HIGH ACCURACY FREQUENCY ANALYSIS USING INSTANTANEOUS FREQUENCY ATTRACTORS

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ABSTRACT

In this paper an improved version of the instantaneous frequency attractors (IFAs) algorithm is presented, by introducing several solutions which reduce the number of spurious components especially for low signal to noise ratios (SNRs) and increase the accuracy of the frequency estimator. We perform multilevel comparison tests with both previous versions of the IFA algorithm and another high precision frequency estimation method based on the derivative of the signal. The test results confirm the superiority of the IFAs' frequency accuracy, in different conditions: low SNRs, frequency distance between components, analysis frame length. Also, the number of spurious components is significantly reduced.

Index Terms— instantaneous frequency attractor, high frequency accuracy

1. INTRODUCTION

The most popular tool for frequency analysis is without any doubt the Fourier transform, namely its numeric implementation the fast Fourier transform (FFT). While there are still numerous application where this mathematical tool finds its utility, for certain applications where high frequency accuracy is required, the classic FFT solution is insufficient (mainly due to its leakage and smearing effects). Radar application, acoustics, communication or multimedia application are only a few examples where a very good frequency separation is pursued.

Through time, the preoccupation for the high frequency resolution problem materialized in two types of approaches: the parametric and the nonparametric ones [1]. The reduced range of applicability of the first ones, led us to focus our efforts on the nonparametric methods, which have a higher degree of generality, they do not employ a signal model, but their performances depend on the compromise between the statistical variability of the applied estimator and the spectral resolution [1]. A part of these methods approach the FFT spectrum through a band-pass filter bank interpretation. Such a solution can be found in [2], where the traditional spectrum was replaced with the more accurate instantaneous frequencies (IF). An even more sophisticated and accurate method for extracting the tonal components from an audio signal resulted in [3], where the concept of instantaneous frequency attractor (IFA) was first introduced. In [4] a modified version of the IFAs is introduced along with the implementation algorithm for discrete signals. In [5] the IFAs are combined with the group delay attractors, resulting a more appropriate solution for tracking time varying sinusoids, while keeping the number of spurious components to an acceptable level.

Given the promising results of the IFAs, in this paper we focus on improving their accuracy and boost their noise robustness, especially in low SNR conditions. We bring forward the IFA parameters which have a strong impact on the algorithm's performances and we discuss the best value settings for them. We also outline the performances of the upgraded IFAs through extensive comparison tests which include different SNRs, analysis frame lengths, distances between spectral components. The current algorithm is compared with both previous versions of the IFA and another high precision nonparametric method used for frequency estimation [6].

The structure of the paper includes a brief description of the IF analysis found in Section 2. In Section 3 a short presentation of the IFAs extraction algorithm is described along with a refined calculus of the attractor's frequency. In addition a confidence index is introduced followed by a temporal tracking of the IFA, both aimed at reducing the number of spurious components, especially for low SNRs. Also, in the same section, considerations regarding the choice of the IFAs parameters can be found. Section 4 is entirely dedicated to simulation tests meant to reveal the frequency accuracy performances of the IFAs algorithm in different conditions. Comments regarding the overall performances of the IFAs are made in Section 5, while Section 6 is reserved for conclusion.

2. IF ANALYSIS

The input data for the IFA estimation is given by the instantaneous frequencies spectrum obtained through an elegant approach which avoids the numerical derivative.

The main idea of this solution adopted in [3] is to decompose the analyzed signal into single component signals or narrow frequency band signals, by using a complex band pass filter (BPF) bank. For each signal obtained through complex filtering the IF is estimated. This frequency is equivalent to the frequency of the best cosine wave that approximates the real part of the band pass filtered signal. The filter bank is elegantly built by modu-

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lating a causal and real prototype low-pass filter (LPF), $w(t)$. The impulse response function of the p^{th} filter will be characterized by the central frequency Ω_p and the shifted LPF prototype, $w(t)e^{j\Omega_p t}$. Assuming that in the p^{th} channel we have a tonal component, the output of the p^{th} filter is completely characterized by its instantaneous amplitude and its instantaneous phase (IP). Traditionally, the IF information is extracted as the derivative of the IP with respect to time. However, mathematical calculus allows the transfer of the signal's derivative to the analysis window. Obviously, it is more convenient to compute the derivative of the window (which is identical to the LPF prototype), since in most cases its analytical expression is a priori known. Thus the IF spectrum is obtained. (For details on this section see [3-4]).

3. IF ATTRACTORS' ESTIMATION

Next we concentrate our efforts on estimating with great accuracy the frequencies of the tonal components, even in harsh SNR conditions. The solution we adopt is based on the IF attractor concept. According to [3], the IF attractors are the points which satisfy the following two conditions:

$$\mu(\Omega,t)=0, \quad (1)$$

$$\frac{\partial\mu(\Omega,t)}{\partial\Omega}=\frac{\partial\omega_{\text{inst}}(\Omega,t)}{\partial\Omega}-1<0, \quad (2)$$

where $\mu(\Omega,t)=\omega_{inst}(\Omega,t)-\Omega$, Ω and t represent the continuous frequency and time variables, while $\omega_{inst}(\Omega,t)$ is the evolution of the instantaneous frequency resulted from the analysis in Section 2.

The second condition (see (2)) states that whenever a tonal component exists in the signal, then the variation of $\omega_{inst}(\Omega,t)$ around the frequency of the tonal component should be less than the variation of Ω. Therefore, in an IF-frequency representation, for a fixed moment $t=t_a$, we can spot an IF attractor by searching for the areas with relatively constant variation (flat areas). However, condition (2) allows for many spurious tonal components to appear, which led us to the more restrictive definition [4]:

$$\left|\frac{\partial\omega_{\text{inst}}(\Omega,t)}{\partial\Omega}\right|<\varepsilon. \quad (3)$$

The parameter ε, with values between 0 and 1, controls the degree of variation we allow for the attractor and at the same time influences the number of spurious components. A value close to 0 drastically reduces the number of false tonal components. When $\varepsilon=1$, (3) equals (2).

The algorithm for the IF attractor estimation will be presented next, considering the case of discrete signals. We also bring forward the parameters which have a strong influence on the outcome of the algorithm and propose solutions for increasing the robustness of the estimator for low SNRs.

3.1. Algorithm for IFA extraction

The input data for the IFA extraction algorithm is given by the set of IFs, $F(k), k=1..\text{N}$ resulted from the analysis mentioned in Section 2 and the amplitude spectrum, $M(k), k=1..N$, with N the number of BPF channels.

The main idea of the algorithm is to see if a certain frequency was "attracted" by more than one channel in the IF analysis procedure. An IF-frequency representation makes it possible to observe if such a situation occurred. In Figure 1.a we can observe that around the 60Hz frequency, the slope of the representation is close zero, which means that more than one channel attracted the same component.

The algorithm continues with the calculus of the numeric derivative for the instantaneous frequencies $F(k)$ and then with the search of the areas with slopes less than ε.

We denote with B_i and E_i the frequency bins for the beginning and the end of a flat area (slope close to zero) and we index it by i. In order to ensure that in the detected area we have a true frequency attractor, we consider only the flat areas spread over a minimum length, L_{min}. This condition also contributes to the reduction of the spurious components as it will be shown in Section 3.2. Finally for the areas fulfilling the length condition we compute the frequency of the tonal components. The original solution determines this frequency as the point of intersection between the curve $\omega_{\text{inst}}(\Omega,t_a)$ and the bisector of the first quadrant angle. Instead, we propose to compute the frequency of the attractor as the gravity center of the amplitude spectrum corresponding to the relatively constant variation area surrounding the sought attractor (e.g. the flat area in Figure 1.a):

$$f_i=\frac{\sum_{j=B_i}^{E_i}M(j)F(j)}{\sum_{j=B_i}^{E_i}M(j)}. \quad (4)$$

The new approach in (4) is based on the fact that the amplitudes of the components resulting from the BPF channels, which attract the same frequency, will exhibit an increasing trend up to a point and then a decreasing trend to the end of the flat area in the IF-frequency representation. This behavior occurs due to the symmetry of the BPFs. Hence we expect to find the frequency of the real attractor as the maximum energy concentration point. As it will be later shown in Section 4, with this new approach we obtained a notable increase in the accuracy of the frequency estimator.

3.2. Recommendations for IFA parameters' selection

The performances of the IFA algorithm are strongly influenced by the choice of three of its parameters: the number of BPF channels N, the minimum length of the flat areas, L_{min}, and the slope variation ε.

The choice of N is dictated by the compromise between the length of the analysis frame (denoted with P

samples) and the frequency resolution. If we are restricted to small analysis frames (which is the case for non-stationary signals), then N should be chosen greater than P. Such a solution increases the frequency resolution, but, at the same time, affects the number of bins of the analysis window. Namely, when $N > P$, the number of bins in the main lobe increases, which means that the number of channels tuned on one component will also increase (see the different number of BPF channels represented with dots in Figure 1.b and 1.c). Consequently the parameter L_{min} should be increased as N increases with respect to P. However, a value too large for N with respect to P causes a slight curving of the edges of the sought flat areas (see Figure 1.b), which in turn will decrease the accuracy of the frequency attractor estimator (see Table 2, Section 4.1). Also, the computational load is unnecessarily increased.

Regarding ε, its value should be adjusted according to the SNR values. A small value for ε works well with a high SNR, but for low SNRs, a misdetection of the true tonal component is prone to appear.

From our simulations (see Section 4) we can conclude that, for SNRs greater than 35dB we obtained the best results for $N = 2P$, $L_{min} = 5$ and $\varepsilon = 0.2$. For SNRs less than 35dB, considering the same choice for N, L_{min} should be lowered to 3, and ε increased up to 0.75, depending on how low the SNR is.

3.3. IFA algorithm optimization for low SNRs

The optimized choice of the parameters N, L_{min} and ε reduces significantly the possibility of not detecting a tonal component when in fact it exists. However, the detection of a nonexistent component still manifests quite often, especially when the SNR is less than 35dB. Consequently, we propose to reduce this effect by completing the IFA estimation algorithm with two more steps.

First, we introduce a confidence coefficient, c_i:

$$c_i = 1 - \frac{1}{E_i - B_i} \sqrt{\sum_{j=B_i}^{E_i} \left(\frac{\left| F(j) - f_i \right|}{\varepsilon \cdot \Delta} \right)^2} \left(\frac{L_{min}}{E_i - B_i} \right)^2, \quad (5)$$

where F_s is the sampling frequency.

The c_i coefficient, with $c_i \in [0,1]$, considers on the one hand the normalized frequency deviation of the attractor with respect to the central frequencies of the IF channels, $F(k), k = 1..N$ and on the other hand the length of the flat area with respect to the imposed minimum length, L_{min}. For c_i towards 0, the estimated component is considered noise and it will be discarded.

Second, we propose to validate an attractor by searching for its existence in the previous two analysis frames, in a frequency interval $[f_i - F_s/N; f_i + F_s/N]$. This approach reduces additionally the detection of false components, especially when the distance between consecutive analysis frames is less than 15ms. For highly non-stationary, an alternative solution is to shift the current analysis frame one sample to the left and one to the right, and then perform the IFA analysis. If a certain IFA is not found in all three instances, then it will be discarded.

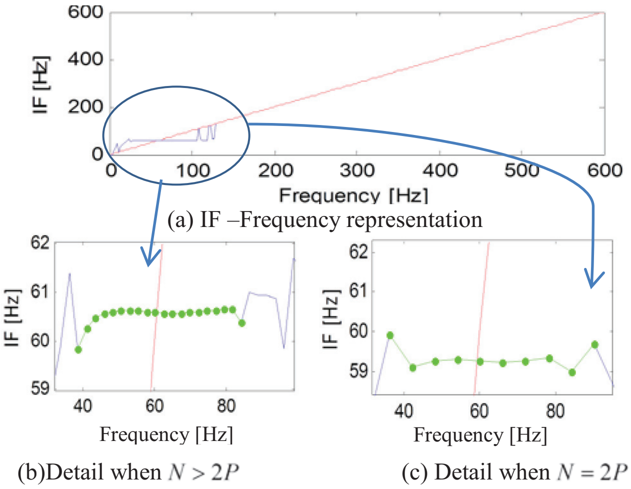

Fig. 1. The IF-frequency representation for a 60Hz tonal component, corrupted by noise (SNR = 60dB)

4. EXPERIMENTAL RESULTS

The performances of the IFA estimation algorithm are outlined through extensive tests performed on sinusoidal signals corrupted by different levels of white Gaussian noise. We compare our current IFA version with its previous ones [3-4] in terms of frequency accuracy. We also compare the IFA method with another high precision frequency estimation method, also based on the signal's derivative [6], later referred to as the DFT[1] method. The comparison includes various SNR conditions, analysis frame lengths, frequency resolutions, frequency discrimination between adjacent components.

Each type of test was performed on a different set of 1000 signals. For all tests we used a Hanning window, at least 100 analysis frames and $F_s = 24\text{kHz}$. The results are returned in terms of mean absolute error frequency ($\overline{e}_f$) and standard deviation (σ_f), both expressed in Hz.

4.1. Current IFA algorithm versus its previous version

First, we validated the use of (4) instead of the classic approach. The tests were performed for a set of 1000 signals, containing only one sinusoidal component with a frequency varying randomly around 60Hz and a 60dB SNR. We used an analysis frame of 40ms, $N = 1920$, $L_{min} = 5$, $\varepsilon = 0.2$. Superior results were obtained when using (4) compared to the solution in [3]: mean absolute frequency error – $2.55 \cdot 10^{-3}$ Hz versus $8.86 \cdot 10^{-3}$ Hz and standard deviation – 0.023 Hz versus 0.078 Hz.

Next, we varied the SNR from 0dB to 60dB and we determined the average number of spurious tonal components, when using/not using the improvements proposed in Section 3.3. Each of the 1000 test signals contained three tonal components placed in the frequency interval

$[200;500]$Hz. The IFA parameters P, N, and L_{min} were set to 1024, 2048, and 5 respectively. As for ε, we started from 0.75 for low SNRs and gradually decreased to 0.2 for high SNRs. The threshold of c_i was set to 0.8.

SNR [dB]	No. of spurious components	
	with the improvements in section 3.3	without the improvements in section 3.3
0	0.1916	9.0715
10	0.186	8.9935
20	0.1865	8.8557
30	0.1096	7.171
40	0.0004	0.5482
50	0.001	0.0015
60	0.0006	0.0027

Table 1. The average number of false detections for different SNRs

The results in Table 1 indicate the opportunity of the proposed solution. The number of spurious components significantly decreased, even for low SNRs. For 0dB SNR, the average number of spurious components fell from approximately 9 to as low as 0.1916. This result appears to be better than the one reported in [5], where an average number of 0.3 spurious IF (for SNR between -2dB and 8dB) can be read from their graphic chart (Fig. 8 in [5]). A frequency accuracy comparison was not possible, since [5] does not report specific results.

Another element worth considering regards the number of true tonal components misdetections. The same tests revealed that for SNRs higher than 15dB, the actual tonal components are always detected. For SNRs close to 0, on average 1 out of 3 components is not detected.

The impact of a proper initialization for the N, L_{min} and ε parameters is investigated next. Each of the 1000 test signal contains three tonal components placed on the frequencies 202.9688Hz, 261.5625Hz and 320.1563.

SNR = 60dB	**N**	$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**
$P = 960$, $L_{min} = 5$, $\varepsilon = 0.2$	840	0.5928	0.4709
	1920	0.1855	0.1658
	2048	0.2366	0.1465
	4096	0.2863	0.1606

Table 2. Frequency estimation results when varying N

The grayed line in Table 2 shows that the best results are obtained when $N = 2P$. Contrary to what one might expect, a greater N (thus better resolution) does not lead to increased frequency accuracy. This is due to the curving effect revealed in Section 3.2, Figure 1b.

	SNR	$\varepsilon = 0.2$		$\varepsilon = 0.75$	
		$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**	$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**
$P = 1024$, $N = 2048$, $L_{min} = 5$	30	0.2142	0.1301	0.3564	0.2767
	40	0.2048	0.1011	0.3576	0.2778
	50	0.2117	0.0954	0.3611	0.2790
	60	0.2115	0.0941	0.3631	0.2780

Table 3. Frequency estimation results for different ε

The importance of a proper value for ε is emphasized in Table 3. A value close to 1 for ε decreases the accuracy of the frequency estimator and increases the number of spurious components. The test is performed for high SNRs, in order to maintain L_{min} and ε constant.

4.2 Current IFA algorithm versus DFT[1] method

For the comparison of the two algorithms we used our own implementation of the DFT[1] based on the details found in [6] and [7]. All the signals used in the comparison tests of this section contained three tonal components.

First we investigated the accuracy of the two frequency estimators in different noise condition. Thus, we varied the SNR from 0 to 60dB and obtained the results in Table 4. The IFA parameters P, N, ε and L_{min} were set to 1024, 2048, 0.2 and 5 respectively.

SNR [dB]	**IFAs**		**DFT[1]**	
	$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**	$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**
0	0.8623	0.6641	1.0071	0.7336
10	0.3258	0.2423	0.7151	0.4943
20	0.2100	0.1471	0.6830	0.4516
30	0.2142	0.1301	0.6794	0.4470
40	0.2048	0.1011	0.6790	0.4465
50	0.2117	0.0954	0.6790	0.4464
60	0.2115	0.0941	0.6789	0.4463

Table 4. Frequency estimation results for the two algorithms when varying the SNR

Examining the data in Table 4, we can easily observe that on average the frequency accuracy of the IFA algorithm is at least two times greater than that of the DFT[1] algorithm, with a standard deviation significantly lower.

Next we inspected the frequency accuracy when varying the bin frequency distance between components. We considered distances ranging from 5 to 10 bins.

Bin distance	**IFA**		**DFT[1]**	
	σ_f **[Hz]**	$\overline{e}_f$ **[Hz]**	σ_f **[Hz]**	$\overline{e}_f$ **[Hz]**
5	0.4616	0.2784	1.3123	1.0075
6	0.5624	0.3002	1.0929	0.8576
7	0.4657	0.3434	0.6248	0.3860
8	0.2496	0.0738	0.6405	0.2966
9	0.2169	0.1357	0.3543	0.2362
10	0.1745	0.1397	0.3736	0.2335

Table 5. Frequency estimation results for the two algorithms when varying the frequency bin distance between consecutive frequency components.

Results in Table 5 show an accuracy at least twice as good for the IFAs in comparison with the DFT[1] method. Furthermore, in the case of IFAs, the accuracy is always less than 0.6Hz, while in the case of DFT[1] exceeds 1Hz when the components are too closely spaced.

Finally, we considered the influence of the analysis frame length over the accuracy of the frequency estimators. We varied the analysis frame length from 20ms to 100ms (see Table 6). We used a 60dB SNR, $N = 2048$,

$L_{min} = 5$ and $\varepsilon = 0.2$. The tonal components were placed on the following frequencies 273.14, 473.54, 673.94Hz.

Frame length[ms]	IFA		DFT[1]	
	σ_f [Hz]	$\overline{e}_f$ [Hz]	σ_f [Hz]	$\overline{e}_f$ [Hz]
20	0.1907	0.1760	0.2690	0.1838
30	0.0253	0.0171	0.1126	0.0775
40	0.0194	0.0167	0.0606	0.0420
50	0.0103	0.0067	0.0376	0.0268
60	0.0111	0.0085	0.0255	0.0192
70	0.0044	0.0035	0.0183	0.0145
80	0.0088	0.0056	0.0146	0.0110

Table 6. Frequency estimation results for the two algorithms when varying the analysis frame length

Regarding the analysis frame length, again, the results obtained with the IFA algorithm are superior to the DFT[1] method. The two algorithms can have similar outputs, however for different frame lengths. For instance, the IFA algorithm requires a 40ms window, as opposed to the DFT[1] method which needs a window almost twice as big (grayed boxes in Table 6) to achieve the same accuracy. This suggests that the IFAs are more suited than the DFT[1] for the frequency analysis of non-stationary signals.

5. DISCUSSION

Regarding the results presented in Section 4, several remarks are in order. First, it is important to note that we used only steady sinusoidal signals, corrupted by different levels of noise. This allowed us to have a very good control over the content of the signal, and therefore over the estimator's performances. The different levels of noise helped to simulate the characteristics of an audio signal with moderate time frequency variation. For highly non-stationary audio signals preliminary test results showed a good behavior of the IFAs, but more tests are required.

Second, it should be stressed that the IFAs' performances are significantly higher for medium and high frequencies (the mean absolute frequency error is less than 10^{-4} Hz and the standard deviation less than 10^{-3} Hz, for high SNRs). For the tests reported in Section 4 we used only low frequencies and spread spectrum (24kHz sampling frequency). If we restrict the tonal components' frequency searching interval, then the error frequency definitely decreases.

Lastly, an observation in relation to the estimation of the phases and amplitudes of the tonal components is needed. Out of the two parameters, the phase is the most important one, since it is known to introduce audible artifacts when inaccurately estimated. The IFA algorithm is able to extract with great accuracy both the amplitudes and phases. For the latter one, on average we obtained a mean absolute phase error (in rad/sec) ten times smaller than its corresponding mean absolute frequency error.

6. CONCLUSIONS AND FUTURE WORK

This paper addressed the problem of high accuracy frequency estimation. Instantaneous frequency attractors proved to be a very good solution for solving the aforementioned problem. We upgraded a previous version of the IFAs' extraction algorithm by refining the calculus of the attractor' frequency and introducing a confidence index doubled by a temporal tracking of the attractor in order to reduce the number of spurious components, especially for low SNR conditions. Extensive comparative tests showed that the upgraded version of the IFAs outperforms its previous versions. The number of spurious components was reduced with up to 97% for low SNRs, and the accuracy increased with approximately 72% compared to the previous IFA version. The IFAs also surpass the DFT[1] method, which is reported as a high precision frequency estimation method. In various SNR conditions the IFAs produce an accuracy at least double than DFT[1]. The same is true when varying the distance between frequency components. The experiments also revealed that the IFAs require a much smaller analysis frame than the DFT[1] in order to produce the same frequency accuracy.

Finally, a commentary regarding the need for such pinpoint frequency accuracy is called for. As resulted from Section 4, the difference between the error frequency reported by our algorithm and the other methods is maximum 0.5Hz. In certain applications such a small difference might be completely unimportant. However, this is not the case when dealing with signal enhancement, signal restoration, signal modeling or other applications which require a very fine decomposition of the analyzed signal. In this sort of applications the smallest frequency deviation introduces phase deviations which may further lead to unwanted audible artifacts. These effects can be drastically reduced with a high accuracy frequency estimation algorithm such as the IFAs algorithm.

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