LIKELIHOOD-BASED BLIND SEPARATION OF QAM SIGNALS IN TIME-VARYING DUAL-POLARIZED CHANNELS

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ABSTRACT

This paper presents a new method for separating QAM signals in time-varying dual-polarized channels. The system applies an adaptive blind source separation (BSS) method based on the likelihood functions of the amplitude of the transmitted signals to recover the input signals and to track the time-varying polarization coefficients. The results demonstrate that the likelihood-based adaptive BSS method is able to recover the source signals of different modulation types for a wide range of input SNRs. The symbol error rate (SER) of estimated signals is close to the theoretical SER of different modulation types at lower SNRs. At high SNRs, the SERs are dominated by the source separation errors. The results also show that this algorithm tracks the time-varying polarization channels coefficients with small errors.

***Index Terms*—** Likelihood function, blind source separation, time-varying dual-polarization

1. INTRODUCTION

In many optical and RF communication systems, information is transmitted through dual-polarization of the carrier to improve transmission rates. For example, in dual-polarized systems, information may be transmitted through the horizontal and vertical polarizations of the carrier. In most practical systems, the received signals may not maintain the relative separation of the polarization angles due to rotation of the polarizations and other distortions. In such situations, the two information-bearing signals reconstructed at the receiver may be a mixture of the source signals. Additional processing is required to separate the original transmissions before they can be demodulated.

In this paper, we present a method for blind source separation(BSS) of dual-polarized signals with time-varying polarization angles. As far as the authors are aware of, there are no papers dealing with this topic applying BSS algorithms in the literature. The majority of the literature dealing with dual-polarization are focused on recovering the signal from dual-polarized channels and demodulating the signals assuming known modulation types. In [3], the author developed a decision-directed polarization algorithm to estimate the dual polarized channel coefficients. This paper used the correlation matrix between the estimated signal and the output signal to update the channel coefficients. In [1], the author developed a least-mean-square (LMS) algorithm to update the channel coefficients. They assumed that the symbols transmitted were known.

In this paper, we develop an adaptive likelihood-based blind source separation method, which implements the likelihood functions of the amplitude of the received signals to estimate the channel coefficients and the source signals. We assume that the two transmitted signals are independent of each other but no other information about the communication system. We also assume the transmitted signals belong to QAM signals, but no knowledge of the modulation type is assumed. This method is different from other BSS algorithms available in the literature in the sense that the separation is achieved using a likelihood-based approach that utilizes the probability density function(PDF) of the amplitude of the transmitted signals.

The rest of the paper is organized as follows. Section 2 introduces a model for the received signals with dual-polarization. In Section 3, the adaptive likelihood-based BSS algorithm is developed. The performance including the symbol error rate(SER) of recovered signals after applying BSS and the BSS system's capability to track the channel coefficients is presented in Section 4. Finally, the concluding remarks are made in Section 5.

2. SYSTEM MODEL

In this section, the system model of a time-varying dual-polarization is described. Figure 1 displays the basic model. The modulated signals are transmitted through the time-varying dual polarized channels. Let $X(t) = [x_1(t), x_2(t)]$

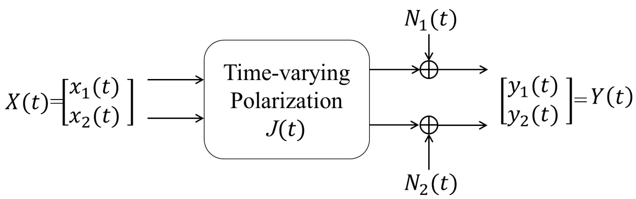


Fig. 1. Block diagram of a dual-polarized channel.

and $Y(t) = [y_1(t), y_2(t)]$ represent the transmitted and received signals respectively. Note that the existence of two channels of transmission is explicitly shown here. The relation between the transmitted and received signals of the channel can be expressed as

$$Y(t) = J(t)X(t) + N(t) \tag{1}$$

where $J(t)$ is a 2×2 matrix with elements representing the time-varying dual-polarization at each time and $N(t) = [N_1(t), N_2(t)]$ is a 2×1 matrix with elements denoting the additive white Gaussian noise in the two channels at each time. The signal model and dual-polarization model will be described next.

2.1. Signal Model

We assume a general model for the modulated signal [8] is

$$y(t) = Re\{\sum_k (s_k g_T(t - kT_b))e^{j2\pi f_c t}\} \tag{2}$$

where s_k is a complex symbol sequence with $s_k = a_k + jb_k$, a_k and b_k are the real and imaginary parts, T_b is the symbol period, $g_T(t)$ is the pulse shape filter and f_c is the carrier frequency. Applying Hilbert transformation to the received signals, an appropriately sampled complex version of this signal is given by

$$y(n) = \sum_k (s_k g_T(nT_s - kT_b))e^{j2\pi f_c nT_s} \tag{3}$$

where T_s is the sampling period.

2.2. Dual Polarized Channels Model

The polarization during transmitting the signals is changed by the variations of the channel. Therefore the state of polarization of a received signal is not known at the receiver. The unitary Jones matrix shown below describes a common model for a time-varying dual-polarization channel [3].

$$J(t) = \begin{pmatrix} \cos\{\upsilon(t)\}e^{j\delta(t)/2} & -\sin\{\upsilon(t)\}e^{-j\varepsilon(t)/2} \\ \sin\{\upsilon(t)\}e^{j\varepsilon(t)/2} & \cos\{\upsilon(t)\}e^{-j\delta(t)/2} \end{pmatrix} \tag{4}$$

In the above equation, $\upsilon(t)$ represents the cross-talk between the two polarization modes and $\delta(t)$ and $\varepsilon(t)$ describe the phase difference introduced by each channel.

3. LIKELIHOOD-BASED ADAPTIVE BLIND SOURCE SEPARATION ALGORITHM FOR COMPLEX SIGNALS

Blind source separation has been widely applied to separate mixtures of source signals. The received signals in a dual-polarized channel can be regarded as time-varying mixtures of the two transmitted signals. The goal of the BSS is to find a matrix that de-mixes the dual-polarized signals so that the transmitted signals can be recovered. The problem can be formalized as follows. We wish to find a time-varying matrix $Q(t)$ such that

$$Z(t) = Q(t)Y(t) \tag{5}$$

where $Q(t) = \begin{bmatrix} q_{11}(t) & q_{12}(t) \\ q_{21}(t) & q_{22}(t) \end{bmatrix}$ is 2×2 matrix and $Z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} q_{11}(t)y_1(t) + q_{12}(t)y_2(t) \\ q_{21}(t)y_1(t) + q_{22}(t)y_2(t) \end{bmatrix}$ is the estimation of the input signals $X(t)$.

The goal of BSS is to make the estimated signals $Z(t)$ comparable to the source signals $X(t)$. One way to achieve good estimation of the source signals is to maximize the likelihood functions of $Z(t)$ applying the probability density function of $X(t)$. Assuming the source signals belong to QAM signals, the PDF of the amplitude of different modulation type is known to be [6]

$$\begin{aligned} p(R|H_M) &= \sum_{i=1}^{N} (p(R|S_{M,i})w_M[i])), R \geq 0, \\ &= \sum_{i=1}^{N} w_M[i] \frac{R}{\sigma^2} e^{\frac{-(R^2+S_{M,i}^2)}{2\sigma^2}} I_0(\frac{RS_{M,i}}{\sigma^2}) \end{aligned} \tag{6}$$

where R is the amplitude of the signal, $p(R|H_M)$ is the conditional PDF of the signal amplitude given that the modulation type is M, $\{S_M[i]; i = 1, 2, \cdots, N\}$ is the amplitude values, $w_M[i]$ is the probability of the ith amplitude value for the Mth modulation type, σ^2 is the noise variance and $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

Applying the criteria of likelihood function, the cost function for BSS can be written as

$$C(t) = \sum_t \{\log(p(|z_1(t)||H_{M_1})) + \log(p(|z_2(t)||H_{M_2}))\} \tag{7}$$

where $p(\cdot)$ is the probability density function(PDF) of input signal $X(t)$.

We use amplitude distribution rather than the distributions of the source signals for two reasons: (1) The distribution functions of the amplitude are well defined, and (2) One of our applications involves the blind modulation identification using received signals amplitudes.

Substituting (6) in (7), the cost function becomes

$$C(t) = \sum_t \log\{\sum_{i=1}^{N} \frac{w_{M_1}[i]}{\sigma^2}|Z_1(t)|e^{\frac{-(|Z_1(t)|^2+S^2_{M_1}[i])}{2\sigma^2}} I_0(\frac{|Z_1(t)|S_{M_1}[i]}{\sigma^2})\} + \sum_t \log\{\sum_{i=1}^{N} \frac{w_{M_2}[i]}{\sigma^2} |Z_2(t)|e^{\frac{-(|Z_2(t)|^2+S^2_{M_2}[i])}{2\sigma^2}} I_0(\frac{|Z_2(t)|S_{M_2}[i]}{\sigma^2})\} \quad (8)$$

In order to find the maximum of the likelihood function, the elements of $Q(t)$ are updated by taking the gradient of the cost function in the following manners.

$$\begin{aligned} Q(t+1) &= Q(t) + \mu \frac{\partial C(t)}{\partial Q(t)} \\ &= Q(t) + \mu \frac{\partial C(t)}{\partial |Z(t)|}\frac{\partial |Z(t)|}{\partial Q(t)} \end{aligned} \quad (9)$$

The gradient $\frac{\partial C(t)}{\partial |Z(t)|}\frac{\partial |Z(t)|}{\partial Q(t)}$ will be derived with respect to each element of the matrix $Q(t)$. As an example, we differentiate with respect to q_{11} as shown below:

$$\frac{\partial C(t)}{\partial |Z(t)|}\frac{\partial |Z(t)|}{\partial q_{11}(t)} = \frac{y_1(t)}{p(|z_1(t)|)}\frac{\partial p(|z_1(t)|)}{\partial |z_1(t)|}\frac{\partial |z_1(t)|}{\partial q_{11}(t)} \quad (10)$$

First, we note that

$$\begin{aligned} &\frac{\partial p(|z_1(t)|)}{\partial |z_1(t)|} = \frac{\partial p(R)}{\partial R}|_{R=|z_1(t)|} \\ &= \sum_{i=1}^{N} \frac{w_{M_1}[i]}{\sigma^2}\frac{\partial R}{\partial R} e^{\frac{-(R^2+S^2_{M_1}[i])}{2\sigma^2}} I_0(\frac{RS_{M_1}[i]}{\sigma^2}) \\ &+ \sum_{i=1}^{N} \frac{w_{M_1}[i]}{\sigma^2} R \frac{\partial e^{-(R^2+S^2_{M_1}[i])/2\sigma^2}}{\partial R} I_0(\frac{RS_{M_1}[i]}{\sigma^2}) \\ &+ \sum_{i=1}^{N} \frac{w_{M_1}[i]}{\sigma^2} Re^{\frac{-(R^2+S^2_{M_1}[i])}{2\sigma^2}} \frac{\partial I_0(\frac{RS_{M_1}[i]}{\sigma^2})}{\partial R} \\ &= \sum_{i=1}^{N} \frac{w_{M_1}[i]}{\sigma^2} e^{\frac{-(R^2+S^2_{M_1}[i])}{2\sigma^2}} [I_0(\frac{RS_{M_1}[i]}{\sigma^2}) \\ &- \frac{R^2}{\sigma^2} I_0(\frac{RS_{M_1}[i]}{\sigma^2}) + \frac{S_{M_1}[i]}{\sigma^2} I_1(\frac{RS_{M_1}[i]}{\sigma^2})] \end{aligned} \quad (11)$$

where $I_1(\cdot)$ is the first-order modified Bessel function of the first kind.

Then, applying the gradient to real and imaginary part of $q_{11}(t)$ separately, we obtain

$$\frac{\partial |z_1(t)|}{\partial \Re\{q_{11}(t)\}} = \frac{\partial |z_1(t)|}{\partial \Re\{q_{11}(t)\}} = \frac{\Re\{z_1(t)\}}{|z_1(t)|} y_1^*(t) \quad (12)$$

$$\begin{aligned} \frac{\partial |z_1(t)|}{\partial \Im\{q_{11}(t)\}} &= \frac{\partial |z_1(t)|}{\partial \Im\{q_{11}(t)\}} \\ &= \frac{\Im\{z_1(t)\}}{|z_1(t)|}(\Im\{y_1(t)\} + j\Re\{y_1(t)\}) \end{aligned} \quad (13)$$

Substituting (10),(11), (12) and (13) in (9), the updating equation of q_{11} with respect to real and imaginary parts becomes

$$\begin{aligned} q_{11}(t+1) &= \Re\{q_{11}(t+1)\} + j\Im\{q_{11}(t+1)\} \\ &= (\Re\{q_{11}(t)\} + \mu \frac{\partial C(t)}{\partial \Re\{q_{11}(t)\}}) \\ &\quad + j(\Im\{q_{11}(t)\} + \mu \frac{\partial C(t)}{\partial \Im\{q_{11}(t)\}}) \\ &= \frac{y_1(t)}{p(|z_1(t)|)}\frac{\partial p(|z_1(t)|)}{\partial |z_1(t)|} \\ &\quad [\frac{\partial |z_1(t)|}{\partial \Re\{q_{11}(t)\}} + j\partial \frac{|z_1(t)|}{\partial \Im\{q_{11}(t)\}}] \\ &= \frac{y_1(t)}{p(|z_1(t)|)}\frac{\partial p(|z_1(t)|)}{\partial |z_1(t)|} [\frac{\Re\{z_1(t))\}}{|z_1(t)|} y_1^*(t) \\ &\quad + \frac{\Im\{z_1(t)\}}{|z_1(t)|}(\Im\{y_1(t)\} + j\Re\{y_1(t)\})] \end{aligned} \quad (14)$$

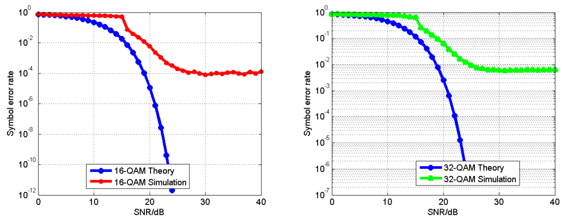
The updating equations for other three elements in $Q(t)$ can be derived similarly as (14).

4. PERFORMANCE EVALUATION

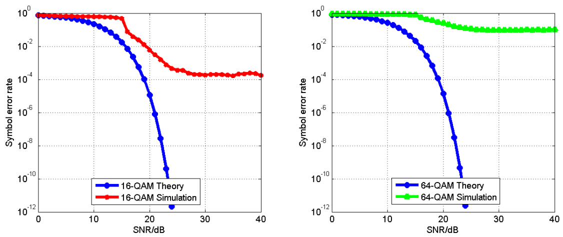
In this section, the performance of the likelihood-based BSS algorithm is demonstrated by comparing the average symbol error rates of the separated signals via Monte Carlo simulations with the theoretical SERs. We also present results evaluating the ability of the system to track the time-varying coefficients of the dual-polarized channel.

Figure 2 shows the comparison between average SERs with the corresponding theoretical results for different modulation types. In Figure 2, the channel coefficients were $\upsilon(t) = 1533\pi t, \delta(t) = 575\pi t$ and $\varepsilon(t) = 767\pi t$. This case corresponds to a rotating polarization with period =1.3 ms. The results are shown for two different cases. The first is when the two input signals were 16-QAM with 32-QAM signals and the second is for 16-QAM with 64-QAM signals. The SNR value is ranged from 0 to 40 dB. We can notice from the reuslts that the symbol error rates (SERs) of the estimated signals are close to the theoretical SER of different modulation types at lower SNRs. At high SNRs, the SERs are dominated by the source separation errors. This results in essentially no reduction in SERs with increasing SNR after some threshold.

This happens because that the BSS algorithm introduces a residual error into the separated signals during the separation process. At higher SNRs, the effect of this error on SER will dominate comparing with the effect of channel noise. The residual error power will depend on the rate of time variations in the dual-polarization channel and the step size of the adaptive separation system. Thus the performance of the receiver or other types of processors of the separated signals is limited by these separation errors. Analysis performance of BSS algorithms and similar behavior at high SNRs can be found in [9] and [10].



(a)



(b)

Fig. 2. The comparison between symbol error rate of separated signals after likelihood-based BSS and theoretical SERs at different SNRs: (a) 16-QAM and 32-QAM; (b) 16-QAM and 64-QAM.

Figure 3 and 4 show the results for the likelihood-based BSS for different modulation types in the likelihood functions. In Figure 3, we present the results of tracking the coefficients by comparing the estimated coefficients with the inverse of dual-polarization matrix. In Figure 4, the product of $J(t)$ and $Q(t)$ is presented. Here the input two signals are a 16-QAM signals and a 32-QAM signals. The parameters of the systems are the same as before. In these two figures, case 1 is when employed the likelihood functions based on the exact modulation types, which are 16-QAM and 32-QAM in this case. Case 2 used 16-QAM based likelihood functions for both input signals regardless of the modulation types.

We observe from Figure 3 that the system can track the amplitude of the coefficients with a constant scaling factor but not the phase of the coefficients. However, from Figure 4 we can see that the two diagonal elements of the matrix for the product are constant and the two off-diagonal elements are almost 0. The phase of the two diagonal elements is linear. This means that the separated signals after the likelihood-based BSS algorithm will be the source signals with constant scaled amplitudes and constant phase shifts. This is as expected for the adaptive source separation algorithms. The constant scaling factor on amplitude and the linear phase shift will not change the characteristics of the source signals.

We also observe that modulation types based cost functions exhibited similar performance of tracking the coefficients as single modulation type based cost function, which indicates that the modulation types used in the cost function will not affect the performance of the separation. The PDF described in (6) belongs to the sub-Gaussian PDF group regardless of the modulation type. This is not a surprising results since BSS algorithms employing contrast functions applicable to whole classes of some distributions have been studied before [4] and [11].

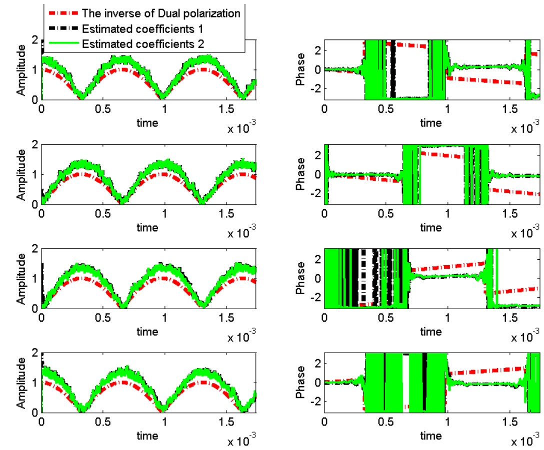


Fig. 3. Comparison of estimated coefficients with actual coefficients at each time.

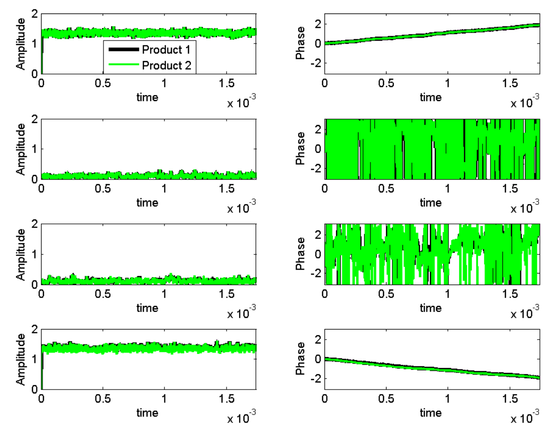


Fig. 4. Product of the dual-polarization and estimated coefficients at each time.

5. CONCLUSION REMARKS

This paper presented a likelihood-based blind separation method for QAM signals in time-varying dual-polarized channels. The performance indicated that the likelihood-based adaptive blind source separation in this paper can recover the signals from the dual-polarized channels at low SNRs. The system recovered the signal with small symbol error rates in a wide range of different SNR value. This algorithm also tracks the time-varying coefficients well. However, we also showed that the performance capabilities of the receiver may be limited by fast variations in the dual-polarization channels and residual separation errors in the separation process.

In order to increase the accuracy of the system, we will improve the adaptive BSS method considering about tracking the phase of the coefficients in the dual-polarized channels. Additional performance evaluation including theoretical analysis of the algorithm will be presented in future papers.

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