SPARSE SUPPORT RECOVERY FOR DOA ESTIMATION IN THE PRESENCE OF MUTUAL COUPLING

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ABSTRACT

Direction-of-arrival (DOA) estimation in the presence of mutual coupling and coherent signals is a hard task for arbitrary sensor arrays including uniform circular array (UCA). While the coherent sources can be resolved using spatial smoothing algorithms for uniform linear and rectangular arrays, it cannot be applied to UCA. In this paper, a new technique is proposed for DOA estimation in UCA using a single snapshot. Joint-sparse recovery algorithm is proposed where the source signal spatial directions and coupling coefficients are embedded into a joint-sparse signal. A dictionary is defined according to restricted isometry and compressed sensing is employed for both DOA and coupling coefficient estimation. It is shown that the proposed method performs better than the alternative sparse recovery techniques.

Index Terms— Compressed sensing, Joint-sparse recovery, Mutual coupling, Multipath, DOA estimation, Uniform circular array.

1. INTRODUCTION

In direction finding (DF) applications, mutual coupling (MC) among the antennas in the array is an important source of error that should be corrected. Coupling effect depends on both antenna type and array geometry. If the antennas in the array are omni-directional, then MC is direction independent and the same for all source directions. The array structure is another factor that should be taken into account in order to model the MC among the antennas. Different MC matrix structures are modelled for different array geometries i.e., Uniform linear arrays (ULA), Uniform circular arrays (UCA) [1] and Uniform rectangular arrays (URA) [2]. There are several works on Direction-of-arrival (DOA) estimation with unknown MC [3], [4], [5], however limited number of previous works for the DF scenario where the sources are fully coherent and the signals are corrupted with mutual coupling [6], [7], [8], [9]. When the source signals are coherent, the array covariance matrix is rank-deficient. In order to overcome this problem, spatial smoothing approaches are proposed for ULA [10] and URA [11]. In [7] and [8], ESPRIT-Like approaches are proposed to estimate the DOA's of coherent sources in the presence of MC using URA and ULA respectively. In [8], parallel-ULA geometry is used for coherent source localization in case of MC where spatial smoothing [10] and ESPRIT methods are utilized. Maximum likelihood estimation is proposed to estimate the coherent source DOA's in [9] using ULA structure. In [12] and [4], convex minimization methods are proposed for DOA estimation in case of MC with ULA. In [13], single snapshot DOA estimation problem is solved using a sparse recovery algorithm without considering the effect of MC. In [14], array interpolation and ESPRIT algorithm are used to estimate the source DOA for UCA while the source signals are assumed to be independent. In [15], rank reduction and Root-MUSIC approach is proposed for the DOA estimation of independent sources under MC.

Spatial smoothing except array interpolation [16] cannot be used for arrays such as UCA. While array interpolation is effective in ideal conditions, it cannot be applied under nonlinearities and array imperfections.

In this paper, we consider the DF problem for UCA where the source signals are fully coherent and corrupted by both antenna couplings and external noise. DOA estimation problem is solved using a joint-sparse recovery algorithm in compressed sensing (CS) framework [17] where the unknown spatial directions of the sources and the MC coefficients are jointly embedded into a sparse signal. In order to recover the joint-sparse signal, circulant structure of MC matrix is utilized and a new overcomplete dictionary space is constructed. Then convex minimization techniques are used to recover both the signal support set and MC coefficients as well as the source DOA angles.

2. SIGNAL MODEL AND PROBLEM FORMULATION

The DOA estimation problem is considered where there are K narrowband source signals impinging on M-element UCA from far-field. The array output can be written as follows

$$\mathbf{y}(t) = \mathbf{C}\bar{\mathbf{A}}\bar{\mathbf{s}}(t) + \mathbf{e}(t) \quad , t = 1, \ldots, E \tag{1}$$

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where E is the number of snapshots, $\mathbf{e}(t)$ is zero-mean, spatially and temporarily white, Gaussian additive noise. $\bar{\mathbf{s}}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is a $K \times 1$ vector composed of coherent source signals, which are related to each other as, $s_k(t) = \beta_{kl} s_l(t)$, $k, l = 1, \ldots, K$ where $k \neq l$ and β_{kl} is a complex number. $\bar{\mathbf{A}}$ is $M \times K$ array steering matrix and defined as

$$\bar{\mathbf{A}} = [\mathbf{a}(\bar{\Theta}_1), \mathbf{a}(\bar{\Theta}_2), \ldots, \mathbf{a}(\bar{\Theta}_K)] \tag{2}$$

where $\bar{\Theta}_k = (\bar{\phi}_k, \bar{\theta}_k)$ represents the azimuth and elevation angle of the k^{th} source direction respectively. The m^{th} element of the array steering vector $\mathbf{a}(\bar{\Theta}_k)$ is given as

$$a_m(\bar{\Theta}_k) = exp\left\{j\frac{2\pi}{\lambda}\mathbf{r}^T\mathbf{p}_m\right\} \tag{3}$$

where $\mathbf{r} = [\cos(\bar{\phi}_k)\sin(\bar{\theta}_k)\ \sin(\bar{\phi}_k)\sin(\bar{\theta}_k)\ \cos(\bar{\theta}_k)]^T$, λ is the wavelength and $\mathbf{p}_m = [x_m\ y_m\ z_m]^T$ is the m^{th} antenna position. $\mathbf{C} \in \mathbb{C}^{M \times M}$ is the mutual coupling matrix which is direction independent, symmetric and circulant [1]. $\mathbf{C}$ can be structured as $\mathbf{C} = T(\bar{\mathbf{c}})$ for Toeplitz operation $T(\cdot)$ and circulant vector $\bar{\mathbf{c}}$ which is defined as

$$\bar{\mathbf{c}} = \begin{cases} [c_1, c_2, \ldots, c_{\frac{M-1}{2}}, c_{\frac{M-1}{2}-1}, \ldots, c_2]^T & , M\ is\ odd \\ [c_1, c_2, \ldots, c_{\frac{M}{2}}, c_{\frac{M}{2}+1}, c_{\frac{M}{2}}, c_{\frac{M}{2}-1}, \ldots, c_2]^T & , M\ is\ even \end{cases} \tag{4}$$

where $\{c_m\}_{m=1}^{\bar{M}}$ are the coupling coefficients. $\bar{M}$ is the number of distinct coupling coefficients and $\bar{M} \in \left\{\frac{M-1}{2}, \frac{M}{2}+1\right\}$ for M is even or odd.

The aim is to find the source DOA's $\{\bar{\Theta}_k\}_{k=1}^K$ and the coupling coefficient vector $\mathbf{c} = [c_1, c_2, \ldots, c_{\bar{M}}]^T$ given that the array output $\mathbf{y}(t)$ for a single snapshot.

3. JOINT-SPARSE RECOVERY OF SUPPORT SET AND MC MATRIX

The conventional CS theory addresses the following problem

$$\min_{\mathbf{x} \in \mathbb{R}^N} ||\mathbf{x}||_0 \quad s.t.\ \mathbf{y} = \mathbf{A}\mathbf{x} \tag{5}$$

where $\mathbf{y} \in \mathbb{C}^M$ is the measurement vector, $\mathbf{x} \in \mathbb{R}^N$ is K-sparse signal, namely, all entries of $\mathbf{x}$ but K are zero. $\mathbf{A} \in \mathbb{C}^{M \times N}$ is the dictionary matrix where $N \gg M$. $||\mathbf{x}||_0 = |\{i : x_i \neq 0\}|$ denotes the number of nonzero elements of $\mathbf{x}$, namely, the supports of $\mathbf{x}$. The support set is defined as $\mathbb{S}_\mathbf{x} = \{x_i\ : x_i \neq 0\}$.

3.1. Estimation of Signal Support Set

Since we consider only a single snapshot, time index in (1) is removed and we have the following measurement vector

$$\mathbf{y} = \mathbf{C}\bar{\mathbf{A}}\bar{\mathbf{s}} + \mathbf{e}. \tag{6}$$

Above expression can be written in the CS context for noise-free case as

$$\mathbf{y} = \mathbf{C}\mathbf{A}\mathbf{s} \tag{7}$$

where $\mathbf{s}$ is $N \times 1$ K-sparse vector and $\mathbf{A}$ is $M \times N$ dictionary matrix defined as

$$\mathbf{A} = [\mathbf{a}(\Theta_1), \mathbf{a}(\Theta_2), \ldots, \mathbf{a}(\Theta_N)] \tag{8}$$

where $\Theta_i = (\phi_i, \theta)$ and the dictionary resolution in azimuth plane is $|\phi_i - \phi_{i+1}| = \Delta_\phi$, $i = 1, \ldots, N-1$ and elevation $\theta = \frac{\pi}{2}$ is selected for simplicity. In order to find the support set $\mathbb{S}_\mathbf{s}$ and $\mathbf{C}$ with CS, the following minimization problem can be considered

$$\min_{\mathbf{s} \in \mathbb{R}^N, \mathbf{C} \in \mathbb{C}^{M \times M}} ||\mathbf{s}||_0\ s.t.\ \mathbf{y} = \mathbf{C}\mathbf{A}\mathbf{s}. \tag{9}$$

Above problem is non-convex and NP-hard. In order to recast (9) into linear form, $\mathbf{s}$ and coupling coefficients $\mathbf{c}$ are embedded into a joint-sparse vector. A new dictionary is defined using the circulant structure of $\mathbf{C}$.

First, we state the MC matrix, $\mathbf{C}$, in the following form,

$$\mathbf{C} = \sum_{m=1}^{\bar{M}} c_m \mathbf{I}_m \tag{10}$$

where $\mathbf{I}_m$ is $M \times M$ matrix whose $(i,j)^{th}$ entry is given as

$$\mathbf{I}_m(i,j) = \begin{cases} 1 & , if\ \mathbf{C}(i,j) = c_m \\ 0 & , otherwise \end{cases} \tag{11}$$

where $m = 1, 2, \ldots, \bar{M}$. Now we can define the new $M \times \bar{M}N$ dictionary matrix $\mathbf{D}$ by stacking the former dictionary matrix $\mathbf{A}$ as

$$\mathbf{D} = [\mathbf{I}_1\mathbf{A}, \mathbf{I}_2\mathbf{A}, \ldots, \mathbf{I}_{\bar{M}}\mathbf{A}]. \tag{12}$$

Then (9) can be written in the following form, i.e.,

$$\min_{\mathbf{x} \in \mathbb{C}^{\bar{M}N}} ||\mathbf{x}||_0\ s.t.\ \mathbf{y} = \mathbf{D}\mathbf{x} \tag{13}$$

where $\mathbf{x} = [\mathbf{s}^T, c_2\mathbf{s}^T, c_3\mathbf{s}^T, \ldots, c_{\bar{M}}\mathbf{s}^T]^T$ for the first coupling coefficient $c_1 = 1$ is assumed which is usually the case in practical applications [1]. If (13) is solved, K-sparse signal $\mathbf{s}$ can be recovered as the first N entries of $\bar{M}K$-sparse signal $\mathbf{x}$. Note that the special structure of $\mathbf{x}$ is used in the following part of this paper.

The restricted isometry property for joint-sparse case (JS-RIP) is investigated in [18, 19] for $2K$-sparse signals. In this study, $\bar{M}K$-sparse signal $\mathbf{x}$ is said to be K-joint sparse if $||\mathbf{x}||_{0,1} = K$ where $||\cdot||_{0,1}$ denotes the joint-sparsity which is defined explicitly as

$$||\mathbf{x}||_{0,1} = |\{i : x_i^{(m)} \neq 0\}|,\ \ m = 1, \ldots, \bar{M} \tag{14}$$

where $x_i^{(m)}$ is the i^{th} entry of the m^{th} block of $\mathbf{x}$.

Definition. *The dictionary* $\boldsymbol{D}$ *is said to obey JS-RIP with sparsity level* K*, if there exists* $\delta_K \in [0, 1)$ *for all joint-sparse (*$\bar{M}K$*-sparse) signal* $\boldsymbol{x} \in \mathbb{C}^{\bar{M}N}$ *such that*

$$(1 - \delta_K)||\boldsymbol{x}||_2^2 \leq ||\boldsymbol{D}\boldsymbol{x}||_2^2 \leq (1 + \delta_K)||\boldsymbol{x}||_2^2 \tag{15}$$

holds for δ_K which can be found as [20],

$$\min_{\delta_K \in [0,1)} \delta_K \, s.t. (1-\delta_K)||\boldsymbol{x}||_2^2 \leq ||\boldsymbol{Dx}||_2^2 \leq (1+\delta_K)||\boldsymbol{x}||_2^2. \quad (16)$$

Theorem 1. *Let $\hat{\mathbf{s}}$ and $\hat{\mathbf{c}}$ be the solution to problem in (13) where the optimal solutions are $\boldsymbol{s}^\star$ and $\boldsymbol{c}^\star$ respectively. If the matrix $\boldsymbol{D}$ obeys JS-RIP with $\delta_{2K} < 1$, then the solution is unique.*

Proof: If $\hat{\mathbf{x}}$ is the solution to (13) with

$$\hat{\mathbf{x}} = [\hat{\mathbf{s}}^T, \hat{c}_2\hat{\mathbf{s}}^T, \hat{c}_3\hat{\mathbf{s}}^T, \ldots, \hat{c}_{\bar{M}}\hat{\mathbf{s}}^T]^T, \quad (17)$$

we can say that $||\hat{\mathbf{x}}||_{0,1} \leq ||\mathbf{x}^\star||_{0,1} \leq K$ since both $\hat{\mathbf{x}}$ and $\mathbf{x}^\star$ are the solution. Using the triangle inequality, the difference is bounded by $2K$ as

$$||\mathbf{x}^\star - \hat{\mathbf{x}}||_{0,1} \leq 2K. \quad (18)$$

Since, both $\hat{\mathbf{x}}$ and $\mathbf{x}^\star$ solves (13) with equality constraint, then, $\mathbf{y} = \mathbf{D}\hat{\mathbf{x}} = \mathbf{D}\mathbf{x}^\star$ which results $\mathbf{D}(\mathbf{x}^\star - \hat{\mathbf{x}}) = 0$. We can use (18) in JS-RIP as follows

$$(1-\delta_{2K})||\mathbf{x}^\star - \hat{\mathbf{x}}||_2^2 \leq ||\mathbf{D}(\mathbf{x}^\star - \hat{\mathbf{x}})||_2^2 = 0. \quad (19)$$

Then we can see that $||\mathbf{x}^\star - \hat{\mathbf{x}}||_2^2 = 0$ since $\delta_{2K} < 1$ which concludes the proof.

When the observation $\mathbf{y}$ is noisy, i.e., $\mathbf{y} = \mathbf{CAs} + \mathbf{e}$ where the noise power is bounded by $||\mathbf{e}||_2^2 \leq \epsilon^2$, the constrained CS problem in (13) can be formulated with an inequality constraint with respect to ϵ^2. Furthermore, mixed $l_{2,1}$-norm can be utilized to relax the minimization problem and ease the computational burden. The final form of the joint-sparse recovery problem can be given as

$$\min_{\mathbf{x} \in \mathbb{C}^{\bar{M}N}} ||\mathbf{x}||_{2,1} \; s.t. \; ||\mathbf{Dx} - \mathbf{y}||_2^2 \leq \epsilon^2 \quad (20)$$

where the residual is bounded by $\epsilon = \sigma_N\sqrt{M + \eta\sqrt{2M}}$ [21]. η is an adjustable parameter which controls the noise power $||\mathbf{e}||_2^2$. The mixed $l_{2,1}$-norm is defined as

$$||\mathbf{x}||_{2,1} = \sum_{i=1}^{N}\left(\sum_{m=1}^{\bar{M}} |x_{N(m-1)+i}|^2\right)^{1/2} \quad (21)$$

which can explicitly be given as

$$||\mathbf{x}||_{2,1} = \sum_{i=1}^{N}\left(\sum_{m=1}^{\bar{M}} |c_m s_i|^2\right)^{1/2}. \quad (22)$$

Using Lagrangian approach, (20) can be written as

$$\min_{\mathbf{x} \in \mathbb{C}^{\bar{M}N}} \zeta||\mathbf{x}||_{2,1} + \frac{1}{2}||\mathbf{Dx} - \mathbf{y}||_2^2 \quad (23)$$

where ζ is the penalty term which determines the trade-off between $l_{2,1}/l_2$-normed terms in the problem. A choice for ζ is $\zeta = \sigma_N\sqrt{2log(K)}$ where σ_N is the noise standard deviation [22].

3.2. Estimation of MC coefficients

Once the convex problem in (23) is solved, the coupling coefficients can be found as $c_m = x_{N(m-1)+i}/s_i$ for $m = 1, \ldots, \bar{M}$ and $i \in \mathbb{I}_{\mathbf{x}}$ which is the set of indices of $\mathbb{S}_{\mathbf{x}}$. It is a suboptimum method for finding MC coefficients since there exist K many solutions of c_m, for $i \in \mathbb{I}_{\mathbf{x}}$. We can find $\{c_m\}_{m=1}^{\bar{M}}$ by first solving (13) for $\mathbf{x}$ and obtain $\mathbf{s}$, then use Least-square solution of the following cost function for $\mathbf{c}$

$$\begin{aligned} J(\mathbf{c}) &= ||\mathbf{y} - \sum_{m=1}^{\bar{M}} c_m \mathbf{I}_m \mathbf{As}||_2^2 \\ &= \mathbf{y}^H\mathbf{y} - \mathbf{y}^H \sum_{m_1=1}^{\bar{M}} c_{m_1}\mathbf{I}_{m_1}\mathbf{As} \\ &\quad - \sum_{m_2=1}^{\bar{M}} c_{m_2}^* \mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{y} \\ &\quad + \sum_{m_1=1}^{\bar{M}}\sum_{m_2=1}^{\bar{M}} c_{m_1}c_{m_2}^*\mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{I}_{m_1}\mathbf{As}. \end{aligned} \quad (24)$$

Taking derivative of $J(\mathbf{c})$ with respect to $c_{m_2}^*$, we obtain the following expression, i.e.,

$$\begin{aligned} \frac{\partial J(\mathbf{c})}{\partial c_{m_2}^*} &= -\sum_{m_2=1}^{\bar{M}} \mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{y} \\ &\quad + \sum_{m_1=1}^{\bar{M}}\sum_{m_2=1}^{\bar{M}} c_{m_1}\mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{I}_{m_1}\mathbf{As} = 0. \end{aligned} \quad (25)$$

This expression is written in the following form,

$$\sum_{m_1=1}^{\bar{M}}\sum_{m_2=1}^{\bar{M}} c_{m_1}\mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{I}_{m_1}\mathbf{As} = \sum_{m_2=1}^{\bar{M}} \mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{y}. \quad (26)$$

Above equation can be expressed as a linear set of equations

$$\mathbf{A}_c\mathbf{c} = \mathbf{b}_c \quad (27)$$

where the elements of $\mathbf{A}_c \in \mathbb{C}^{\bar{M}\times\bar{M}}$ and $\mathbf{b}_c \in \mathbb{C}^{\bar{M}}$ are found as

$$\begin{aligned} A_c(m_2, m_1) &= \mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{I}_{m_1}\mathbf{As} \\ b_c(m_2) &= \mathbf{s}^H\mathbf{A}^H\mathbf{I}_{m_2}^H\mathbf{y}. \end{aligned} \quad (28)$$

Now we can find the MC coefficients as $\mathbf{c} = \mathbf{A}_c^{-1}\mathbf{b}_c$ and construct $\bar{\mathbf{c}}$ in (4).

4. SIMULATION RESULTS

In this section, the performance of the proposed algorithm is compared with Orthogonal Matching Pursuit (OMP) [23] and Basis Pursuit De-noising (BPDN) [22]. The measurement

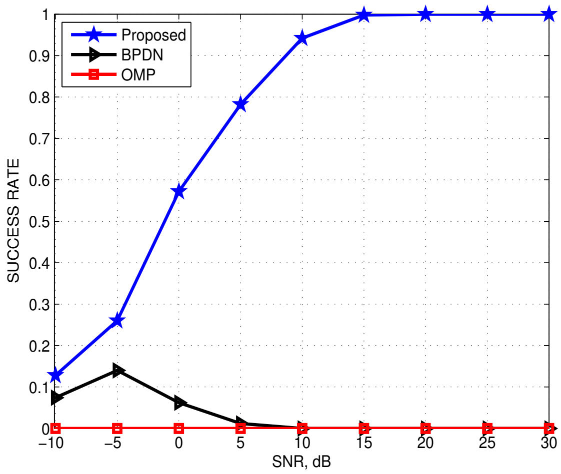

Fig. 1. Succes rate performance vs SNR.

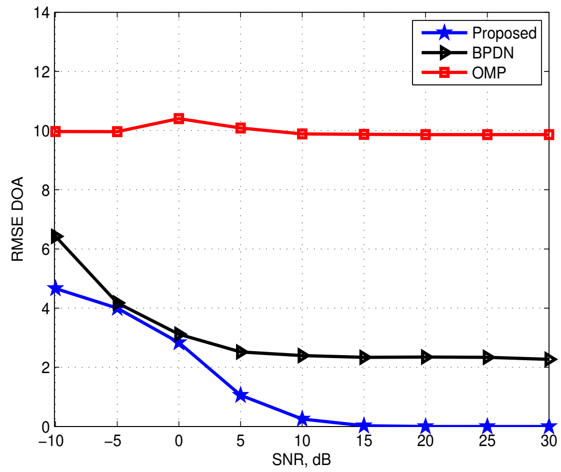

Fig. 2. DOA estimation performance vs SNR.

$\mathbf{y}$ is generated from UCA composed of $M = 16$ antennas with $\lambda/2$ array spacing. Three coherent sources are located in $(32°, 90°)$, $(51°, 90°)$ and $(75°, 90°)$ respectively. The MC coefficients are selected as $\mathbf{c} = [1.0000, 0.4935 + j0.3031, 0.3541 + j0.2264, 0.3658 + j0.2033, 0.3112 + j0.1277, 0.2927 + j0.0941, 0.2628 + j0.0808, 0.2584 + j0.0936, 0.2395 + j0.0858]^T$ for all simulations. The dictionary matrix, $\mathbf{A}$ is generated with $\Delta_\phi = 1°$ resolution. Then $\mathbf{D}$ is constructed according to (12) and $\mathbf{D}$ is used for the proposed approach whereas $\mathbf{A}$ is used for OMP and BPDN algorithms. In order to estimate $\mathbf{s}$ with the proposed aproach, the problem in (23) is solved for $\zeta = 1$ using convex problem solver cvx in MATLAB. The MC coefficients, $\mathbf{c}$, are estimated using (27). The simulations are run for $J = 500$ Monte-Carlo trials and Root-mean-square error (RMSE) for

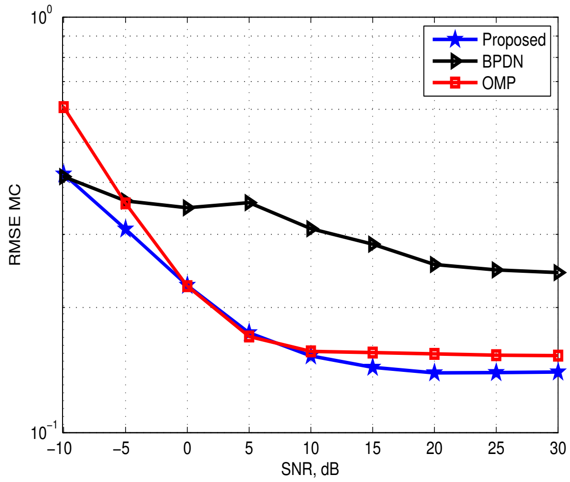

Fig. 3. RMSE for the MC coefficients vs SNR.

DOA angles and MC coefficients are computed as

$$RMSE_{DOA} = \sqrt{\frac{1}{KJ}\sum_{k=1}^{K}\sum_{j=1}^{J}|\hat{\phi}_{k,j} - \bar{\phi}_{k,j}|^2}$$
$$RMSE_{MC} = \sqrt{\frac{1}{\bar{M}J}\sum_{m=1}^{\bar{M}}\sum_{j=1}^{J}|\hat{c}_{m,j} - c^{\star}_{m,j}|^2}.$$

In Fig. 1, the probability of perfect reconstruction of the support set for the proposed algorithm is compared with the alternative methods. As it is seen, the proposed algorithm reaches the best success rate when SNR$\geq$ 15dB, whereas the other algorithms are not able to achieve this rate. This is due to the fact that the MC coefficients corrupt the array data so that the support set cannot be recovered exactly by the alternative methods. Since $\mathbf{A}$ is not structured for MC coefficients, OMP and BPDN algorithms perform as if there is a bias due to the coupling coefficients.

DOA estimation performance is evaluated in Fig. 2. The proposed method achieves the best performance for a very large range of SNR values. While BPDN algorithm does not achieve the perfect support recovery, its DOA error is approximately $2.3°$ for SNR$\geq$5dB whereas OMP gives approximately $10°$ error for the simulated SNR range and its performance does not improve. In Fig. 3, RMSE for the coupling coefficients are given. As it is seen that the proposed approach outperforms the other algorithms.

5. CONCLUSIONS

In this paper, DOA estimation problem is considered for coherent sources in case of mutual coupling in a UCA when there is only a single snapshot. A joint-sparse recovery algorithm is proposed to estimate both source DOA angles and MC coefficients. A new dictionary is defined in CS framework using joint-sparse RIP. It is shown that the proposed approach recovers source DOA angles and MC coefficients with high accuracy.

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