\boldsymbol{x_1}$. The cost function for that is then given by

$$J = \frac{\sum_{k=1}^{N} \|\boldsymbol{e_k}\|^2}{\|\boldsymbol{e_1}\|^2_{\boldsymbol{P_1}^{-1}} + \sum_{k=1}^{N} \left(\|w_k\|^2_{\boldsymbol{Q}^{-1}} + \|v_k\|^2_{R^{-1}} \right)} \quad (16)$$

where $(\boldsymbol{e_1}, \boldsymbol{w_k}, v_k) \neq 0$, $\boldsymbol{\hat{x}_1}$ is an *a priori* estimate of $\boldsymbol{x_1}$ and $\boldsymbol{e_1}$ represents unknown initial condition error, $\boldsymbol{P_1} > 0$, $\boldsymbol{Q} > 0$ and $R > 0$ are the weighting matrices and $\|\boldsymbol{x_k}\|_S = \boldsymbol{x_k}^T S \boldsymbol{x_k}$. Our goal is to find an estimate $\boldsymbol{\hat{x}_k}$ that minimizes J. Since the direct minimization of J is not easy, we choose instead a performance bound and seek an estimation strategy that satisfies a chosen threshold. Therefore, the H_∞ problem consists on finding an estimate $\boldsymbol{\hat{x}_k}$ for among all possible $\boldsymbol{x_k}$ such that

$$supJ \leq 1/\gamma \quad (17)$$

where "Sup" stands for the supreme value and γ is the user-specified performance bound. The problem formulated above shows that the H_∞ estimator guarantees the smallest estimation error over all possible disturbances of finite energy.

Then, the solution of the H_∞ problem is given by the following theorem [14].

Theorem: Let $\gamma > 0$ be the user-specified performance bound. Then, there exists an H_∞ estimation for $\boldsymbol{x_k}$ if and only if there exists a stabilizing symmetric solution $\boldsymbol{P_k} > 0$ to the following discrete-time Riccati equation:

$$\boldsymbol{P_k} = \boldsymbol{P_{k-1}} \left[\boldsymbol{I} - \gamma \boldsymbol{P_{k-1}} + \boldsymbol{h_k} R^{-1} \boldsymbol{h_k}^T \boldsymbol{P_{k-1}} \right]^{-1} + \boldsymbol{Q} \quad (18)$$

Where $\boldsymbol{I}$ is the identity matrix. Then, the H_∞ estimation is given by

$$\boldsymbol{\hat{x}_k} = \boldsymbol{\hat{x}_{k-1}} + \boldsymbol{G_k} \left(y_k - \boldsymbol{h_k}^T \boldsymbol{\hat{x}_{k-1}} \right) \quad (19)$$

where $\boldsymbol{G_k}$ is the H_∞ gain given by

$$\boldsymbol{G_k} = \boldsymbol{P_k} \left[\boldsymbol{I} - \gamma \boldsymbol{P_{k-1}} + \boldsymbol{h_k} R^{-1} \boldsymbol{h_k}^T \boldsymbol{P_{k-1}} \right]^{-1} \boldsymbol{h_k} R^{-1} \quad (20)$$

Note that in this study we take the optimal value γ_{opt} of γ. This optimum corresponds to the greatest value of γ that guarantees the stability of the matrix $\boldsymbol{P_k}$ over all samples. This stability is reached, according to Yaesh and Shaked [15], when the module of $\boldsymbol{P_k}$'s eigenvalues are less than one.

III. Simulation analysis

In this section, a simulated gearbox vibration signal is used to evaluate the effectiveness of the proposed method over the previous scheme in detecting a gear fault, both combined with the H_∞ estimator. The vibration signal, sampled at 10 kHz during one second, is computed using (1). One component of the signal is modeled here and in which the gear meshing frequency varies sinusoidally as $f_{mesh,k} = 500 + 250 \sin(2\pi t_k)$ where t_k is the discretized time. The AM and the PM is composed of the two first harmonics of the rotating frequency such as

$$A_k = \sum_{p=1}^{2} \left(1 + a_p \cos \left(2\pi p \sum_{j=1}^{k} \frac{f_{r,j}}{f_s} \right) \right) \quad (21)$$

$$\phi_k = \sum_{p=1}^{2} \left(b_p \sin \left(2\pi p \sum_{j=1}^{k} \frac{f_{r,j}}{f_s} \right) \right) \quad (22)$$

where $a_1 = 0.8$, $a_2 = 0.5$ and $b_1 = b_2 = 0.7$ and the rotating frequency $f_{r,k} = f_{mesh,k}/20$. A colored and white Gaussian noise is added to the simulated signal such that the input signal-to-noise ratio varies from 6 dB to 0 dB. The colored noise is a white noise passes through a band-stop filter within the interval $[2000, 4000]$ Hz. We use the H_∞ combines with the SSC and the OLPA approach to estimate the AM and the PM. To measure their performance, we evaluate the output signal-to-noise ratio calculated by

$$SNR = 10 \log 10 \frac{\sum_{k=1}^{N} \boldsymbol{s_k}^2}{\sum_{k=1}^{N} (\boldsymbol{s_k} - \boldsymbol{\hat{s}_k})^2} \quad (23)$$

where $\boldsymbol{s_k}$ is the noiseless simulated signal and $\boldsymbol{\hat{s}_k}$ is the estimate or filtered signal. The signal waveform and its time-frequency representation, when the intput $SNR = 0$ dB, are exhibited on Fig. 1.

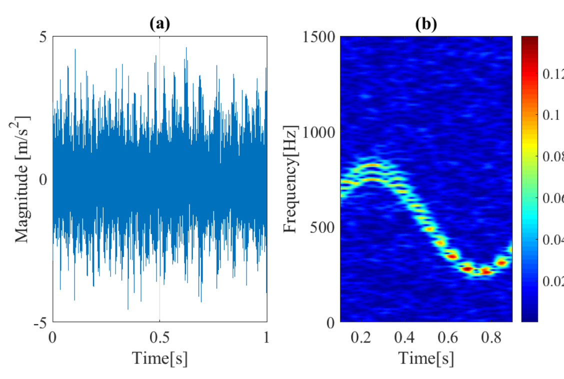

Fig. 1. Simulated signal with colored noise: a) Waveform b) Short-time Fourier spectrogram

In this simulation, $\boldsymbol{Q} = 0.01\boldsymbol{I}$ for the SSC model and $\boldsymbol{Q} = 10^{-5}\boldsymbol{I}$ for the $OLPA$ model. $R = yy^T$, $\gamma_{opt} = 0.01$ and $\boldsymbol{P_1} = 10^{-6}\boldsymbol{I}$ for the two models. The results of the

TABLE I
PERFORMANCE COMPARISON OF THE TWO APPROACHES

Estimation Model	Input SNR (dB)	Output SNR (dB)	
		White Noise	Colored Noise
SSC	6	8.72	7.91
OLPA	6	10.70	9.86
SSC	3	7.77	6.57
OLPA	3	9.43	8.25
SSC	0	6.46	4.86
OLPA	0	7.88	6.41

Table I, obtained after 400 Monte-Carlo simulations, show the performance comparison between the two model of estimating the AM and the PM. Based on the output signal-to-noise ratio, the H_∞ using the Legendre polynomial gives an estimate of $\boldsymbol{x_k}$ better than the use of the SSC. When the $SNR = 0$ dB the use of the OLPA improves the estimation of the SSC by 1.55 dB. In all the situations, the proposed method takes advantage over the previous one.
To examine the details of the estimation, the waveforms are plotted on Fig. 2. By comparing these waveforms, we notice that using the OLPA approach (in black line) performs better than the SSC modeling. The SSC model (in red line) provides a quite good estimation in slow variation regions of modulations. But it increases estimation error in the rapid transient zones. Besides of that, the OLPA approach gives a smooth estimate with a small estimation error whatever the region of the signal. This advantage leads to state that the H_∞ estimation using an orthogonal Legendre polynomial base is more efficient to track the AM and the PM in gearbox vibration signal.

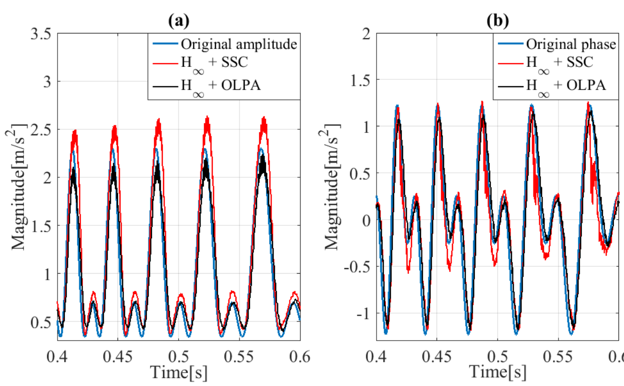

Fig. 2. Comparison between stochastic smoothness constraint and orthogonal Legendre polynomial for $SNR = 0$ dB (a) Estimation of amplitude modulation (b) Estimation of the phase modulation

IV. EXPERIMENTAL SIGNAL ANALYSIS

A. Test rig

The efficiency of the H_∞ estimation using the OLPA modeling for detecting gear fault under non-stationary conditions was tested on a laboratory test-rig. Fig. 3 presents the test rig of the University of Lyon lab used in this experiment. The system is composed of one stage gear in which the input pinion and the output gear have respectively 45 and 26 teeth. Both healthy signal (when all gear are healthy) and faulty signal (when one tooth of the output gear were purposely broken) are measured during a run-up process in which the input pinion rotating frequency f_{r1} increases from 6 to 24 Hz. For each situation a signal of almost 10 s was recorded at 52 kHz. Only the three first meshing frequency has been selected in all the analysis. Our interest is to find in the AM the output gear order and its harmonics, i.e $ratio = 45/26$ of the input shat order. The order is expressed as the number of events per revolution. The AM plotted here is the weighted mean of the three AM around the meshing frequencies.

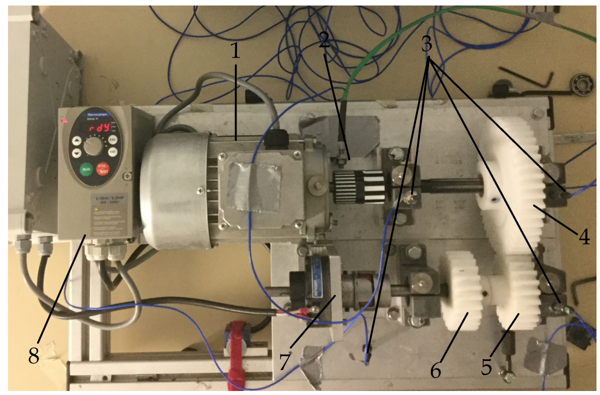

Fig. 3. Test rig: –1 Drive motor, –2 Optical encoder, –3 Accelerometer, –4 Healthy pinion, –5 Healthy gear, –6 Faulty gear, –7 Generator, –8 Speed variator

B. Healthy and faulty signal analysis

Fig. 4 shows the vibration signal and the rotating frequency in both situations. The corresponding rotating frequency f_{r1} (Fig. 4 c) and d)) is estimated from the optical encoder signal. The H_∞ technique has been used to estimate the AM. The weighting matrices are $R = 10^2$, $\boldsymbol{Q} = 60\boldsymbol{I}$, $\boldsymbol{P_1} = 10^{-6}\boldsymbol{I}$ and the performance bound is $\gamma = 10^{-8}$.
Fig. 5 shows the AM in both situation and their squared envelope order spectrum. The order spectrum is obtained by applying the Fourier transform on the amplitude re-sampled in angular domain. More details on this technique can found in [18].
The faulty AM has more energy than the healthy AM. On Fig. 5 c) the order spectrum of the healthy case (blue line) exhibits the two first harmonics of the output gear order. This may be due to a misalignement or a native default on the gear tooth. Compared to the order spectrum of the healthy AM, more harmonics of the output gear order (1xratio, …, 5xratio) emerge when a faulty gear is introduced. In addition, the energy of the faulty AM is much more significant than that of the healthy AM. This results corresponds to the expectation of the experiment, hence the gear fault is successfully diagnosed.

V. CONCLUSION

In this paper, we have introduced a new method to diagnose a gear fault in non-stationary operations based on the H_∞

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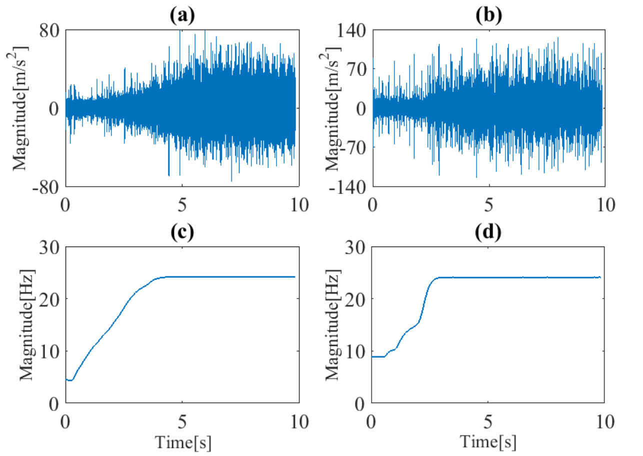

Fig. 4. Measurement signal: (a) Healthy signal (b) Faulty signal (c) Rotating frequency of healthy case (d) Rotating frequency of faulty case

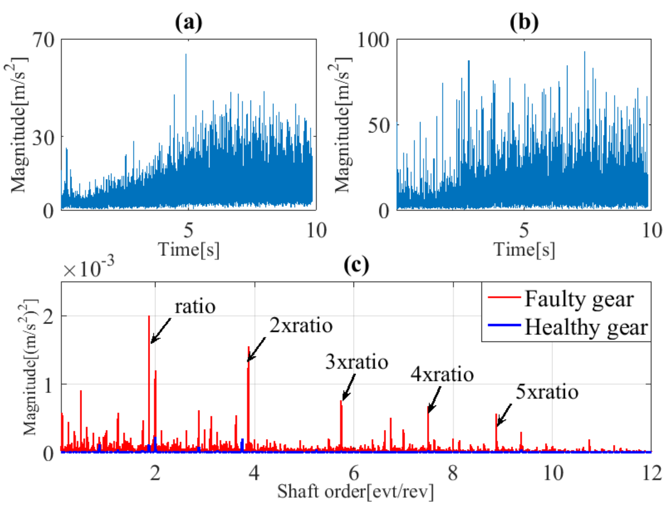

Fig. 5. Amplitude modulation: (a) Healthy case (b) Faulty case (c) Squared envelope order spectrum of amplitude modulation

estimation and the Legendre polynomial. This method is used to enhance a previous approach adopted for modeling the amplitude modulation and phase modulation. The comparison between these approaches is made using a simulated signal with both white and colored noises. The results provided have shown that using the orthogonal Legendre polynomial base highly improves the estimation accuracy over the stochastic smoothness constraint. The improvement is significant mostly in the rapid signal transition zones and in a high level noise environment. The effectiveness of the new approach has been applied to an experimental gearbox vibration signal where the gear fault has been successfully diagnosed. Therefore, the H_∞ estimation using the Legendre polynomial is appropriate in practical gearbox diagnosis. Our future research will concern the diagnosis of bearing faults using the proposed method.

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