DESIGN OF A NEW HIGH–ENERGY CONCENTRATION KERNEL QUADRATIC TFD FOR EEG SPIKE SIGNAL

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ABSTRACT

In this paper, the design of a novel high-energy concentration kernel quadratic TFD for EEG spike signal analysis is presented. Firstly, we show that the suppression of the negative frequency of the signal due of the use of Hilbert transform causes low Time-Frequency Distribution (TFD) resolution in the very low frequency band. To remedy this artifact, a frequency shifting of the signal to the mid frequency band is used so that the negative and positive frequencies are taken into account in the time-frequency domain. This process enhances the TFD resolution in the very low frequency band. Secondly, we derived a new separable kernel TFD with a high auto-terms energy concentration based on the localization of the auto-terms and cross-terms of the EEG spike signal in the ambiguity domain. The proposed kernel uses only two parameters and offers high TFD resolution compared to the existing ones.

***Index Terms*—** EEG Spike signal, Quadratic TFD, Auto-terms and cross-terms.

1 Introduction

Some diseases can be easily detected by a direct observation of cough, raising of temperature, etc. However, advanced equipments are required to detect and monitor the majority of other diseases. To monitor the dysfunction of the nervous system, Electroencephalography (EEG) signal is often used and it represents the recording of electrical activity along the scalp provided by time-varying electrical currents generated by the neurone [1]. Several studies have shown that EEG signals are non-stationary signals, [2] and at least two seizures can be detected where the first one of EEG seizure is modeled as a piecewise Linear FM signals [3] and the second is modeled as a sum of spikes [4].

In this paper, we are interested in EEG spike signals. Due to the non-stationarity feature of these signals, the observation in time domain and frequency domain separately is insufficient to detect the different behaviors of the signal. Several techniques are used to transform the EEG spike signals from time domain into time-frequency domain including matching pursuit method, Quadratic time-frequency distribution (QTFD), and Wavelet (time-scale presentation). In this paper, we focus on the time-frequency presentation (TFP) based on QTFD. The later is known by its high TFD resolution, but suffers from the presence of cross-terms artifacts.

Before using the QTFDs, the signal is often transformed into analytic signal to suppress negative frequencies by using Hilbert transform. Unfortunately, this transformation causes a low resolution in the very low frequency band. As the signal is localized in the low frequency band, we first shift the whole effective signal (negative frequencies included) to the mid frequency band and then, we apply the analytic transform to eliminate some noise that still presents in the negative frequencies. This process allows us to keep the whole characteristics of the signal and to improve the TFD resolution in the very low frequency band.

The QTFD is defined as the convolution product of the Wigner-Ville distribution (WVD) and the kernel [5]. The latter is designed with an objective to maintain high TFD resolution and eliminate the cross-terms artifacts. Classical QTFDs including: CWD [6, 7] and Modified B Distribution (MBD) [8] are often used for the detection of EEG spike seizure. However, none of them is specific to EEG spike signal. In the literature, there are more recent and useful QTFDs including Spectrogram, Extended B distribution (EMBD) [9], and Compact Separable Kernel (CSK) [10] that offer better resolution than CWD and MBD but are not specific to EEG spike signals. In this paper, we introduce a new separable kernel QTFD specific to EEG spike based on the localization of the auto-terms and cross-terms of the EEG spike signal in the ambiguity domain. The proposed kernel refereed to a high energy concentrations (HEC) kernel, is able to conserve the maximum of the auto-terms energy while eliminating the cross-terms artifacts. Therefore, it ensures a high TFD resolution with cross-terms elimination. Moreover, the proposed kernel contains only two parameters that leads a low computational complexity.

The rest of this paper is organized as follows: the frequency shifting and the review of QTFD based on a separable kernel is given in Section 2. In Section 3, the new high energy concentration kernel TFD is proposed. Section 4 is dedicated to show the enhancement given by the frequency shifting and the new separable kernel QTFD. The last Section is devoted for the conclusion and perspectives.

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2 Frequency Shifting and a Review of QTFD Based on Separable Kernel

Based on some observations of EEG spike signals, it is found that the most energy of the signal is localized in the low frequency band. Figs. 1 and 2 show an example of EEG spike signal in the time and frequency domains where it is noticeable in the frequency domain that the most signal energy is localized in the low frequency band. In order to obtain better

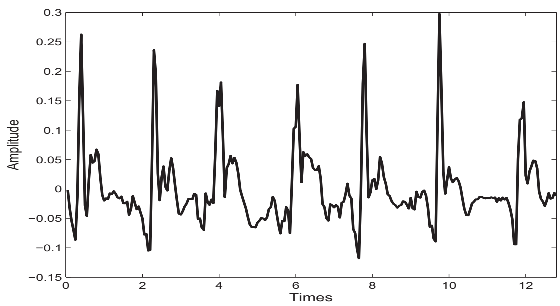

Fig. 1*: An example of EEG spike signal in the time domain*

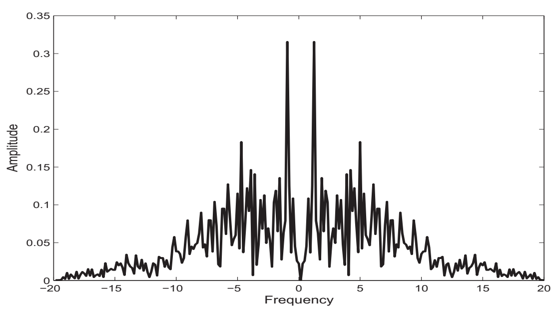

Fig. 2*: Frequency representation of the signal shown in Fig. 1*

TFD resolution of the signal in this region, the signal is shifted in the frequency domain in such a way to position the whole signal in the first middle band (negative frequency included). More precisely, let us denote $s(t)$ as the original EEG spike signal with duration T, the shifted signal $s_s(t)$ is expressed as:

$$s_s(t) = s(t)e^{(2j\pi \frac{f_{max}}{4} t)} \tag{1}$$

where $f_m ax$ is the maximum frequency of the signal. The result of frequency shifting is illustrated in Fig. 3 where it is observed that only the noise exists in the negative frequency zone and the whole signal is localized in the positive frequency. To eliminate this noise, the signal $s_s(t)$ is transformed into analytic signal $z(t)$ using Hilbert transform as follows:

$$z(t) = -\frac{1}{\pi} \lim_{\epsilon \to 0} \int_{\epsilon}^{+\infty} \frac{s_s(t+\tau) - s_s(t-\tau)}{\tau} d\tau \tag{2}$$

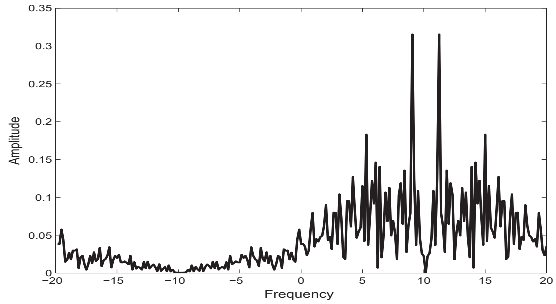

Fig. 3*: The shifted EEG Spike signal in frequency domain.*

In this paper, QTFD based on separable kernel is used. The QTFD is defined as a double convolution with WVD as:

$$\rho_z(t,f) = W_z(t,f) \underset{t,f}{\star\star} g(t,f) \tag{3}$$

where $W_z(t,f)$ is Wigner-Ville distribution (WVD) given by the Fourier transform (FT) of the instantaneous correlation function $K_z(t,\tau)$ given by:

$$\begin{aligned} W_z(t,f) &= int_{-\infty}^{+\infty} z(t+\frac{\tau}{2}) z^*(t-\frac{\tau}{2}) e^{-2j\pi\tau f} d\tau \\ &= \int_{-\infty}^{+\infty} K_z(t,\tau) d\tau \end{aligned} \tag{4}$$

In [5], the authors show that the QTFD can be written as 2D FT (half inverse, half forward) of the filtered ambiguity function (AF) as follows:

$$\rho_z(t,f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(\nu,\tau) A_z(\nu,\tau) e^{2j\pi(\nu t - f\tau)} d\nu d\tau \tag{5}$$

where the AF is given by:

$$A_z(\nu,\tau) = \int_{-\infty}^{+\infty} z(t+\tau/2) z^*(t-\tau/2) e^{-2j\pi\nu t} dt \tag{6}$$

and $g(\nu,\tau)$ is the kernel. Eq. 5 shows that the design of the kernel is easier in the ambiguity domain than in the time-frequency domain. In fact, the QTFD is a 2D FT of the multiplication of the kernel with the AF rather than a double convolution of the kernel with the WVD and thus, the design of the kernel based on the multiplication operator is simpler than the one that is based on the convolution operator. We note that the AF has a quadratic form and thus, if the signal contains several components, then the lag-Doppler plane is divided into two regions: 1) auto-terms, and 2) cross-terms artifacts. The first one is generally passed by the origin and also localized close to the origin, and the second one is mostly localized far from the origin with the same directions of the auto-terms. A separable kernel represents a specific case of the kernel that is used to conserve the auto-terms and eliminate the cross-terms where $g(\nu,\tau)$ is equal to:

$$g(\nu,\tau) = g_1(\nu) g_2(\tau) \tag{7}$$

and thus, the QTFD can be written as:

$$\rho_z(t,f) = G_1(t) \star W_z(t,f) \star G_2(f) \quad (8)$$

where $G_1(t)$ and $G_2(f)$ are FT of $g_1(\nu)$ and $g_2(\tau)$, respectively. Before designing a suitable kernel for the EEG spike signal, it is interesting to observe the localization of the auto-terms and cross-terms artifacts in the ambiguity domain. Fig.

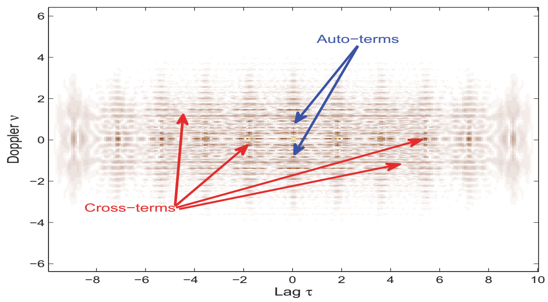

Fig. 4: *The ambiguity function of an example of an EEG spike signal*

4 illustrates an example of EEG spike signal in the ambiguity domain where it can be noticed that the energy of the auto-terms is concentrated in the Doppler direction and that cross-terms artifacts follow the same direction but far from the origin. We note also the absence of other directions in the lag-Doppler (angle) plane different from the lag and the Doppler directions. We conclude that the use of QTFD based on the separable kernel is suitable to obtain a high TFD resolution without cross-terms artifacts.

3 A Novel High-Energy Concentration QTFD (HEC QTFD)

It is shown in the previous section that the suitable kernel for EEG spike signals should be a separable kernel. In the literature, several separable kernels exist including MBD, EMBD and CSK. For examples, the expression of the kernel of EMBD which represents an extension of modified B distribution (MBD) is given by:

$$g(\nu,\tau) = \frac{|\Gamma(\beta + j\pi\nu)|^2}{\Gamma^2(\beta)} \frac{|\Gamma(\alpha + j\pi\nu)|^2}{\Gamma^2(\alpha)} \quad (9)$$

where $-0.5 \le \nu \le 0.5, \; -0.5 \le \tau \le 0.5, \; 0 \le \beta \le 1$ and $0 \le \alpha \le 1$. The disadvantage of MBD and EMBD is that the size and shape of the kernel are not independent. This independence is observed in the CSK TFD but we note that the kernel size on the lag direction and the Doppler direction have the same width size:

$$g(\nu,\tau) = \begin{cases} e^{2c} e^{\frac{cD^2}{\nu^2 - D^2} + \frac{cD^2}{\tau^2 - D^2}}, & \texttt{if} \begin{cases} \nu^2 < D^2 \\ \tau^2 < D^2 \end{cases} \\ 0, & \texttt{otherwise} \end{cases} \quad (10)$$

We note also that the authors used the conditions $\nu^2 < D^2$ and $\tau^2 < D^2$ to ensure that CSK kernel is a compact kernel. These conditions are equivalent to multiplying the kernel by rectangular windows. It is known that the FT of these windows are a $sinc$ function. So, artifacts due to the effect of lobes may appear. In order to remedy different drawbacks of EMBD TFD and CSK TFD, we proposed a new separable kernel based on the hyperbolic tangent function (tanh). This function is continuous, symmetric, and also compact. This function varies between -1 and 1 and it converges rapidly to its limits. Thus, the function $\tanh(|f|^{-\alpha})$ is close to the ideal filter when $\alpha >> 1$ and thus, the filter has high energy concentration in this zone. In order to integrate the size parameter of the filter window, the filter is modified as follows:

$$H_c(f) = \left(\tanh\left(\left|\frac{f}{f_c}\right|^{-\alpha}\right)\right)^{\alpha} \quad (11)$$

where f_c is the cut-off frequency that controls the minimal size of the filter and α is the parameter that controls the shape of the kernel as shown in Fig 5. In our context, the kernel

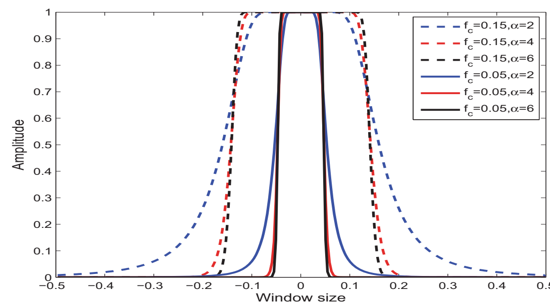

Fig. 5: *High concentration energy filter*

is designed in the AF and should be a separable kernel for EEG spike signal. We proposed the following high energy concentration kernel based on the filter in the Eq.11:

$$\begin{aligned} g(\nu,\tau) &= g_1(\nu) g_2(\tau) \\ &= \left(\tanh\left(\left|\frac{\nu}{D_1}\right|^{-\alpha}\right)\right)^{\alpha} \left(\tanh\left(\left|\frac{\tau}{D_2}\right|^{-\beta}\right)\right)^{\beta} \end{aligned} \quad (12)$$

where α and β control the shape of the kernel in the lag and Doppler directions. D_1 and D_2 represent the minimal size of the length of the windows g_1 and g_2,respectively. The proposed kernel contains four parameters and tuning thereof requires significant computational complexity. Based on the location of the cross-terms and auto-terms, we aim to reduce the number of the parameters from four to two. First, we observe that length of the window of the kernel in the Doppler direction ν in the AF of the example EEG spike signal should be higher than the window of the kernel in the lag direction τ to enclose the maximum of the auto-terms energy. This means that $g_1(\nu)$ has a large window compared to $g_2(\tau)$ and therefore, we can associate a low number of α to reduce the presence of the lobes as shown in Fig. 6. For the filter $g_2(\tau)$, the value β should be high to avoid the presence of cross-terms artifacts. Finally, the values $\alpha = 2$ and $\beta = 4$ represent a

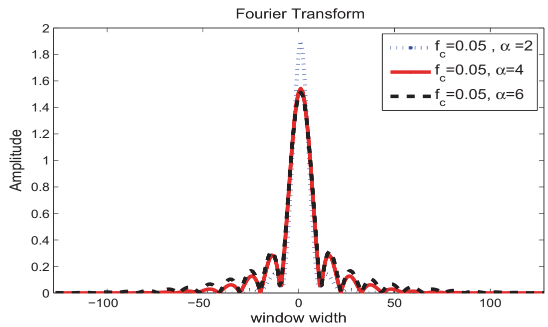

Fig. 6: *TF High concentration energy filter*

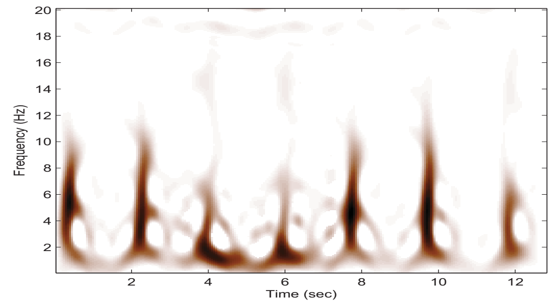

Fig. 7: *HEC without shifting signal with parameters:* $D = 0.09$ *and* $D1 = 0.038$

good trade-off between TFD resolution and cross-terms suppression. The proposed high-energy concentration TFD for EEG spike signal based on only two parameters becomes:

$$g(\nu,\tau) = \left(\tanh\left(\left|\frac{\nu}{D_1}\right|^{-2}\right)\right)^2 \left(\tanh\left(\left|\frac{\tau}{D_2}\right|^{-4}\right)\right)^4 \quad (13)$$

4 Simulation results

In this section, we discuss first the effect of the frequency shifting to enhance the TFD resolution in the low frequency band. Fig. 7 shows a TFP of the analytic EEG spike signal showed in Fig.1 by using the proposed HEC TFD. After frequency shifting to the mid frequency band, the TFP of the whole signal (including negative frequency) is shown in Fig. 8. The positive frequency part of Fig. 8 is given by Fig. 9. By comparing Fig. 7 and Fig. 9, it can be observed that the effect of the frequency shifting is noticeable as Fig. 9 shows better resolution than Fig. 7 precisely in very low frequency zone. Moreover, Fig. 8 illustrates more details than both figures in the TFP.

Now, a comparison between the proposed HEC TFD and the recent exiting QTFDs, such as EMBD, CSK and Spectrogram. is illustrated. The same EEG spike signal shown in Fig. 1 is used for this comparison. All parameters are exhaustively optimized to find the best TFP of the EEG spike signals. Figs. 10, 11, and 12 show the TFD of the shifted EEG spike signals using, Spectrogram, EMBD, and CSK TFD, respectively. We observe the absence of the cross-terms artifacts but the resolution is low when the Spectrogram TFD is used. A better TFD resolution is observed when EMBD is used, Fig 11, but the cross-terms artifacts exist. For the CSK TFD, the presence of cross-terms artifacts is less than when using the EMBD but also the TFD resolution is reduced. Compared to Spectrogram, EMBD, and CSK, the proposed HEC TFD has the best compromise between TFD resolution and cross-terms artifacts elimination as illustrated in the Fig. 8. In fact, it can be noticed that when using HEC TFD, the cross-terms artifacts disappear while the TFD resolution is still high.

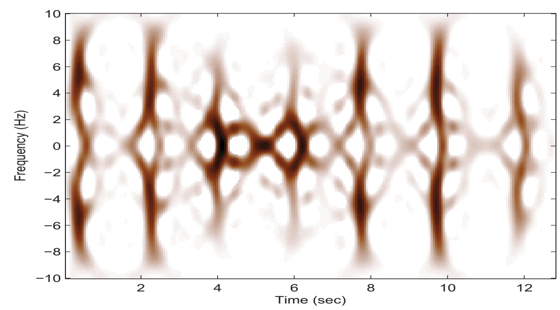

Fig. 8: *HEC TFD with shifting signal of the whole signal (positive and negative frequencies included) with parameters:* $D = 0.09$ *and* $D1 = 0.038$

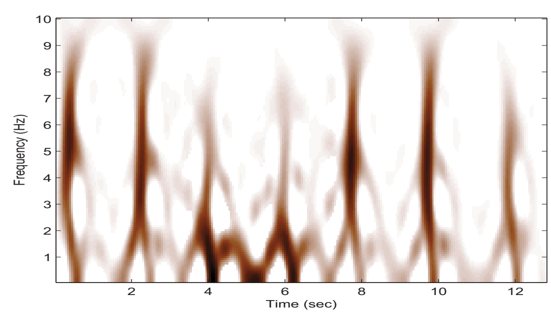

Fig. 9: *Proposed TFD: High Concentration energy TFD with shifting signal with parameters:* $D = 0.09$ *and* $D1 = 0.038$

5 Conclusion

In this paper, a new process based on shifting the frequency of the signal to enhance the TFD resolution in low frequency band is proposed. This process allows to enclose the negative and positive frequencies in the QTFD. In addition, the localization of the auto-terms and cross-terms of the EEG spike signal in the ambiguity domain is analyzed. Based on this analysis, a new QTFD refereed to HEC TFD specific for EEG spike signals is derived. HEC TFD contains only two param-

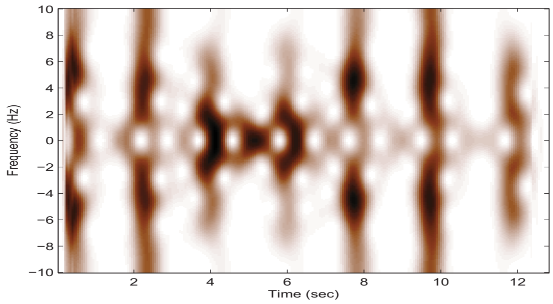

Fig. 10: *Spectrogram TFD with parameter: Hamming window* $L = 27$

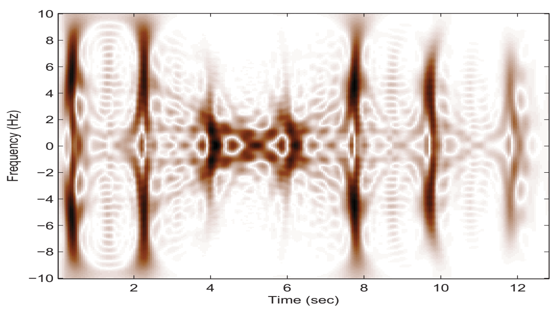

Fig. 11: *Extended Modified B distribution with parameters:* $\alpha = .08$ *and* $\beta = 0.3$*;*

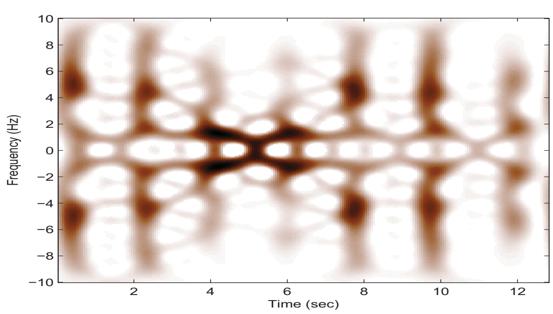

Fig. 12: *Compact Separable kernel with parameters:* $c = 2$ *and* $D = 0.09$

eters which result in low computational complexity. Simulations results proved that the proposed HEC kernel QTFD is more suitable QTFD for EEG spike signal than the existing ones. Future work aims to use HEC kernel QTFD to enhance the automatically detection of the EEG spike.

REFERENCES

[1] F. da Silva, "EEG: Origin and measurement," in *EEG-fMRI*, C. Mulert and L. Lemieux, Eds. Springer Berlin Heidelberg, 2010, pp. 19–38.

[2] A. Y. Kaplan, A. A. Fingelkurts, A. A. Fingelkurts, S. V. Borisov, and B. S. Darkhovsky, "Nonstationary nature of the brain activity as revealed by eeg/meg: Methodological, practical and conceptual challenges," *Signal Processing*, vol. 85, no. 11, pp. 2190 – 2212, 2005.

[3] J. O'Toole, M. Mesbah, and B. Boashash, "Neonatal eeg seizure detection using a time-frequency matched filter with a reduced template set," in *Signal Processing and Its Applications, 2005. Proceedings of the Eighth International Symposium on*, vol. 1, August 2005, pp. 215–218.

[4] S. Mukhopadhyay and G. Ray, "A new interpretation of nonlinear energy operator and its efficacy in spike detection," *Biomedical Engineering, IEEE Transactions on*, vol. 45, no. 2, pp. 180–187, Feb 1998.

[5] B. Boashash, *Time-Frequency Signal Analysis and Processing: A Comprehensive Reference*. Elsevier, Oxford, 2003.

[6] H.-I. Choi and W. Williams, "Improved time-frequency representation of multicomponent signals using exponential kernels," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 37, no. 6, pp. 862–871, Jun 1989.

[7] H. Hassanpour and M. Mesbah, "Neonatal eeg seizure detection using spike signatures in the time-frequency domain," in *Signal Processing and Its Applications, 2003. Proceedings. Seventh International Symposium on*, vol. 2, July 2003, pp. 41–44 vol.2.

[8] B. Barkat and B. Boashash, "A high-resolution quadratic time-frequency distribution for multicomponent signals analysis," *Signal Processing, IEEE Transactions on*, vol. 49, no. 10, pp. 2232–2239, 2001.

[9] B. Boashash and T. Ben-Jabeur, "Design of a high-resolution separable-kernel quadratic TFD for improving newborn health outcomes using fetal movement detection," in *in Proceedings of the 11th International Conference on Information Science, Signal Processing and their Applications (ISSPA)*, July 2012, pp. 354–359.

[10] M. Abed, A. Belouchrani, M. Cheriet, and B. Boashash, "Time-frequency distributions based on compact support kernels: Properties and performance evaluation," *Signal Processing, IEEE Transactions on*, vol. 60, no. 6, pp. 2814–2827, June 2012.