SOURCE ENUMERATION IN LARGE ARRAYS USING CORRECTED RAO'S SCORE TEST AND RELATIVELY FEW SAMPLES

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***Abstract*—We focus on the problem of source enumeration in large arrays with relatively few samples, which is solved in this paper by using a statistic of corrected Rao's score test (CRST) via the generalized Bayesian information criterion (GBIC). Under the white noise assumption, the covariance matrix of the noise subspace components of the observations is proportional to an identity matrix, and this structure can be tested by the CRST statistic for the sphericity hypothesis test. The observations are decomposed into signal and noise subspace components by unitary coordinate transformation under a presumptive number of sources. Only when there is no signal in the presumptive noise subspace components, the corresponding CRST statistic is asymptotic normal distribution. The CRST statistic of the presumptive noise subspace components also is a statistic of the sample eigenvalues, and can be used as the statistic in the GBIC for estimating the number of sources. Simulation results demonstrate that the proposed method can achieve more accurate detection of the number of sources in the case of a large number of sensors with relatively few samples, especially when the number of samples is smaller than the number of sensors.**

I. INTRODUCTION

Estimating the number of source signals is a fundamental problem of the signal parameter estimation in array signal processing, radar and wireless communication. It is often the first step or a prior information in the direction of arrival (DOA) estimation, targets tracking, etc. The information theoretic criterion (ITC) like methods are common and effective approaches for source enumeration in white Gaussian noise when N is large enough compared with M, where N and M are the number of samples and the number of sensors, respectively [1]–[4]. However, in some projects such as the multiple-input multiple-output (MIMO) radar system which contains a large number of sensors, the number of available samples may be restricted and the value N is on the same order of magnitude as M, or even N is smaller than M. The ITC-like methods will be degraded or out of work in the case that a large number of sensors with relatively few samples, since these methods actually depend on the sample eigenvalues which are heavily biased with respect to the true ones [5].

The conventional ITC-like methods have been developed for the large-scale sensor array in the framework of random matrix theory, which considers the asymptotic condition $M, N \to \infty$ with $M/N \to c \in (0, \infty)$ and provides more accurate descriptions for the sample eigenvalues of the high dimensional observations [6]. B. Nadler modified the Akaike information criterion (AIC) via increasing the penalty term based on the probability distribution of the largest sample eigenvalue [7]. Using the moments of the sample eigenvalues, R. R. Nadakuditi and A. Edelman devised the random matrix theory based AIC (RMT-AIC) [8], and E. Yazdian *et al.* proposed a minimum description length (MDL) based method [9]. L. Huang and H. C. So provided the linear shrinkage MDL (LS-MDL) by making new estimations of the noise eigenvalues via the linear shrinkage technique [10]. In addition, L. Huang *et al.* reformulated the Bayesian information criterion (BIC) for the large-scale adaptive antenna array [11]. Z. Lu and A. M. Zoubir derived a generalized Bayesian information criterion (GBIC), which is a general rule for constructing the BIC by adding more information from the available data such as the probability distribution or the statistic of sample eigenvalues [12]. Although these methods improve the ITC-likes method, their performance are not satisfactory when N is smaller than M. Moreover, some criterion functions (such as [7], [11]) are not applicable in the case of $M > N$ due with the existence of $M - N$ zero sample eigenvalues.

This paper proposes a method to overcome the insufficient of the sample size by introducing a statistic of corrected Rao's score test (CRST) when we estimate the number of sources in the case of a large array with relatively few samples. The CRST is proposed in [13], which is used to test the structure of a high-dimensional covariance matrix. The proposed method based on that the covariance matrix of the white noise observations can be tested via the CRST statistic for the sphericity hypothesis test. The observation data are decomposed into signal and noise subspace components by unitary coordinate transformation under a presumptive number of sources. If the presumptive noise subspace components do not contain signals, its covariance matrix will be proportional to an identity matrix, and can be tested via the CRST statistic for sphericity test. Only when there is no signal in the presumptive noise subspace components, the corresponding CRST statistic is of the normal distribution. Moreover, the CRST statistic also is a statistic of sample eigenvalues and used as the statistic in GBIC for estimating the number of sources.

The article is structured as follows. The signal model and the GBIC are presented in section II. The CRST statistic is described in section III. The proposed method is introduced in section IV. Numerical simulation results are shown in section

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V. The principle conclusion is summarized in section VI.

II. Signal Model and The GBIC

A. Signal model

We assume K narrow-band signals with DOAs $\theta_1, \cdots, \theta_K$ impinging on an array of M sensors. At discrete time t, the observed vector $\mathbf{y}(t) \in \mathbb{C}^{M\times 1}$ is usually modeled as

$$\begin{aligned}\mathbf{y}(t) &= \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{w}(t) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t),\end{aligned} \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is the steering matrix with unit norm directional vectors $\mathbf{a}(\theta_k) \in \mathbb{C}^{M\times 1}, k = 1, \cdots, K$, and $\mathbf{s}(t) = [s_1(t), \cdots, s_K(t)]^T \in \mathbb{C}^{K\times 1}$ contains source signals, and $\mathbf{w}(t) \in \mathbb{C}^{M\times 1}$ is the white additive noise. We assume there are N samples collected in the observed sample matrix $\mathbf{Y}_N$, and

$$\mathbf{Y}_N = \mathbf{A}\mathbf{S}_N + \mathbf{W}_N, \tag{2}$$

where $\mathbf{Y}_N = [\mathbf{y}(1), \cdots, \mathbf{y}(N)]$, $\mathbf{S}_N = [\mathbf{s}(1), \cdots, \mathbf{s}(N)]$, and $\mathbf{W}_N = [\mathbf{w}(1), \cdots, \mathbf{w}(N)]$. The model (1) satisfies the following assumptions.

A1. The number of source signals K is unknown and satisfies $K < \min(M, N)$.

A2. The signals are incoherent Gaussian sequences with zero mean and covariance $E\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \mathrm{diag}\{p_1, \cdots, p_K\} \triangleq \mathbf{P}_S$, where p_i is the received power of the i-th signal.

A3. The noise is assumed to be the complex white Gaussian noise with zero mean and power σ^2, and is independent of signals.

Under the assumptions *A1*∼*A3*, the covariance matrix of $\mathbf{y}(t)$ is $E\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \mathbf{A}\mathbf{P}_S\mathbf{A}^H + \sigma^2\mathbf{I}_M \triangleq \mathbf{R}$, and $\mathbf{I}_M$ is an M-dimensional identity matrix. We denote the eigenvalues and the corresponding eigenvectors of $\mathbf{R}$ as $\lambda_1 \geqslant \cdots \geqslant \lambda_K > \lambda_{K+1} = \cdots = \lambda_M = \sigma^2$ and $\mathbf{u}_1, \cdots, \mathbf{u}_M$, respectively. The sample covariance matrix of the observation $\mathbf{Y}_N$ is

$$\hat{\mathbf{S}} = \frac{1}{N}\mathbf{Y}_N\mathbf{Y}_N^H. \tag{3}$$

The eigenvalues and eigenvectors of $\hat{\mathbf{S}}$ are denoted as $\hat{\lambda}_1 \geqslant \cdots \geqslant \hat{\lambda}_M$ and $\hat{\mathbf{u}}_1, \cdots, \hat{\mathbf{u}}_M$, respectively, and also called as sample eigenvalues and sample eigenvectors. Consequently, the source enumeration is to estimate the number of the sources K from the matrix $\mathbf{Y}_N$.

B. The GBIC

The GBIC improves the BIC by incorporating the probability density or the statistic of sample eigenvalues. There are two different expressions of the GBIC, namely, *GBIC*$_1$ and *GBIC*$_2$ (see Eq. (17) and Eq. (19) in [12], respectively). The BIC and *GBIC*$_1$ consider the density of the observations and have a common log-likelihood term which is not applicable when there are zero sample eigenvalues. Therefore, the BIC as well as *GBIC*$_1$ is not applicable when $M > N$. While *GBIC*$_2$ drops the density of the observations from *GBIC*$_1$, and just utilizes the statistic of sample eigenvalues. In this paper, we use *GBIC*$_2$ with a statistic of sample eigenvalues to estimate the number of sources. We assume $\mathcal{Z}$ is a statistic of sample eigenvalues corresponding to the unknown parameter vector $\mathbf{\Theta}_z^{(k)}$ with the possible number of sources k. We denote $f(\mathcal{Z}|\mathbf{\Theta}_z^{(k)})$ is the probability density function (pdf) of the statistic $\mathcal{Z}$ under the unknown parameter vector $\mathbf{\Theta}_z^{(k)}$. The expression of the *GBIC*$_2$ is

$$\textit{GBIC}_2(k) = -2\log f(\mathcal{Z}|\hat{\mathbf{\Theta}}_z^{(k)}) + n^{(k)}\log N, \tag{4}$$

where $\hat{\mathbf{\Theta}}_z^{(k)}$ is the maximum likelihood estimation of $\mathbf{\Theta}_z^{(k)}$, and $n^{(k)}$ is the number of free parameters in $\mathbf{\Theta}_z^{(k)}$.

III. The Corrected Rao's Score Test

The proposed method is inspired by the corrected Rao's score test (CRST) which is used to test the structure of a covariance matrix of high dimensional observations [13]. Let $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_N]$ be an observation matrix with independently and identically distributed (i.i.d.) samples from an M-dimensional random vector $\mathbf{x}$ with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$. We assume the observations $\mathbf{x}_1, \cdots, \mathbf{x}_N$ have the representation $\mathbf{x}_j = \mathbf{\Sigma}^{\frac{1}{2}}\boldsymbol{\xi}_j + \boldsymbol{\mu}$, where the $M \times N$ table $\{\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_N\} = \{\xi_{ij}\}_{1\le i\le M, 1\le j\le N}$ are made with an array of i.i.d. standardized random variables (mean 0 and variance 1). We introduce the parameter κ with values 1 and 2 for the complex and real variable ξ_{ij}, respectively. Also, we define the kurtosis coefficient $\beta = E\{|\xi_{ij}|^4\} - 1 - \kappa$ for both case, and note that $\beta = 0$ for Gaussian variable. We assume ξ_{ij} satisfies $E\{|\xi_{ij}|^4\} < \infty$ and the condition

$$\frac{1}{NM}\sum_{ij} E\{|\xi_{ij}|^4\} I(|\xi_{ij}| \ge \sqrt{N}\eta) \to 0$$

for any fixed $\eta > 0$.

To test the structure of the matrix $\mathbf{\Sigma}$, the Rao's score test (RST) considers the hypothesis

$$\mathcal{H}_0 : \mathbf{\Sigma} = \mathbf{\Sigma}_0 \quad \text{vs.} \quad \mathcal{H}_1 : \mathbf{\Sigma} \neq \mathbf{\Sigma}_0, \tag{5}$$

where $\mathbf{\Sigma}_0$ is the objective matrix, for example, $\mathbf{\Sigma}_0 = \mathbf{I}_M$ for the identity hypothesis test and $\mathbf{\Sigma}_0 = \gamma\mathbf{I}_M$ (γ is a positive constant) for the sphericity hypothesis test. Under the null hypothesis $\mathcal{H}_0$, the RST statistic is defined as

$$\textit{RST}(\mathbf{X}, \mathbf{\Sigma}_0) = \frac{N}{2} tr\{(\mathbf{\Sigma}_0^{-1}\hat{\mathbf{\Sigma}} - \mathbf{I}_M)^2\}, \tag{6}$$

where $\hat{\mathbf{\Sigma}}$ is the sample covariance matrix of the observation $\mathbf{X}$, and $\hat{\mathbf{\Sigma}} = \frac{1}{N}\sum_{j=1}^{N}(\mathbf{x}_j - \hat{\boldsymbol{\mu}})(\mathbf{x}_j - \hat{\boldsymbol{\mu}})^H$ with $\hat{\boldsymbol{\mu}} = \frac{1}{N}\sum_{j=1}^{N}\mathbf{x}_j$.

The CRST statistic modified the RST statistic and made it can be applicable for the high-dimensional observations. Under the above conditions and $M/N = c_N \to c \in (0, +\infty)$, the CRST statistic is described as (the Theorem 3.1 in [13])

$$\textit{CRST}(\mathbf{X}, \mathbf{\Sigma}_0) = \tilde{\upsilon}^{-\frac{1}{2}}\{\frac{2}{N}\textit{RST}(\mathbf{X}, \mathbf{\Sigma}_0) - M\tilde{c}_N - \tilde{\mu}\}, \tag{7}$$

where $\tilde{\upsilon} = 2\kappa c^2(1 + 2c) + 4\beta c^3$, $\tilde{\mu} = (\kappa - 1)c + \beta c$ and $\tilde{c}_N = M/(N-1) \neq 1$. When $N \to \infty$, the CRST statistic is asymptotic normal distribution under the hypothesis $\mathcal{H}_0$, i.e.,

$$\textit{CRST}(\mathbf{X}, \mathbf{\Sigma}_0) \to \mathcal{N}(0, 1). \tag{8}$$

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If $\mathbf{\Sigma}_0 = \gamma \mathbf{I}_M$, the hypothesis test (5) will be the sphericity hypothesis test. Then the CRST statistic in (7) will become

$$\mathit{CRST}(\mathbf{X}, \gamma \mathbf{I}_M) = \tilde{\upsilon}^{-\frac{1}{2}}\{tr[(\hat{\gamma}^{-1}\hat{\mathbf{\Sigma}} - \mathbf{I}_M)^2] - M\tilde{c}_N - \tilde{\mu}\}, \quad (9)$$

where $\hat{\gamma} = \frac{tr(\hat{\mathbf{\Sigma}})}{M}$ is the maximum likelihood estimation of γ.

IV. The Proposed Source Enumeration Method

The key point of the proposed method is how to use the CRST statistic for the sphericity hypothesis test to estimate the number of sources. If there are k signals in the vector $\mathbf{y}(t)$, the signal eigenvalues of the covariance matrix $\mathbf{R}$ will be $\lambda_1, \cdots, \lambda_k$ and the noise eigenvalues will be $\lambda_{k+1}, \cdots, \lambda_M$. We denote the signal subspace and noise subspace as $\mathbf{U}_k \triangleq [\mathbf{u}_1, \cdots, \mathbf{u}_k]$ and $\mathbf{U}_{M-k} \triangleq [\mathbf{u}_{k+1}, \cdots, \mathbf{u}_M]$, respectively. Using the unitary coordinate transformation, the model $\mathbf{y}(t)$ is decomposed as

$$\begin{bmatrix} (\mathbf{U}_k)^H \\ (\mathbf{U}_{M-k})^H \end{bmatrix} \mathbf{y}(t) \triangleq \begin{bmatrix} \mathbf{y}_S^{(k)}(t) \\ \mathbf{y}_W^{(k)}(t) \end{bmatrix}, \quad (10)$$

where $\mathbf{y}_S^{(k)}(t) = (\mathbf{U}_k)^H \mathbf{y}(t) \in \mathbb{C}^{k \times 1}$ and $\mathbf{y}_W^{(k)}(t) = (\mathbf{U}_{M-k})^H \mathbf{y}(t) \in \mathbb{C}^{(M-k)\times 1}$ are the signal and the noise subspace components of $\mathbf{y}(t)$, respectively [8]. Hence, the covariance matrices of the signal and the noise subspace components are

$$E\{\mathbf{y}_S^{(k)}(t)[\mathbf{y}_S^{(k)}(t)]^H\} = \mathrm{diag}\{\lambda_1, \cdots, \lambda_k\} \triangleq \mathbf{R}_S^{(k)},$$

and

$$E\{\mathbf{y}_W^{(k)}(t)[\mathbf{y}_W^{(k)}(t)]^H\} = \mathrm{diag}\{\lambda_{k+1}, \cdots, \lambda_M\} \triangleq \mathbf{R}_W^{(k)},$$

respectively.

If the presumptive noise subspace components $\mathbf{y}_W^{(k)}(t)$ do not contain signals, the $M-k$ smallest eigenvalues of $\mathbf{R}$ are equal and the covariance matrix $\mathbf{R}_W^{(k)}$ is proportional to an identity matrix, i.e., $\mathbf{R}_W^{(k)} = \sigma_k^2 \mathbf{I}_{M-k}$, where σ_k^2 is the noise power under the k signals assumption. If $\mathbf{y}_W^{(k)}(t)$ contain signals, the $M-k$ smallest eigenvalues of $\mathbf{R}$ are not equal and $\mathbf{R}_W^{(k)}$ is just a diagonal matrix. Therefore, we can infer the number of sources based on that the structure of the k-th presumptive noise subspace covariance matrix $\mathbf{R}_W^{(k)}$ can be tested via the CRST statistic for sphericity test.

Using the sample eigenvectors $\hat{\mathbf{u}}_1, \cdots, \hat{\mathbf{u}}_M$, the presumptive noise subspace components of the observation matrix $\mathbf{Y}_N$ is estimated by

$$\hat{\mathbf{Y}}_W^{(k)} = (\hat{\mathbf{U}}_{M-k})^H \mathbf{Y}_N, \quad (11)$$

where $\hat{\mathbf{U}}_{M-k} = [\hat{\mathbf{u}}_{k+1}, \cdots, \hat{\mathbf{u}}_M]$ is the sample noise subspace. The estimation of $\mathbf{R}_W^{(k)}$ is

$$\hat{\mathbf{S}}_W^{(k)} = \frac{1}{N}\hat{\mathbf{Y}}_W^{(k)}[\hat{\mathbf{Y}}_W^{(k)}]^H = \mathrm{diag}\{\hat{\lambda}_{k+1}, \cdots, \hat{\lambda}_M\}. \quad (12)$$

Under the hypothesis $\mathbf{R}_W^{(k)} = \sigma_k^2 \mathbf{I}_{M-k}$, the CRST statistic $\mathit{CRST}(\hat{\mathbf{Y}}_W^{(k)}, \sigma_k^2 \mathbf{I}_{M-k}) \triangleq T^{(k)}$ can be used to test the sphericity of the covariance matrix of the k-th presumptive noise subspace components $\hat{\mathbf{Y}}_W^{(k)}$. When the observations are with the complex Gaussian distributions, the parameters in the CRST statistic are $\kappa = 1$ and $\beta = 0$. Hence, the statistic $T^{(k)}$ under the assumptions of model (1) is

$$T^{(k)} = \frac{1}{\sqrt{\tilde{\upsilon}^{(k)}}}\{tr[(\frac{1}{\hat{\sigma}_k^2}\hat{\mathbf{S}}_W^{(k)} - \mathbf{I}_{M-k})^2] - (M-k)\tilde{c}_N^{(k)}\}, \quad (13)$$

where $\tilde{\upsilon}^{(k)} = 2(\tilde{c}_N^{(k)})^2(1 + 2\tilde{c}_N^{(k)})$, $\tilde{c}_N^{(k)} = (M-k)/(N-1)$ and $\hat{\sigma}_k^2 = \frac{1}{M-k}tr(\hat{\mathbf{S}}_W^{(k)})$ is the estimation of σ_k^2.

If there is no signal in the k-th presumptive noise subspace components $\hat{\mathbf{Y}}_W^{(k)}$, the hypothesis $\mathbf{R}_W^{(k)} = \sigma_k^2 \mathbf{I}_{M-k}$ is established and the statistic $T^{(k)}$ is of the normal distribution. Otherwise, the covariance matrix $\mathbf{R}_W^{(k)}$ is not spherical and the statistic $T^{(k)}$ doesn't have the normal distribution. Moreover, the statistic $T^{(k)}$ also is a statistic of the $M-k$ smallest sample eigenvalues of the observations $\mathbf{Y}_N$. Therefore, the statistic $T^{(k)}$ can be used as the statistic $\mathcal{Z}$ in GBIC_2 (Eq.(4)). The number of sources can be estimated via GBIC based on the principle that $T^{(k)}$ is of the normal distribution only when there is no signal in the k-th presumptive noise subspace components $\hat{\mathbf{Y}}_W^{(k)}$.

Under k signals assumption, the unknown parameter vector $\mathbf{\Theta}_z^{(k)}$ in GBIC_2 is $[\lambda_1, \cdots, \lambda_k, \sigma_k^2] \triangleq \mathbf{\Theta}^{(k)}$, and $\hat{\mathbf{\Theta}}^{(k)} = [\hat{\lambda}_1, \cdots, \hat{\lambda}_k, \hat{\sigma}_k^2]$ is the estimation of $\mathbf{\Theta}^{(k)}$. The pdf of the statistic $T^{(k)}$ under $\hat{\mathbf{\Theta}}^{(k)}$ is

$$f(T^{(k)}|\hat{\mathbf{\Theta}}^{(k)}) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(T^{(k)})^2\right). \quad (14)$$

Substituting (14) into (4) with $f(\mathcal{Z}|\hat{\mathbf{\Theta}}_z^{(k)}) = f(T^{(k)}|\hat{\mathbf{\Theta}}^{(k)})$ and the parameters number $n^{(k)} = k+1$ and ignoring the constant term, we have the proposed CRST-GBIC function

$$\mathit{CRST\text{-}GBIC}(k) = (T^{(k)})^2 + (k+1)\log N. \quad (15)$$

Minimizing (15) with respect to k yields the estimate of the number of sources

$$\begin{aligned} \hat{k}_{new} &= \arg\min_k \mathit{CRST\text{-}GBIC}(k), \\ k &\in \mathbb{N} : 0 \leqslant k \leqslant \min(M,N) - 1. \end{aligned} \quad (16)$$

The steps of proposed CRST-GBIC method for source enumeration is summarized as follows.

Step 1: Perform eigenvalue decomposition on $\hat{\mathbf{S}}$ (calculating by (3)) and obtain the sample eigenvalues $\hat{\lambda}_1 \geqslant \cdots \geqslant \hat{\lambda}_M$.

Step 2: For $k = 0, \cdots, \min(M,N) - 1$, let $\hat{\mathbf{S}}_W^{(k)} = \mathrm{diag}\{\hat{\lambda}_{k+1}, \cdots, \hat{\lambda}_M\}$. Then calculate $T^{(k)}$ and the function $\mathit{CRST\text{-}GBIC}(k)$ with (13) and (15), respectively.

Step 3: The source number estimate $\hat{k}_{new}$ is the k corresponding to the minimum value of $\mathit{CRST\text{-}GBIC}(k)$.

Remark: If the parameter β in the CRST statistic can be estimated, the proposed method is applicable for the non-Gaussian observations with the white noise assumption.

V. Numerical Evaluation

There are $K = 8$ narrow-band signals impinging upon a uniform linear array, which consists of M sensors with half-wavelength element separation. The DOA of the signals are $\{-55°, -40°, -25°, -10°, 5°, 20°, 35°, 50°\}$. Hence, the

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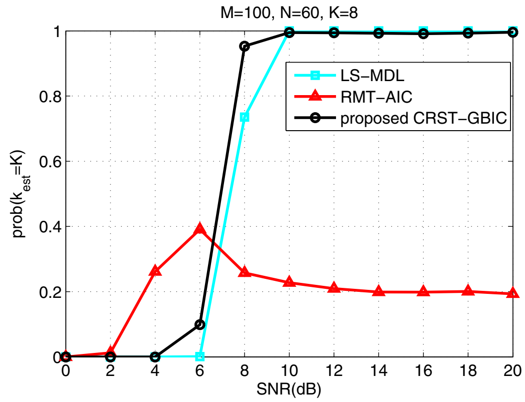

Fig. 1. Probabilities of correct detection versus the SNR.

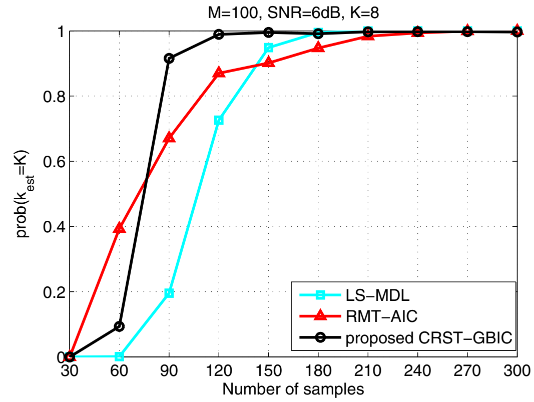

Fig. 2. Probabilities of correct detection versus the sample size.

directional vectors in the model (1) are $\mathbf{a}(\theta_k) = \frac{1}{\sqrt{M}}[1, e^{j\pi \sin\theta_k}, \cdots, e^{j\pi(M-1)\sin\theta_k}]^T, k = 1, \cdots, K$. The signals are generated by i.i.d. Gaussian sequences with mean zero and unit variance. The additive noise is complex white Gaussian noise with power σ^2. The signal-to-noise ratio (SNR) is defined by SNR$= -10\lg\sigma^2$. We compare the proposed method with the RMT-AIC method in [8] and the LS-MDL method in [10], which are suitable for the scenario of a large array in the white Gaussian noise. The empirical probabilities of correct detection (i.e. the probability of $\hat{k} = K$) are calculated from 2000 independent numerical trials.

Fig.1 shows the empirical probabilities of correct detection versus SNR with $M = 100$ and $N = 60$. Under the condition of $M > N$, the proposed method and the LS-MDL can achieve the correct detection with probability 1 when $SNR \geq 10dB$, while the RMT-AIC fails to estimate the number of sources even with a high SNR. It illustrates that the RMT-AIC is out of work when the number of samples is smaller than the number of sensors. Since the basic mathematical result of the RMT-AIC is the distribution of sample eigenvalues which is defined under the condition of $M/N < 1$.

Fig.2 shows the empirical probabilities of correct detection versus the number of samples under the condition of $M = 100$ and SNR=6dB. Among of all methods, the proposed method detects all sources with the smallest sample size. Compared Fig.2 with Fig.1, the RMT-AIC works only in the condition of $M < N$, and the LS-MDL performs well when the SNR is high. However, the proposed method has a satisfactory performance not only in the case of insufficient samples but also in the case of low SNR.

Fig.3 shows the empirical probabilities of correct detection versus the number of sensors at SNR=6dB with a fixed ratio $M/N = c_N = 1.25$. The correct detection probability of the proposed method increases quickly and is the first one to reach 1 when $M > N$ and M, N increase with a fixed ratio. Moreover, the proposed method is outstanding of all methods. Although the number of sensors is large enough, the RMT-AIC is still out of work as long as the sample size is less than the sensors number.

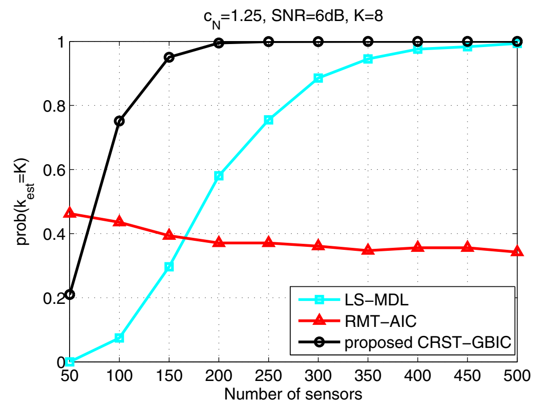

Fig. 3. Probabilities of correct detection versus the number of sensors.

VI. Conclusion

This paper has provided a method of source enumeration in the case of a large sensor array with relatively few samples in the white Gaussian noise. The proposed method overcomes the insufficient samples by introducing the corrected Rao's score test (CRST) statistic for sphericity test, which is of normal distribution only when there is no signal in the presumptive noise subspace components of the observations. The CRST statistic of the presumptive noise subspace component also is a statistic of the sample eigenvalues and used as the statistic in the generalized Bayesian information criterion (GBIC) for estimating the source number. Compared with the pervious works, the proposed method achieves a higher correct detection probability when the number of samples is smaller than the number of sensors.

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