A Sparsity-Aware Proportionate Normalized Maximum Correntropy Criterion Algorithm for Sparse System Identification in Non-Gaussian Environment

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***Abstract*—A sparsity-aware proportionate normalized maximum correntropy criterion (PNMCC) algorithm with l_p-norm penalty, which is named as l_p-norm constraint PNMCC (LP-PNMCC), is proposed and its crucial parameters, convergence speed rate and steady-state performance are discussed via estimating a typical sparse multipath channel and an typical echo channel. The LP-PNMCC algorithm is realized by integrating a l_p-norm into the PNMCC's cost function to create an expected zero attraction term in the iterations of the presented LP-PNMCC algorithm, which aims to further exploit the sparsity property of the sparse channels. The presented LP-PNMCC algorithm has been derived and analyzed in detail. Experimental results obtained from sparse channel estimations demonstrate that the proposed LP-PNMCC algorithm is superior to the PNMCC, PNLMS, RZA-MCC, ZA-MCC, NMCC and MCC algorithms according to the convergence speed rate and steady-state mean square deviation.**

I. Introduction

The quality of modern communication is largely dependent on channel state information that is always implemented by channel estimation [1]. Furthermore, the early studies showed that multipath channel and echo channel are typical models in practical applications [1], [2], [3], which are usually sparse. As a sparse channel, it has a typical feature that the major of the channel responses are close to zeros or equal to zeros, while only few channel responses are non-zero ones [4]. Moreover, channel estimations and system identifications have attracted a great concern in recent decades. Adaptive filter technology is a very effective method to estimate or identify these channels [5], [6], which has been widely studied. However, most of the adaptive filter algorithms are mainly presented for non-sparse systems and Gaussian noise environment. To estimate the sparse channels, a lot of adaptive filter algorithms were reported [7], [8], [9], [10], [11], [12], [13], [14] to estimate sparse channels. Least mean square (LMS) algorithm [7] has attracted much attention owing to its simple and low computational complexity. However, the LMS algorithm is sensitive to the scaling of the input training signal and it has a poor performance in low signal to noise ratio (SNR) environment. Then, least mean fourth (LMF) [8], [9], normalized LMS (NLMS) [10], [11] and affine projection algorithm (APA) [5], [12] were presented to improve LMS's estimation performance. Although these sparse adaptive filters have achieved good performance for estimating sparse channels in Gaussian environment, they cannot well exploit the sparseness of sparse channels. After that, zero attracting techniques inspired by compressed sensing (CS) [15] have been introduced into the exiting classical adaptive filtering algorithms, which are realized by using different norm penalizes such as l_1-norm, reweighted l_1-norm and l_p-norm. As a result, the zero attracting LMS (ZA-LMS), reweighted ZA-LMS (RZA-LMS) and l_p-norm constrained LMS were presented by incorporating the zero attracting technique into LMS's cost function, respectively [16], [17]. However, these sparse adaptive filter algorithms are obtained in Gaussian environment, while their performance might be deteriorated in non-Gaussian environment.

Recently, a maximum correntropy criterion (MCC) algorithm has been presented based on information theoretic quantity in [18], which achieves robust performance in non-Gaussian environment. However, the MCC has a similar drawback with the well-known LMS algorithm, which is sensitive to the scaling of the input signal. Then, normalized MCC (NMCC) and proportionate NMCC (PNMCC) algorithms have been presented in [19] to improve the performance of the MCC algorithm. Similarly, the MCC algorithm cannot utilize the sparseness characteristics of these sparse channels. Motivated by the ZA- and RZA- LMS algorithms, ZA- and RZA- MCC have been presented in [20], [21].

A l_p-norm penalized proportionate normalized MCC algorithm (LP-PNMCC) is proposed, which is realized by incorporating a l_p-norm constraint term into the PNMCC's cost function. The proposed LP-PNMCC algorithm converges faster and has smaller steady-state error than the PNMCC, ZA- and RZA- MCC algorithms for estimating sparse channels. The LP-PNMCC algorithm has an extra parameter p which is

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ranging from 0 to 1. The key parameters, convergence speed rate and steady-state error behavior of the LP-PNMCC algorithm are investigated and discussed in detail. The simulation results give that the presented LP-PNMCC algorithm significantly improves both the convergence speed rate and steady-state performance compared with the PNMCC algorithm for estimating sparse channels.

II. Review of the PNMCC algorithm

In sparse system identification, an unknown impulse response $\mathbf{w}(n)$=$[\mathrm{w}_0, w_1, ..., w_{N-1}]^T$ and an input signal $\mathbf{x}(n)$ are considered to get the estimation signal $\hat{\mathbf{w}}(n)$ which is close to $\mathbf{w}(\mathbf{n})$. Here, normalized maximum correntropy criterion algorithm (NMCC) is used to estimate $\mathbf{w}(n)$, and the NMCC's updating equation is given by [19]

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \chi_{\mathrm{NMCC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)}{\|\mathbf{x}(n)\|^2} e(n)\,\mathbf{x}(n), \quad (1)$$

where $e(n) = d(n) - \mathrm{y}(n)$ is the estimation error which is the difference between the expected signal $d(n) = \mathbf{x}^T(n)\mathbf{w}(n) + v(n)$ and the estimated signal $\mathrm{y}(n) = \mathbf{x}^T(n)\hat{\mathbf{w}}(n)$. Then, a gain assignment matrix $\mathbf{G}(n) = \mathrm{diag}(g_0(n), g_1(n), g_2(n), \cdots, g_{N-1}(n))$ [22] is introduced into the equation (1) to carry out the PNMCC algorithm whose updated equation is

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \chi \frac{\mathbf{G}(n)\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)}{\mathbf{x}^T(n)\,\mathbf{G}(n)\,\mathbf{x}(n) + \theta} e(n)\,\mathbf{x}(n). \quad (2)$$

Here, θ denotes a very small positive parameter and the elements $g_i(n)$ is obtained by

$$g_i(n) = \frac{\varphi_i(n)}{\sum_{i=0}^{N-1} \varphi_i(n)}, 0 \le i \le N-1 \quad (3)$$

with

$$\varphi_i(n) = \max\left[\gamma_g \max\left[\begin{array}{c} \rho_p, |\hat{w}_o(n)|, |\hat{w}_1(n)|, \\ \cdots, |\hat{w}_{N-1}(n)| \end{array}\right], |\hat{w}_i(n)|\right], \quad (4)$$

where $\rho_p = 0.01$ and $\gamma_g = 5/N$. It is found that there is an extra gain assignment matrix $\mathbf{G}(\mathbf{n})$ in the PNMCC algorithm in comparison with the NMCC algorithm. However, the PNMCC's convergence speed may be worse than the NMCC algorithm in the steady-state stage.

III. The proposed LP-PNMCC algorithm

Similarly to the sparse LMS algorithms [16], [17], a sparse-aware proportionate normalized maximum correntropy criterion algorithm with l_p-norm constraint [17], [23], [24] is proposed. The LP-PNMCC algorithm is implemented by introducing a l_p-norm constraint into the PNMCC's cost function, which is to form a zero attractor. The proposed LP-PNMCC aims to further exploit the sparsity property of the sparse channels by the designed zero attractor. The proposed LP-PNMCC solves the following problem

$$\begin{aligned} &\min\ (\hat{\mathbf{w}}(n+1) - \hat{\mathbf{w}}(n))^T \mathbf{G}^{-1}(n)(\hat{\mathbf{w}}(n+1) - \hat{\mathbf{w}}(n)) \\ &+\gamma_{\mathrm{LP}}\mathbf{G}^{-1}(n)\|\hat{\mathbf{w}}(n+1)\|_p \\ &\text{subject to } \widehat{e}(n) = \left[1 - \xi\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n), \end{aligned} \quad (5)$$

where $\widehat{e}(n) = d(n) - \mathbf{x}^T(n)\,\hat{\mathbf{w}}(n+1)$, γ_{LP} denotes a very small constant for trading off the sparsity and the estimation error, and $0 < p < 1$. The modified cost function of the LP-PNMCC is

$$\begin{aligned} J_{\mathrm{LP}} =& (\hat{\mathbf{w}}(n+1) - \hat{\mathbf{w}}(n))^T \mathbf{G}^{-1}(n)(\hat{\mathbf{w}}(n+1) - \hat{\mathbf{w}}(n)) \\ &+ \gamma_{\mathrm{LP}}\mathbf{G}^{-1}(n)\|\hat{\mathbf{w}}(n+1)\|_p \\ &+ \lambda\left(\widehat{e}(n) - \left[1 - \xi\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n)\right), \end{aligned} \quad (6)$$

where λ is a Lagrange multiplier.

Based on Lagrange multiplier method [25], we obtain the gradients of J_{LP}, which are given by

$$\frac{\partial J_{\mathrm{LP}}}{\partial \hat{\mathbf{w}}(n+1)} = 0 \text{ and } \frac{\partial J_{\mathrm{LP}}}{\partial \lambda} = 0, \quad (7)$$

Then, we have

$$\begin{aligned} \hat{\mathbf{w}}(n+1) =& \hat{\mathbf{w}}(n) + \lambda\mathbf{G}(n)\,\mathbf{x}(n) \\ &- \gamma_{\mathrm{LP}} \frac{\|\hat{\mathbf{w}}(n+1)\|_p^{1-p}\,\mathrm{sgn}(\hat{\mathbf{w}}(n+1))}{|\hat{\mathbf{w}}(n+1)|^{1-p}} \end{aligned} \quad (8)$$

and

$$\widehat{e}(n) - \left[1 - \xi\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n) = 0 \quad (9)$$

By multiplying $\mathbf{x}^T(n)$ on both sides of (8) and considering equation (9), we get

$$\begin{aligned} \lambda =& \frac{\xi\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n)}{\mathbf{x}^T(n)\,\mathbf{G}(n)\,\mathbf{x}(n)} \\ &+ \frac{\gamma_{\mathrm{LP}}\mathbf{x}^T(n)\frac{\|\hat{\mathbf{w}}(n+1)\|_p^{1-p}\mathrm{sgn}(\hat{\mathbf{w}}(n+1))}{|\hat{\mathbf{w}}(n+1)|^{1-p}}}{\mathbf{x}^T(n)\,\mathbf{G}(n)\,\mathbf{x}(n)} \end{aligned} \quad (10)$$

By substituting (10) into (8) and rounding it using the rule in [19], the updating equation of the proposed LP-PNMCC algorithm is

$$\begin{aligned} \hat{\mathbf{w}}(n+1) =& \hat{\mathbf{w}}(n) + \chi_1 \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n)\,\mathbf{G}(n)\,\mathbf{x}(n)}{\mathbf{x}^T(n)\,\mathbf{G}(n)\,\mathbf{x}(n) + \theta} \\ &- \rho_{\mathrm{LP}} \frac{\|\hat{\mathbf{w}}(n)\|_p^{1-p}\,\mathrm{sgn}(\hat{\mathbf{w}}(n))}{|\hat{\mathbf{w}}(n)|^{1-p} + \phi_{\mathrm{LP}}}, \end{aligned} \quad (11)$$

where $\chi_1 = \xi\mu$, $\rho_{\mathrm{LP}} = \gamma_{\mathrm{LP}}\mu$ and ϕ_{LP} denotes a positive constant with a small value to avoid dividing by zero. Our proposed LP-PNMCC algorithm with expected zero attractor can significantly improve the convergence at the steady-state stage for estimating sparse channels.

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IV. Experimental results

The estimation performance of the LP-PNMCC is presented for estimating sparse channels. A multipath channel with its length of $N = 16$ and K non-zero coefficients is used to estimate the performance of the LP-PNMCC algorithm first. Herein, mean square deviation (MSD) is used as a metric to evaluate the behavior of the developed LP-PNMCC algorithm, giving by $\mathrm{MSD}\left(\hat{\mathbf{w}}\left(n\right)\right) = \mathrm{E}\left[\left\|\mathbf{w}\left(n\right) - \hat{\mathbf{w}}\left(n\right)\right\|^2\right]$. A mixed Gaussian noise $(1-\theta)N(\iota_1, \nu_1^2) + \theta N(\iota_2, \nu_2^2) = (0, 0.01, 0, 20, 0.05)$ is used to model the noise signal $v\left(n\right)$, where $N(\iota_i, \nu_i^2)(i = 1, 2)$ are the Gaussian distributions with their means of ι_i and variances of ν_i^2. Here, θ is used to get a balance of the two mixed noises. Parameters χ_1, ρ_{LP} and p have significant effects on the behaviors of the proposed LP-PNMCC algorithm. The parameter effects of χ_1 is shown in Fig. 1. The parameters used in this experiment are $K = 1$, $\sigma = 1000$, $\theta = \phi_{\mathrm{LP}} = 0.01$, $\rho_{\mathrm{LP}} = 9 \times 10^{-6}$, and $p = 0.65$. It is found that χ_1 is similar to the step size in the LMS algorithm and it can control the convergence of the LP-PNMCC algorithm. With an increasing of χ_1, the MSD is deteriorated, while the convergence speed rate is increased.

Next, the effects of ρ_{LP} is investigated and its performance is demonstrated in Fig. 2. It is observed that the MSD decreases when ρ_{LP} decreases form 9×10^{-4} to 5×10^{-5}. When ρ_{LP} is less than 5×10^{-5}, the MSD is becoming worse. The performance of p is given in Fig. 3. We can see that a smaller steady-state MSD is achieved when a large p is used in the LP-PNMCC algorithm. Thus, proper parameters should be chosen for achieving a good performance in sparse channel estimation.

Based on the above parameter selections, the channel in the above experiment with $K = 1, 2, 4, 6$ are adopted to evaluate the behaviors of the LP-PNMCC algorithm and its

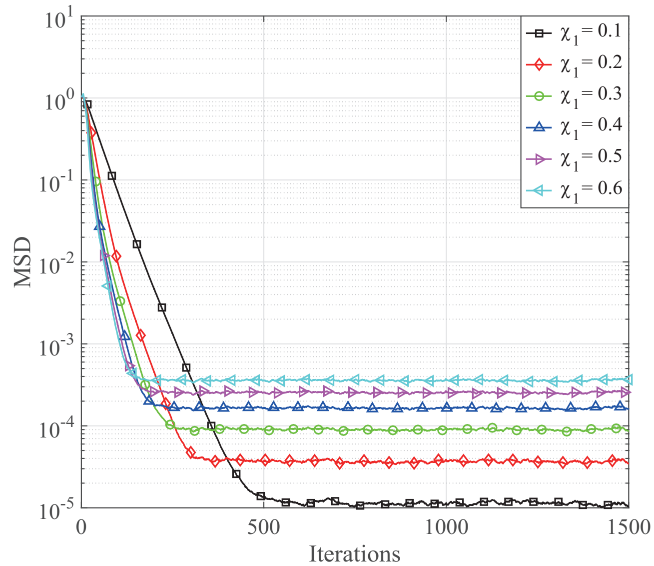

Fig. 1. Effects of χ_1 on the proposed LP-PNMCC algorithm.

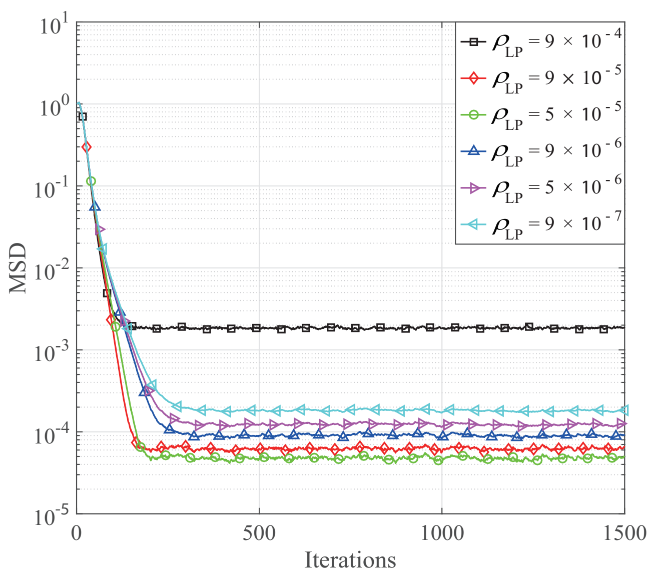

Fig. 2. Effects of ρ_{LP} on the proposed LP-PNMCC algorithm.

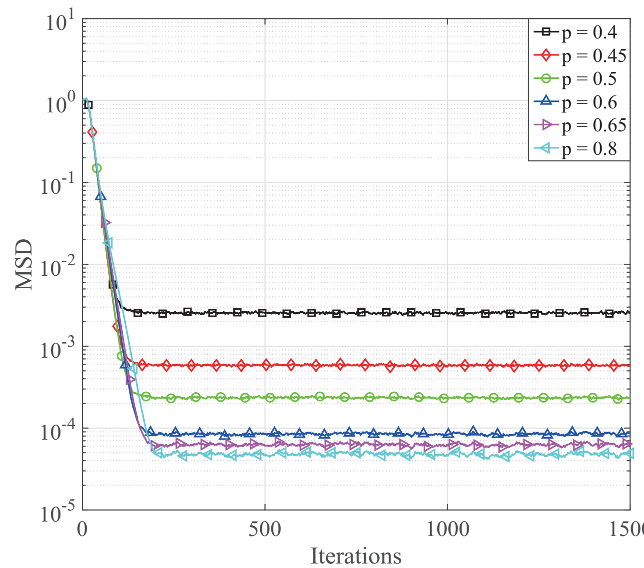

Fig. 3. Effects of p on the proposed LP-PNMCC algorithm.

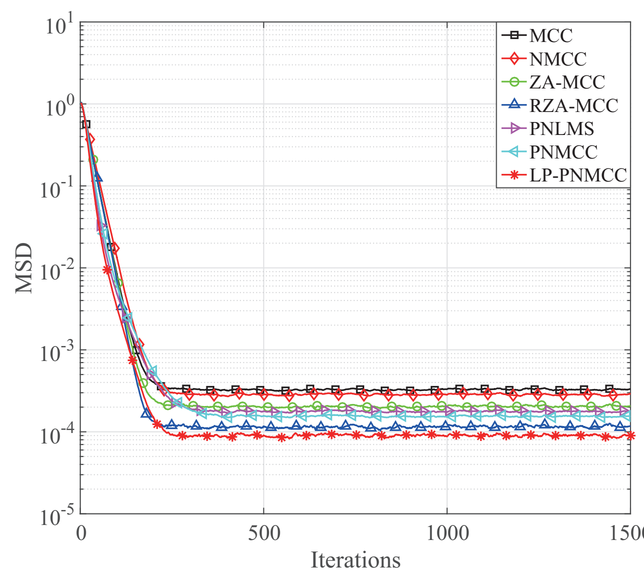

Fig. 4. MSD of the LP-PNMCC algorithm for K=1.

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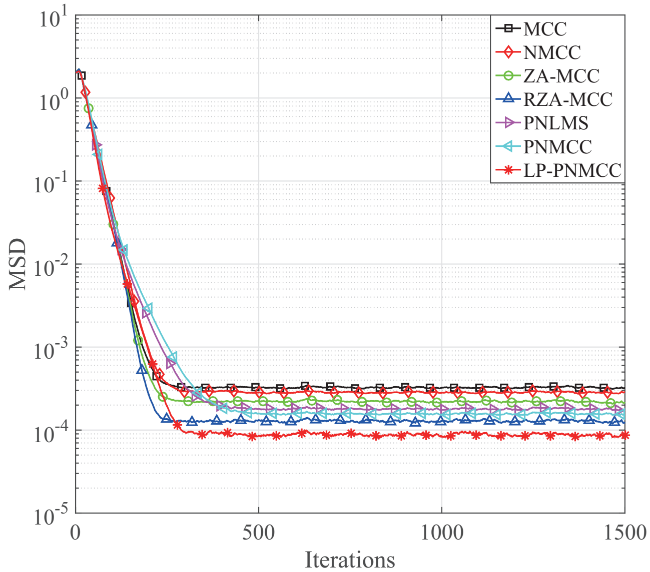

Fig. 5. MSD of the LP-PNMCC algorithm for K=2.

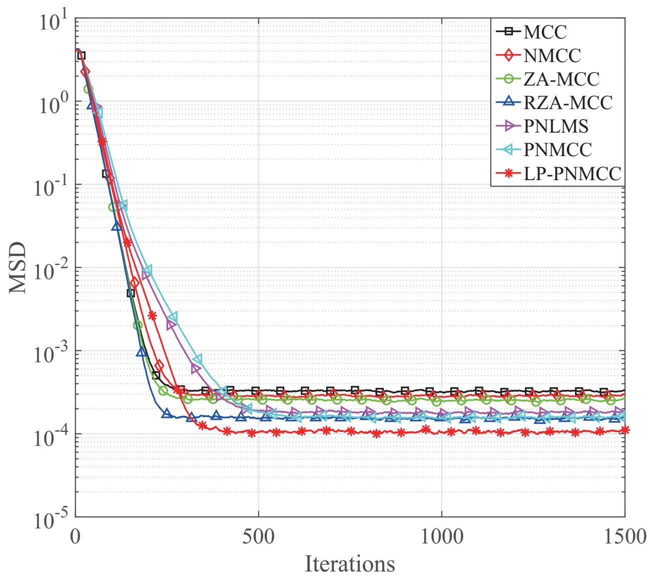

Fig. 6. MSD of the LP-PNMCC algorithm for K=4.

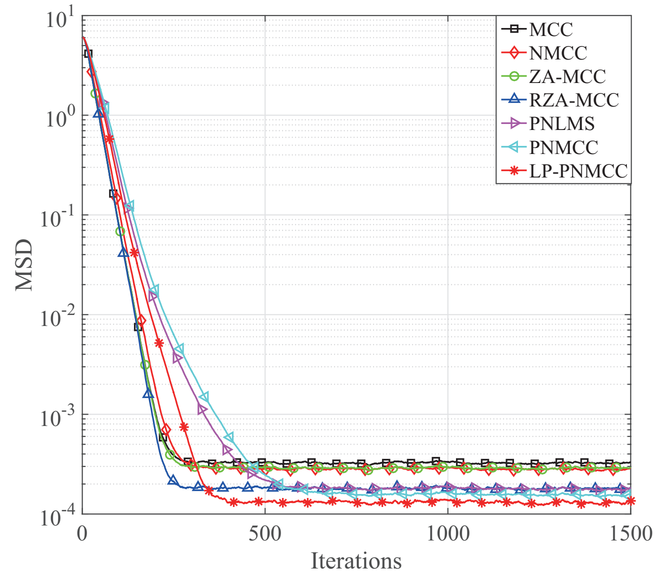

Fig. 7. MSD of the proposed LP-PNMCC algorithm for K=6.

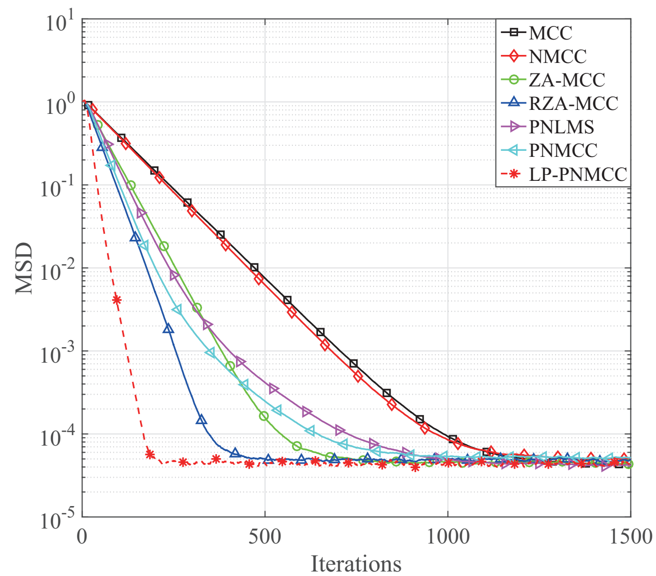

Fig. 8. MSD of the LP-PNMCC algorithm.

behaviors are described in Figs. 4, 5, 6 and 7, respectively. The channel estimation behaviors of the well devised LP-PNMCC algorithm is compared with the early reported MCC, NMCC, PNLMS, PNMCC, ZA- and RZA- MCC algorithms. In this simulations, the parameters are set to be $\chi_{\mathrm{MCC}}=\chi_{\mathrm{ZA}}=\chi_{\mathrm{RZA}}=0.03$, χ_{NMCC}=0.4, $\rho_{\mathrm{ZA}}=8\times10^{-5}$, $\rho_{\mathrm{RZA}}=2\times10^{-4}$, $\mu_{\mathrm{PNLMS}}=0.27$, χ=0.24, χ_1=0.3, $\rho_{\mathrm{LP}}=9\times10^{-6}$ and $p=0.65$. χ_{MCC}, χ_{ZA} and χ_{RZA} are the step sizes of the MCC, ZA- and RZA- MCC algorithms, while ρ_{ZA} and ρ_{RZA} are the penalty controlling parameters of the ZA-MCC and RZA-MCC algorithms. It is found that the developed LP-PNMCC algorithm achievs the fastest convergence speed rate and lowest steady-state misalignment for $K=1$. With the increasing of K, the convergence speed of LP-PNMCC is getting worse. However, the steady-state MSD of the developed LP-PNMCC algorithm is still the lowest for different K. Then, the convergence analysis is investigated with the same estimation error in Fig. 8. The simulation parameters are χ_{MCC}= 0.0052, χ_{NMCC}=0.085, $\chi_{\mathrm{ZA}}=0.01$, $\chi_{\mathrm{RZA}}=0.015$, $\rho_{\mathrm{ZA}}=3\times10^{-5}$, $\rho_{\mathrm{RZA}}=7\times10^{-5}$, $\mu_{\mathrm{PNLMS}}=0.072$, χ=0.088, χ_1=0.3, $\rho_{\mathrm{LP}}=9\times10^{-6}$ and $p=0.45$. From the convergence performance shown in Fig. 8, we can see that the LP-PNMCC algorithm achieves the fastest convergence speed, which depends on the proposed zero attractor that improves the convergence at the steady-state stage.

To further understand the performance of the proposed LP-PNMCC algorithm, an echo channel with different sparsity levels is investigated. The echo channel with a length of 256 and a sparseness of $\zeta_{12}(\mathbf{w})$ are used to discuss the performance with respect to the MSD, where the sparsity measurement is defined by $\zeta_{12}(\mathbf{w})=\frac{N}{N-\sqrt{N}}\left(1-\frac{\|\mathbf{w}\|_1}{\sqrt{N}\|\mathbf{w}\|_2}\right)$ [1]. In this setup, the first 8000 iterations, $\zeta_{12}(\mathbf{w})=0.8222$ is used. After 8000

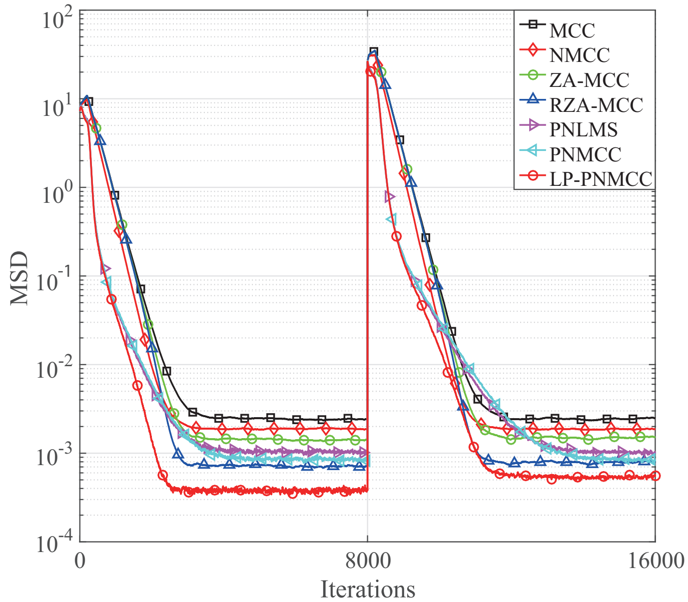

Fig. 9. MSD of the LP-PNMCC algorithm for estimating network echo channel.

iterations, $\zeta_{12}(\mathbf{w})$ is changed to be $\zeta_{12}(\mathbf{w}) = 0.7362$. Here, the simulation parameters are χ_{MCC}=$\chi_{\text{ZA}} = \chi_{\text{RZA}} = 0.0055$, χ_{NMCC}=1.3, $\rho_{\text{ZA}} = 4 \times 10^{-6}$, $\rho_{\text{RZA}} = 1 \times 10^{-5}$, $\mu_{\text{PNLMS}} = 1$, χ=0.9, χ_1=0.8, $\rho_{\text{LP}} = 1 \times 10^{-6}$ and $p = 0.8$. The echo channel estimation performance based on the proposed LP-PNMCC algorithm is shown in Fig. 9. It is found that the proposed LP-PNMCC algorithm achieves the fastest convergence and smallest MSD for estimating the echo channel under different sparsity levels.

V. Conclusion

A l_p-norm constrained proportionate normalized maximum correntropy criterion algorithm has been proposed and its performance has been discussed in term of convergence speed and steady-state performance for estimating sparse channels. The proposed LP-PNMCC algorithm is derived and analyzed in detail. The LP-PNMCC algorithm is realized by giving an expected zero attractor in its iteration, which can significantly improve its performance. The simulation results demonstrated that the proposed LP-PNMCC algorithm is superior to the PNMCC and previously presented ZA- and RZA-NMCC algorithms.

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