# MOVING TARGET LOCALIZATION USING DISTRIBUTED DUAL-FREQUENCY RADARS AND SPARSE RECONSTRUCTION

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## ABSTRACT

In this paper, we present a sparsity-based approach for target location and velocity estimation using a network of distributed dual-frequency radar units. A single dual-frequency radar can only estimate the range and radial velocity component of the moving target. The distributed configuration permits not only target localization in cross-range and downrange, but also provides Doppler velocity diversity, which enables the estimation of the horizontal and vertical target velocity components. We develop a linear signal model for the distributed radar network configuration under dual-frequency operation, and perform joint optimization for simultaneously recovering the target location and motion parameters. Supporting simulation results are provided, which validate the effectiveness of the proposed method.

***Index Terms***— Dual-frequency, moving target, localization, sparse reconstruction

## 1. INTRODUCTION

Target detection and localization are highly desirable in urban radar sensing applications, such as surveillance and reconnaissance, survivor search in natural disasters, and hostage rescue missions [1]. To this end, ground-based urban radar systems typically employ physical or synthetic apertures, which are deployed in a co-located configuration on a vehicular platform. More recently, distributed configurations of man-portable radar units, equipped with a limited number of transmitters/receivers, have emerged as an effective and flexible alternative to vehicle-mounted large-aperture systems for localizing targets in urban sensing applications.

For moving targets, a distributed network of man-portable radar units provides Doppler velocity diversity, which enables the estimation of the horizontal and vertical target velocity components. The individual radar units can be wideband radars, such as linear frequency modulated or pulse-Doppler radars, or dual-frequency radars [2, 3]. The latter provides a viable solution to address the operational constraints on cost and system complexity in urban radar sensing [4, 5].

Sparse signal reconstruction techniques have been extensively applied to radar applications, including urban sensing [6]. Such techniques have been shown to outperform least squares and backprojection based approaches when the scene being interrogated is sparse [7-9]. The scene can be sparse in space and/or Doppler. The former includes sparsity in downrange and crossrange, and for the latter, the sparsity can be in velocity and/or acceleration. Sparse reconstruction techniques can be applied when electromagnetic sensing is performed using either co-located or distributed sensor configurations [6].

In this paper, we consider a multiplicity of dual-frequency radars for moving target localization and velocity estimation in urban environments using sparsity-based techniques. The radar units are deployed in a distributed configuration around a sparse scene of a few moving targets, which allows the sensors to view the targets from different aspect angles. This provides Doppler velocity diversity, which enables estimation of the horizontal and vertical target velocity components. The measurements are made at each individual unit in a stand-alone fashion, which are then communicated to a central processing unit where they are combined using sparse reconstruction. Specifically, we develop a linear signal model for the distributed dual-frequency radar network and perform joint optimization for simultaneously recovering the target location and motion parameters. Supporting simulation results are provided, which validate the effectiveness of the proposed method.

The remainder of the paper is organized as follows. The signal model is presented in Section 2. Section 3 describes the proposed sparse reconstruction approach for target localization and motion estimation. Supporting simulation results are provided in Section 4, followed by concluding remarks in Section 5.

## 2. SIGNAL MODEL

### 2.1. Single Radar Unit

Consider a dual-frequency radar unit, operating at the two carrier frequencies, $f_1$ and $f_2$, and located at position vector $\boldsymbol{x}_m = (x_m, y_m)$. Assume a single target is present at initial position $\boldsymbol{x_0} = (x_0, y_0)$, moving with uniform velocity $v$ in the direction $\theta$, as shown in Fig. 1. Let $v_x = v\cos(\theta)$ and $v_y = v\sin(\theta)$ denote the horizontal and vertical target velocity components, respectively. The target is assumed to be undergoing linear motion, with the range-to-motion given by

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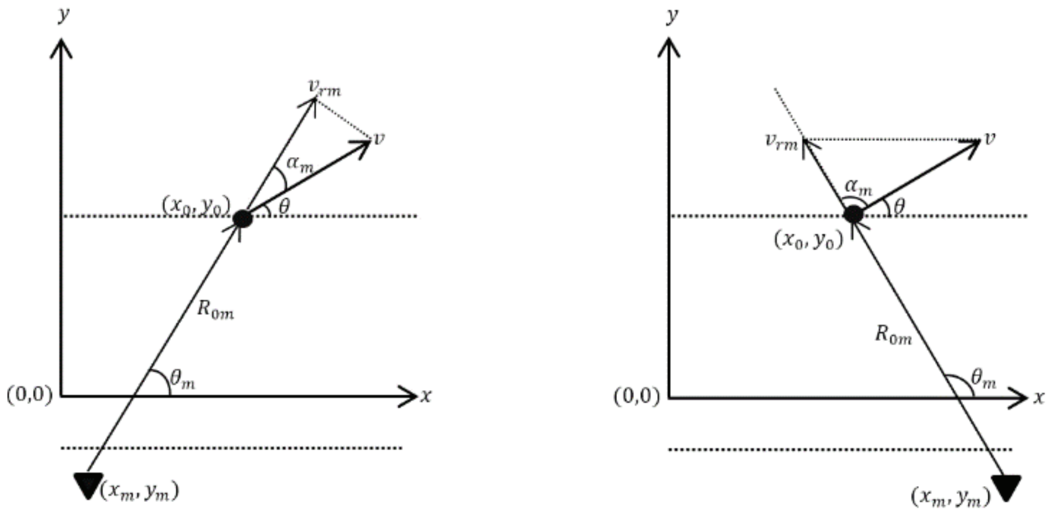


(a) (b)

**Fig. 1.** Free-space scene geometry, (a) Case 1, (b) Case 2.

$$R_m(t) = R_{0m} + v_{rm}t,, \tag{1}$$

where $R_{0m}$ is the initial target range

$$R_{0m} = \sqrt{(x_0 - x_m)^2 + (y_0 - y_m)^2}, \tag{2}$$

and $v_{rm}$ is the target radial velocity

$$v_{rm} = v\cos(\alpha_m). \tag{3}$$

In (3), $\alpha_m$ is the angle between the direction of target motion and the radar line-of-sight (LOS), as shown in Fig. 1, and is given by

$$\alpha_m = \theta_m - \theta, \tag{4}$$

where $\theta_m$ is the angle between the radar LOS and the $x$-axis. For the case depicted in Fig. 1(a), $\theta_m$ is given by

$$\theta_m = \tan^{-1}\left(\frac{y_m - y_0}{x_m - x_0}\right), \tag{5}$$

whereas that corresponding to Fig. 1(b) is

$$\theta_m = \pi - \tan^{-1}\left(\frac{y_m - y_0}{x_m - x_0}\right). \tag{6}$$

The baseband return, corresponding to the $i$th frequency, can be expressed as [10]

$$s_{im}(t) = \rho_m \exp\left(-j\frac{4\pi f_i}{c}(R_{0m} + v_{rm}t)\right),\ i = 1,2 \tag{7}$$

where $\rho_m$ is the amplitude of the return, which is assumed to be frequency-independent over the bandwidth considered, and $c$ is the speed of propagation in free-space. For the case of $K$ moving targets, the radar return is a superposition of the individual target returns.

### 2.2. Multiple Radar Units

Let there be $M$ dual-frequency radar units, each operating at frequencies $f_1$ and $f_2$, distributed around the target in the $(x, y)$-plane (See Fig. 2). Since the radar unit measures the returns from different sides, the $M$ radar returns have to be registered to a single reference coordinate system before the data can be combined for target localization and motion parameter estimation. For the radar units deployed along the bottom of Fig. 2, the *xy*-coordinate system, as defined in Fig. 2, is used, with the corresponding radar return given by eq. (7). The *xy*-coordinate system is chosen as the reference for registration. Let $x'y'$ be the radar specific coordinate system. For the radar units interrogating the scene from the top of Fig. 2, the corresponding radar return is expressed as

$$s'_{im}(t) = \rho_m \exp\left(-j\frac{4\pi f_i}{c}(R'_{0m} + v'_{rm}t)\right),\ i = 1,2 \tag{8}$$

where $R'_{0m}$ and $v'_{rm}$ are given by eqs. (2)-(4) with $(x_m, y_m)$, $(x_0, y_0)$, and $\theta$ replaced by $(x'_m, y'_m)$, $(x'_0, y'_0)$, and $\theta'$. Using the *xy*-coordinate system as the reference, the registration process is simply an identity mapping from the signal $s'_{im}(t)$ to a corresponding signal $s_{im}(t)$, where the radar specific coordinates $(x', y')$ are related to the $(x, y)$ coordinates by a 180° counter-clockwise rotation,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\pi) & \sin(\pi) \\ -\sin(\pi) & \cos(\pi) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}, \tag{9}$$

and the target motion direction is $\theta' = \pi + \theta$.

Likewise, for the radars deployed to the right and left sides in Fig. 2, the registered signal $s_{im}(t)$ corresponding to the signal $s'_{im}(t)$ (defined in eq. (8) with respect to the target specific coordinate system) is obtained by exploiting the respective coordinate system transformations: 90° and 270° counter-clockwise rotations relating the $(x', y')$ and $(x, y)$ coordinates. The corresponding target motion angles in the radar specific coordinate system can also be readily calculated through geometric considerations. Once the radar returns from the $M$ dual-frequency radar units have been registered, sparsity-based combining can proceed to jointly estimate the target position and velocity components.

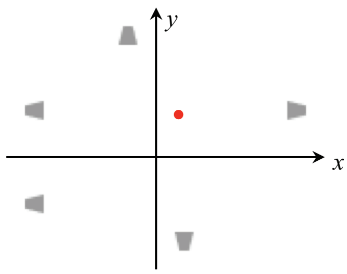


**Fig. 2.** Distributed radar configuration.

## 3. SPARSITY-BASED TARGET LOCATION-VELOCITY ESTIMATION

Assume that the scene of interest is divided into $N_x \times N_y$ pixels in crossrange ($x$) and downrange ($y$). Likewise, the horizontal and vertical velocity components are sampled on a discrete grid with $N_{v_x} \times N_{v_y}$ values. In other words, an image with $N_x \times N_y$ pixels is associated with each horizontal and vertical velocity pair considered, resulting in a four-dimensional (4D) target space. Since each dual-frequency radar observes the target space from a different aspect angle, we assume an individual target state vector of length $N_x N_y N_{v_x} N_{v_y}$ for each radar unit. Let $\boldsymbol{r}^{(m)}$ denote the target state vector for the $m$th radar unit. If a target exists at a specific point in the 4D space, the corresponding element of $\boldsymbol{r}^{(m)}$ assumes a non-zero value; otherwise, it is zero.

The baseband radar return $s_{im}(t)$ after registration is sampled at times $\{t_n\}_{n=0}^{N-1}$. Appending the $N$ samples at each of the two frequencies $f_1$ and $f_2$, we form a $2N\times 1$ measurement vector $\boldsymbol{s}^{(m)}$ corresponding to the $m$th radar unit, which using (7) can be expressed as,

$$\boldsymbol{s}^{(m)} = \boldsymbol{\Psi}^{(m)}\boldsymbol{r}^{(m)},\ m=1,2,\ldots,M \tag{10}$$

where

$$\boldsymbol{\Psi}^{(m)} = \left[\left(\boldsymbol{\Psi}_1^{(m)}\right)^T \quad \left(\boldsymbol{\Psi}_2^{(m)}\right)^T\right]^T \tag{11}$$

is of dimension $2N \times N_x N_y N_{v_x} N_{v_y}$ and the $(p, q)$th element of $\boldsymbol{\Psi}_i^{(m)}, i = 1,2$ , is equal to the complex exponential term in eq. (8) after registration.

Stacking the measurement vectors corresponding to all $M$ radar units results in a $2MN\times 1$ measurement vector

$$\boldsymbol{s} = \left[\left(\boldsymbol{s}^{(1)}\right)^{\boldsymbol{T}} \quad \cdots \quad \left(\boldsymbol{s}^{(M)}\right)^{\boldsymbol{T}}\right]^T, \tag{12}$$

we obtain the linear model

$$\boldsymbol{s} = \boldsymbol{\Psi}\boldsymbol{r} \tag{13}$$

where $\boldsymbol{\Psi} \in \mathbb{C}^{2MN\times MN_x N_y N_{v_x} N_{v_y}}$ is the dictionary matrix given by

$$\boldsymbol{\Psi} = \text{blkdiag}(\boldsymbol{\Psi}^{(1)} \quad \cdots \quad \boldsymbol{\Psi}^{(M)}), \tag{14}$$

bdiag(·) denotes a block diagonal matrix, and

$$\boldsymbol{r} = \left[\left(\boldsymbol{r}^{(1)}\right)^{\boldsymbol{T}} \quad \cdots \quad \left(\boldsymbol{r}^{(M)}\right)^{\boldsymbol{T}}\right]^T \in \mathbb{C}^{MN_x N_y N_{v_x} N_{v_y}\times 1}. \tag{15}$$

Given the combined measurements $\boldsymbol{s}$ in (13), we aim at recovering the target state information $\boldsymbol{r}$ using sparse reconstruction. Since the target state vectors corresponding to the $M$ radar units all describe the same underlying 4D target space, the vector $\boldsymbol{r}$ exhibits a group sparse structure. That is, if a certain element in, say $\boldsymbol{r}^{(1)}$, assumes a non-zero value, the corresponding elements in $\boldsymbol{r}^{(m)}, m = 2,3,\cdots,M$, should be also non-zero. In other words, the target state vectors corresponding to the $M$ radar units share the same sparsity pattern. As such, the target state vector can be recovered using a group sparse reconstruction approach, such as SparSA [11], group LASSO [12], or greedy algorithms [13, 14]. In this paper, we use Simultaneous Orthogonal Matching Pursuit (SOMP) for recovering the target space vector from the data measurements [14].

## 4. SIMULATION RESULTS

Consider three dual-frequency radar units located, respectively, at (-1 m, -0.2 m), (-0.1 m, -0.1 m), and (1 m, -0.1 m) in the $(x, y)$-plane. The carrier frequencies used in the simulation are 990 MHz and 1 GHz, leading to an unambiguous range of 15 m. Two point targets are considered; the first target is at an initial position (1.5, 1) m and moving with velocities (0.5, 0) m/s, while the second target is initially located at (3 m, 2 m) and moving with velocities (0, -0.3) m/s. White Gaussian noise at 10 dB SNR is added to the radar returns at each frequency. A total of $N = 40$ samples are collected with 10 Hz sampling rate.

The spatial region of interest is of dimensions 5 m × 5 m, with the crossrange and downrange discretized on a square grid with a step size of 0.1 m. The horizontal and vertical velocity components ranging between -1 m/s are and 1 m/s are sampled with a grid size of 0.1 m/s. Therefore, $N_x = N_y = 51$ and $N_{v_x} = N_{v_y} = 21$. Fig. 3 shows the joint position-velocity estimates of the two moving targets with noisy data using SOMP. Clearly, SOMP provides accurate estimates of position and velocity components. For comparison, we also employ the conventional Doppler filtering based least squares approach, wherein $M$ range estimates are first obtained by comparing the phases at the peaks of the complex Doppler spectra at the two carrier frequencies [5, 15], followed by position and velocity estimation through solving a non-linear least squares problem [16]. For the considered simulation example, the conventional approach provides target position estimates of (1.5512 m, 0.9815 m) and (2.9751 m, 1.8765 m), whereas the corresponding velocity component estimates are determined to be (0.5132 m/s, -0.0023 m/s) and (-0.0128 m/s, -0.2985 m/s). Unlike the sparse reconstruction based results, the conventional approach provides biased position and velocity estimates.

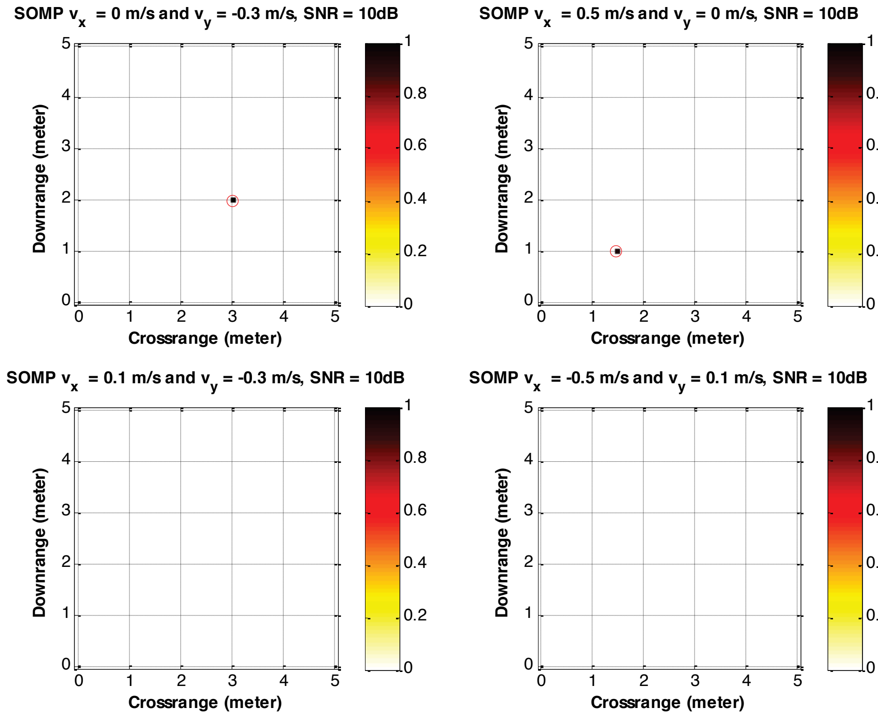


**Fig. 3.** SOMP Joint range-velocity estimates of two moving targets. Red circles indicate the true target parameters.

## 5. CONCLUSION

A sparse reconstruction technique has been proposed to provide target motion and position parameter estimation using a distributed network of dual-frequency radars for urban sensing applications. The diversity provided by the distributed configuration allows not only the target to be localized in crossrange and downrange, but also enables estimation of the horizontal and vertical target velocity components. We developed a linear signal model for the distributed radar system under dual-frequency operation. The measurements from the individual radar units are combined using sparse reconstruction techniques for simultaneously recovering the target location and motion parameters. Supporting simulation results were provided, which demonstrated the superior performance of the proposed approach as compared to the conventional Doppler filtering based least squares combining method.

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