JOINT OPTIMIZATION OF TRANSMIT AND RELAY BEAMFORMER FOR SINGLE GROUP MULTICAST TRANSMISSION

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ABSTRACT

In this paper, single group multicasting is considered for the cooperative relay network. Two-phase transmission is performed and amplify-and-forward relay protocol is used. In the first phase, the base station broadcasts the common signal to the single antenna relays by employing beamforming. Each relay multiplies its received signal by a complex weight and retransmits it to the single antenna users. The aim is to find the beamformer weights at the base station and the relays jointly to minimize the total transmitted power while satisfying signal-to-noise ratio constraint for each user. Nonconvex joint problem is firstly relaxed and then converted to an equivalent biconvex problem by using exact penalty approach. The equivalent problem is solved iteratively using alternating minimization. To the best of our knowledge, this is the first work that considers joint beamforming at the base station and the relays for single group multicasting scenario.

Index Terms— Multicast beamforming, distributed beamforming, convex optimization, exact penalty function

1. INTRODUCTION

Signal fading is one of the major challenges for communication over wireless channels [1]. Using multiple antennas at the base station is an efficient approach to combat signal fading and increase the diversity [2]. If the channel state information (CSI) of the users is available at the base station, beamforming can be used to in order to steer power in the desired directions. In this paper we consider single group multicast beamforming where a single beamforming weight vector is designed for the transmission of a common signal to the multiple destinations [2]. Recently cooperative communication with relays is used to increase coverage and capacity [3], [4]. The relays and users have a single antenna. In cooperative relaying, single antenna relays work as a distributed beamformer to take advantage of spatial diversity.

In [3], several relaying protocols are introduced and amplify-and-forward (AF) has been shown to achieve full diversity. In this paper, AF protocol is used for simplicity. After receiving the signals transmitted by the base station, each relay multiplies its received signal by a complex weight such that quality of service (QoS) constraints are satisfied for the users while minimum transmission power is used.

Multicast beamforming is investigated for different scenarios in the literature. In [2], multicast beamforming is employed without using cooperative relaying. In [4], distributed beamforming is presented for multi-group multicasting relay network where the transmitter is a single antenna node and hence beamforming is not used at the source. The joint design of beamformer weights at the base station and the relays is introduced in [1] and [5]. However these works employ unicast transmission which cannot be used for broadcasting a common signal to users. This paper introduces the first example of a broadcast scenario where the beamformer weights at the base station and the relays are jointly designed.

The optimization problem for multicast beamforming even without distributed beamforming is NP hard [2]. In this paper, some constraints of the joint nonconvex problem are relaxed and then the problem is converted to a biconvex problem using exact penalty function. This conversion is then exploited to implement alternating minimization where the resulting biconvex problem is solved iteratively. These iterations are guaranteed to converge. Furthermore, to overcome the infeasibility introduced by the relaxation, iterative channel rotation proposed in [4] is applied. Simulations show that small number of iterations is needed for the proposed algorithm.

2. SYSTEM MODEL

Consider a wireless relay network with R relays in a single group multicast (broadcast) transmission from a base station to N destination users. Base station is equipped with M antennas whereas all relays and users have a single antenna. It is assumed that there is no direct link between the source and destination users due to path losses. We consider the two-hop data transmission in half-duplex mode. In the first phase of the transmission, the base station broadcasts its signal to the relays and in the second phase, all relays amplify and forward the broadcast signal to the destination users. Let s be the common transmitted symbol and $\mathbf{b} \in \mathbb{C}^{M\times 1}$ denote the beamforming weight vector at the base station. The signal transmitted by the antenna array is given as,

$$\mathbf{x} = \mathbf{b}s \tag{1}$$

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Assume that s is temporally white, zero mean and unit variance, $\sigma_s^2 = 1$ without loss of generality. Then the total power transmitted from the base station is $P_B = \mathbf{b}^H\mathbf{b}$. The received relay signal can be written as,

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{v} = \mathbf{H}\mathbf{b}s + \mathbf{v} \tag{2}$$

where $\mathbf{r} = [\ r_1\ r_2\ ...r_R\ ]^T$ and r_i is the received signal at the i^{th} relay. $\mathbf{v} = [\ v_1\ v_2\ ...\ v_R\ ]^T$ is the relay noise vector and $\mathbf{H} \in \mathbb{C}^{R\times M}$ denotes the frequency-flat quasi-static channel between the base station and the relay network. The noise vector $\mathbf{v} \in \mathbb{C}^{R\times 1}$ is assumed to be white Gaussian, i.e, $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \sigma_v^2\mathbf{I})$ and uncorrelated with the transmitted signal.

In the second phase of the transmission, the i^{th} relay multiplies its received signal, r_i, by a complex weight $w_i^\star$ and transmits the resulting signal, $u_i = w_i^\star r_i$, to the destination users. The transmitted signal from the relays is given as, $\mathbf{u} = \mathbf{W}^H\mathbf{r}$ where $\mathbf{u} = [\ u_1\ u_2\ ...\ u_R\ ]^T$ and $\mathbf{W}$ is a diagonal matrix whose elements are complex conjugates of the complex weights, i.e., $\mathbf{W} = diag\{w_1, w_2,\ ..., w_R\}$. The received signal at the k^{th} user is,

$$y_k = \mathbf{g_k}^T\mathbf{u} + n_k = \mathbf{g_k}^T(\mathbf{W}^H\mathbf{H}\mathbf{b}s + \mathbf{W}^H\mathbf{v}) + n_k \tag{3}$$

where $\mathbf{g_k} = [\ g_{k,1}\ g_{k,2}\ ...\ g_{k,R}\ ]^T$ and $g_{k,i}$ denotes the complex channel gain between the i^{th} relay and k^{th} user. n_k is the zero mean noise at the k^{th} destination with variance σ_k^2. Defining $\mathbf{G}_k = diag\{g_{k,1}, g_{k,2}, ..., g_{k,R}\}$ and $\mathbf{w} = [\ w_1\ w_2\ ...\ w_R\ ]^T$, the received signal at the k^{th} user can be written as,

$$y_k = \mathbf{w}^H\mathbf{G}_k\mathbf{H}\mathbf{b}s + \mathbf{w}^H\mathbf{G}_k\mathbf{v} + n_k \tag{4}$$

It is assumed that the information symbol s, the relay and the receiver noises are mutually uncorrelated in accordance with [4]. Furthermore, CSI is available at both the transmitter and the relay network [4]. In this paper, transmit and relay beamformers are designed by using QoS approach [4]. Hence, it is desired to minimize the total power transmitted from the base station and the relays by ensuring that signal-to-noise ratio (SNR) constraints at the users are satisfied. The total transmitted relay power can be written as,

$$\begin{aligned} P_R &= \sum_{i=1}^{R}\mathbb{E}\{|u_i|^2\} = \sum_{i=1}^{R}|w_i|^2\mathbb{E}\{|r_i|^2\} \\ &= \sum_{i=1}^{R}|w_i|^2(|\mathbf{h}_i^H\mathbf{b}|^2 + \sigma_v^2) \\ &= \sum_{i=1}^{R}(\mathbf{b}w_i^*)^H\mathbf{h}_i\mathbf{h}_i^H(\mathbf{b}w_i^*) + \sigma_v^2\mathbf{w}^H\mathbf{w} \end{aligned} \tag{5}$$

where $\mathbf{h}_i^H$ is the i^{th} row of $\mathbf{H}$, i.e. it is the channel vector between the i^{th} relay and the base station.

The SNR of the k^{th} user is given by,

$$SNR_k = \frac{\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{H}\mathbf{b}s|^2\}}{\mathbb{E}\{|\mathbf{w}^H\mathbf{G}_k\mathbf{v}|^2\} + \mathbb{E}\{n_k^2\}} \tag{6}$$

In case of perfect channel state information, SNR can be written as,

$$SNR_k = \frac{|\mathbf{w}^H\mathbf{G}_k\mathbf{H}\mathbf{b}|^2}{\sigma_v^2||\mathbf{G}_k^H\mathbf{w}||_2^2 + \sigma_k^2} = \frac{|Tr(\mathbf{b}\mathbf{w}^H\mathbf{G}_k\mathbf{H})|^2}{\sigma_v^2||\mathbf{G}_k^H\mathbf{w}||_2^2 + \sigma_k^2} \tag{7}$$

In this case, QoS based beamforming problem can be formulated by using P_B, P_R (5) and (7) as,

$$\min_{\mathbf{b},\mathbf{w}} \quad \mathbf{b}^H\mathbf{b} + \sum_{i=1}^{R}(\mathbf{b}w_i^*)^H\mathbf{h}_i\mathbf{h}_i^H(\mathbf{b}w_i^*) + \sigma_v^2\mathbf{w}^H\mathbf{w} \tag{8.a}$$

$$s.t. \quad \frac{|Tr(\mathbf{b}\mathbf{w}^H\mathbf{G}_k\mathbf{H})|^2}{\sigma_v^2||\mathbf{G}_k^H\mathbf{w}||_2^2 + \sigma_k^2} \geq \gamma_k, \quad k = 1, 2, ..., N \tag{8.b}$$

$$(\mathbf{b}w_i^*)^H\mathbf{h}_i\mathbf{h}_i^H(\mathbf{b}w_i^*) + \sigma_v^2|w_i|^2 \leq p_i, \quad i = 1, 2, ..., R \tag{8.c}$$

where γ_k is the desired SNR threshold for the k^{th} user. (8.c) is used to keep the individual relay power below a threshold, i.e., p_i. The fact that the relays may not want to use too much power due to their limited battery lifetime motivates us to include the individual power constraints in (8.c) to the original QoS relay beamforming problem [4]. Let us define $\mathbf{Z}$ as,

$$\mathbf{Z} \triangleq \begin{bmatrix}\mathbf{b}\\ 1\end{bmatrix}[\ \mathbf{w}^H\ 1\ ] = \begin{bmatrix}\mathbf{b}\mathbf{w}^H & \mathbf{b}\\ \mathbf{w}^H & 1\end{bmatrix} = \begin{bmatrix}\tilde{\mathbf{Z}} & \mathbf{b}\\ \mathbf{w}^H & 1\end{bmatrix} \tag{9}$$

where $\tilde{\mathbf{Z}} = \mathbf{b}\mathbf{w}^H$. If we denote the i^{th} column of $\tilde{\mathbf{Z}}$ as $\tilde{\mathbf{z}}_i = \mathbf{b}w_i^*$, the problem in (8) can be expressed as,

$$\min_{\mathbf{Z}} \quad \mathbf{b}^H\mathbf{b} + \sum_{i=1}^{R}\tilde{\mathbf{z}}_i^H\mathbf{h}_i\mathbf{h}_i^H\tilde{\mathbf{z}}_i + \sigma_v^2\mathbf{w}^H\mathbf{w} \tag{10.a}$$

$$s.t. \quad |Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})| \geq \left\|\begin{matrix}\sqrt{\gamma_k}\sigma_v\mathbf{G}_k^H\mathbf{w}\\ \sqrt{\gamma_k}\sigma_k\end{matrix}\right\|_2, \quad k = 1, 2, ..., N \tag{10.b}$$

$$\tilde{\mathbf{z}}_i^H\mathbf{h}_i\mathbf{h}_i^H\tilde{\mathbf{z}}_i + \sigma_v^2|w_i|^2 \leq p_i, \quad i = 1, 2, ...R \tag{10.c}$$

$$\mathbf{Z} = \begin{bmatrix}\tilde{\mathbf{Z}} & \mathbf{b}\\ \mathbf{w}^H & 1\end{bmatrix} \tag{10.d}$$

$$rank(\mathbf{Z}) = 1 \tag{10.e}$$

The nonconvex constraints in (10) are (10.b) and (10.e). We can use the relaxation approach in [4] to express (10.b) as a convex expression of a second order cone. For this purpose, we use the following approximation,

$$|Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})| \geq \Re\{Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})\} \tag{11}$$

Then the following convex constraint can be used for (10.b), i.e.,

$$\Re\{Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})\} \geq \left\|\begin{matrix}\sqrt{\gamma_k}\sigma_v\mathbf{G}_k^H\mathbf{w}\\ \sqrt{\gamma_k}\sigma_k\end{matrix}\right\|_2 \tag{12}$$

Note that, if (12) is satisfied, (10.b) is automatically satisfied by (11). Hence, we restrict the feasible set of the problem in (10). Therefore, replacing (10.b) by (12) may make the original feasible problem infeasible [4]. In order to smooth over this problem, we will use the channel rotation approach [4] during the iterations of our proposed algorithm. In the following section, rank constraint in (10.e) is converted to a more manageable equivalent constraint.

3. EQUIVALENT RANK ONE CONSTRAINT

The following theorem is used for obtaining the equivalent rank constraint.

Theorem 1: Given a $P \times R$ matrix $\mathbf{Z} \in \mathbb{C}^{P\times R}$, $Tr(\mathbf{Z}^H\mathbf{Z}) = \sum_{i=1}^{S} \varsigma_i^2$ is upper bounded by the square of its nuclear norm, $||\mathbf{Z}||_\star^2$, i.e., $Tr(\mathbf{Z}^H\mathbf{Z}) \leq ||\mathbf{Z}||_\star^2 = (\sum_{i=1}^{S} \varsigma_i)^2$ where $S = \min(P, R)$ and ς_i's are the singular values. The upper bound is reached if and only if $\mathbf{Z}$ is a rank one matrix.

Proof: Consider the singular value decomposition of $\mathbf{Z}$, i.e., $\mathbf{Z} = \sum_{i=1}^{S} \varsigma_i \tilde{\mathbf{u}}_\mathbf{i} \tilde{\mathbf{v}}_\mathbf{i}^H$ where ς_i's are the ordered singular values, i.e., $\varsigma_1 \geq \varsigma_2 \geq ... \geq \varsigma_S \geq 0$. $\tilde{\mathbf{u}}_\mathbf{i}$'s and $\tilde{\mathbf{v}}_\mathbf{i}$'s are the corresponding left and right singular vectors, respectively. Then $Tr(\mathbf{Z}^H\mathbf{Z})$ can be written as,

$$\begin{aligned} Tr(\mathbf{Z}^H\mathbf{Z}) &= Tr(\sum_{i=1}^{S} \varsigma_i \tilde{\mathbf{v}}_\mathbf{i} \tilde{\mathbf{u}}_\mathbf{i}^H \sum_{j=1}^{S} \varsigma_j \tilde{\mathbf{u}}_\mathbf{j} \tilde{\mathbf{v}}_\mathbf{j}^H) \\ &= Tr(\sum_{i=1}^{S} \varsigma_i^2 \tilde{\mathbf{v}}_\mathbf{i} \tilde{\mathbf{v}}_\mathbf{i}^H) = \sum_{i=1}^{S} \varsigma_i^2 \\ \leq \sum_{i=1}^{S} \varsigma_i^2 + \sum_{i=1}^{S-1} \sum_{j=i+1}^{S} 2\varsigma_i\varsigma_j &= (\sum_{i=1}^{S} \varsigma_i)^2 = ||\mathbf{Z}||_\star^2 \end{aligned} \quad (13)$$

If $\mathbf{Z}$ is a rank one matrix, it has only one positive singular value ς_1 and it is given as $\varsigma_1 = ||\mathbf{Z}||_\star$. Therefore $Tr(\mathbf{Z}^H\mathbf{Z})$ reaches its upper bound for this case, i.e., $Tr(\mathbf{Z}^H\mathbf{Z}) = \sum_{i=1}^{S} \varsigma_i^2 = \varsigma_1^2 = ||\mathbf{Z}||_\star^2$. Assume that $\mathbf{Z}$ is a matrix whose rank is higher than one. In this case, $\sum_{i=1}^{S-1} \sum_{j=i+1}^{S} 2\varsigma_i\varsigma_j > 0$. From (13), $Tr(\mathbf{Z}^H\mathbf{Z}) < ||\mathbf{Z}||_\star^2$. Therefore, a matrix whose rank is higher than one cannot reach the upper bound.

Corollary 1: The following constraint in (14) guarantees that $rank(\mathbf{Z}) = 1$, i.e., it is equivalent to (10.e).

$$||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z}) \leq 0 \quad (14)$$

Using Corollary 1 and (12), the problem in (10) can be expressed as,

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \mathbf{b}^H\mathbf{b} + \sum_{i=1}^{R} \tilde{\mathbf{z}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}}_i + \sigma_v^2 \mathbf{w}^H \mathbf{w} \quad (15) \\ & s.t. \quad (12),\ (10.c),\ (10.d),\ (14) \end{aligned}$$

The nonconvex constraint in (14) can be moved into the objective function using exact penalty approach [6]. This modification does not change the optimum solution of the problem in (15). In the following theorem, the equivalency of the new form and (15) is established.

Theorem 2: The problem in (15) is equivalent to the problem in (16) for $\mu > \mu_0$ with μ_0 being a finite positive value in the sense that both problems have the same optimum solution. Furthermore any local minimum of the problem in (16) is also a local minimum of the problem in (15) if (15) is feasible.

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \mathbf{b}^H\mathbf{b} + \sum_{i=1}^{R} \tilde{\mathbf{z}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}}_i + \sigma_v^2 \mathbf{w}^H \mathbf{w} \\ & +\mu(\max(0, ||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z}))) \quad (16) \\ & s.t. \quad (12),\ (10.c),\ (10.d) \end{aligned}$$

Proof: Assume that (15) is feasible and the optimum objective value is finite. Constraints in (16) are all continuous functions. The feasible sets of (15) and (16) are both closed and bounded and hence they are compact due to the finite dimensional space [7]. Therefore $\max(0, ||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z}))$ corresponds to an exact penalty function [6]. As a consequence of the definition of exact penalty function, Theorem 2 becomes a valid statement.

Note that $\max(0, ||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z})) = ||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z})$ and (16) can be expressed as,

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \mathbf{b}^H\mathbf{b} + \sum_{i=1}^{R} \tilde{\mathbf{z}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}}_i + \sigma_v^2 \mathbf{w}^H \mathbf{w} \\ & +\mu(||\mathbf{Z}||_\star^2 - Tr(\mathbf{Z}^H\mathbf{Z})) \quad (17) \\ & s.t. \quad (12),\ (10.c),\ (10.d) \end{aligned}$$

Theorem 3: The optimum solution of the following optimization problem in (18) and (17) are the same, namely $\mathbf{Z}^{\mathbf{I}\star} = \mathbf{Z}^{\mathbf{II}\star} = \mathbf{Z}^\star$ where $\mathbf{Z}^\star$ is the optimum solution of (16):

$$\begin{aligned} \min_{\mathbf{Z}^\mathbf{I}, \mathbf{Z}^\mathbf{II}} \quad & \mathbf{b}^{\mathbf{I}H}\mathbf{b}^\mathbf{I} + \sum_{i=1}^{R} \tilde{\mathbf{z}^\mathbf{I}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}^\mathbf{I}}_i + \sigma_v^2 \mathbf{w}^{\mathbf{I}H} \mathbf{w}^\mathbf{I} \\ & +\mathbf{b}^{\mathbf{II}H}\mathbf{b}^\mathbf{II} + \sum_{i=1}^{R} \tilde{\mathbf{z}^\mathbf{II}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}^\mathbf{II}}_i + \sigma_v^2 \mathbf{w}^{\mathbf{II}H} \mathbf{w}^\mathbf{II} \\ & +\mu(||\mathbf{Z}^\mathbf{I}||_\star ||\mathbf{Z}^\mathbf{II}||_\star - \Re\{Tr(\mathbf{Z}^{\mathbf{I}H}\mathbf{Z}^\mathbf{II})\}) \quad (18.a) \\ & s.t.\ \Re\{Tr(\tilde{\mathbf{Z}^\mathbf{I}}\mathbf{G}_k\mathbf{H})\} \geq \left\| \begin{matrix} \sqrt{\gamma_k}\sigma_v \mathbf{G}_k^H \mathbf{w}^\mathbf{I} \\ \sqrt{\gamma_k}\sigma_k \end{matrix} \right\|_2 \quad (18.b) \\ & \Re\{Tr(\tilde{\mathbf{Z}^\mathbf{II}}\mathbf{G}_k\mathbf{H})\} \geq \left\| \begin{matrix} \sqrt{\gamma_k}\sigma_v \mathbf{G}_k^H \mathbf{w}^\mathbf{II} \\ \sqrt{\gamma_k}\sigma_k \end{matrix} \right\|_2 \quad (18.c) \\ & k = 1, 2, ..., N \\ & \tilde{\mathbf{z}^\mathbf{I}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}^\mathbf{I}}_i + \sigma_v^2 |w_i^I|^2 \leq p_i \quad (18.d) \\ & \tilde{\mathbf{z}^\mathbf{II}}_i^H \mathbf{h}_i \mathbf{h}_i^H \tilde{\mathbf{z}^\mathbf{II}}_i + \sigma_v^2 |w_i^{II}|^2 \leq p_i,\ i = 1, 2, ...R \quad (18.e) \\ & \mathbf{Z}^\mathbf{I} = \begin{bmatrix} \tilde{\mathbf{Z}^\mathbf{I}} & \mathbf{b}^\mathbf{I} \\ \mathbf{w}^{\mathbf{I}H} & 1 \end{bmatrix},\ \mathbf{Z}^\mathbf{II} = \begin{bmatrix} \tilde{\mathbf{Z}^\mathbf{II}} & \mathbf{b}^\mathbf{II} \\ \mathbf{w}^{\mathbf{II}H} & 1 \end{bmatrix} \quad (18.f) \end{aligned}$$

Proof: Let S be the minimum of $\{M+1, R+1\}$.

$$\Re\{Tr(\mathbf{Z^I}^H\mathbf{Z^{II}})\} \le |Tr(\mathbf{Z^I}^H\mathbf{Z^{II}})| \le \sum_{i=1}^{S} \varsigma_i^I \varsigma_i^{II} \quad (19)$$

where ς_i^I's and ς_i^{II}'s are the ordered singular values of $\mathbf{Z^I}$ and $\mathbf{Z^{II}}$ respectively. In (19), we used the lemma in [8] (page 176). In this case,

$$||\mathbf{Z^I}||_\star ||\mathbf{Z^{II}}||_\star - \Re\{Tr(\mathbf{Z^I}^H\mathbf{Z^{II}})\}$$
$$\ge \sum_{i=1}^{S}\varsigma_i^I \sum_{j=1}^{S}\varsigma_j^{II} - \sum_{i=1}^{S}\varsigma_i^I\varsigma_i^{II} = \sum_{i=1}^{S-1}\sum_{j=i+1}^{S} 2\varsigma_i^I\varsigma_j^{II} \quad (20)$$

$||\mathbf{Z^I}||_\star ||\mathbf{Z^{II}}||_\star - \Re\{Tr(\mathbf{Z^I}^H\mathbf{Z^{II}})\}$ is lower bounded by 0 and it is equal to zero if and only if $\mathbf{Z^I} = \alpha\mathbf{Z^{II}}$ where α is a positive scalar and $\mathbf{Z^I}$ and $\mathbf{Z^{II}}$ are rank one matrices from (19) and (20). Since $\mathbf{Z^I}$ and $\mathbf{Z^{II}}$ independently solve the same problem, $\mathbf{Z^I}^\star = \mathbf{Z^{II}}^\star = \mathbf{Z}^\star$.

Alternating minimization can be used to solve the problem in (18) [9]. At the iteration r, with the fixed $\mathbf{Z^{I^{r-1}}}$, we can obtain a new $\mathbf{Z^{II^r}}$ to minimize the objective function while satisfying the SNR conditions. Then we alternate the fixed variable and update $\mathbf{Z^{I^r}}$ while fixing $\mathbf{Z^{II^r}}$. This alternating optimization is continued until convergence. The proposed method of alternating minimization is guaranteed to converge [10].

4. ITERATIVE CHANNEL ROTATION

The constraint in (12) restricts the feasible region of the problem in (10). It approximates the absolute value of $Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})$ with its real part. This approximation may render the problem infeasible. Hence the performance of the approximation depends on how accurately absolute values of complex terms are represented by their real parts. Note that the solution of (10) does not change if $\mathbf{w}$, $\mathbf{b}$, or $\tilde{\mathbf{Z}}$ is multiplied by a rotation factor of $e^{j\varphi}$. In case of single receiver, one can choose this rotation factor such that $Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})$ is real valued and nonnegative, i.e., $|Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})| = \Re\{Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})\}$ without changing the optimum value. However, when there are more than one receiver, it may be impossible to have $|Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})| = \Re\{Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})\}$ for all $k = 1, 2, ..., N$. This is due to the fact that $\tilde{\mathbf{Z}}$ is a common term for all SNR constraints. In [4], an effective method is proposed to increase the performance of this approximation, which is iterative channel rotation. In this method, instead of beamforming weight vector, channel vectors are rotated at each iteration to improve the solution. In our case, channel matrices $\mathbf{G}_k$'s are rotated in order to have $Tr(\tilde{\mathbf{Z}}\mathbf{G}_k\mathbf{H})$ real and nonnegative after each iteration. In the following part, channel rotation is used to improve feasibility and convergence for the proposed approach.

5. ALTERNATING MINIMIZATION ALGORITHM

The steps of the proposed algorithm can be given as,

Alternating Minimization Algorithm with Channel Rotation (AMACR)

Let $\varsigma_1(\mathbf{Z})$ be the maximum singular value of the matrix $\mathbf{Z}$, $\tilde{\mathbf{u}}_\mathbf{1}(\mathbf{Z})$ and $\tilde{\mathbf{v}}_\mathbf{1}(\mathbf{Z})$ denote the corresponding left and right singular vector respectively.

Initialization: $r = 0$,

i) Set proper μ (Ex: $\mu = 1$) and solve the relaxed version of (10) by removing (10.e) and replacing (10.b) by (12). Let $\mathbf{Z}^\star$ denote the solution and $\tilde{\mathbf{Z}}^\star$ be its upper left block. The initializations are made as $\mathbf{Z^{I,0}} = [\ \sqrt{\varsigma_1(\tilde{\mathbf{Z}}^\star)}\tilde{\mathbf{u}}_\mathbf{1}(\tilde{\mathbf{Z}}^\star)\ \ 1\ ]^T$ $[\ \sqrt{\varsigma_1(\tilde{\mathbf{Z}}^\star)}\tilde{\mathbf{v}}_\mathbf{1}(\tilde{\mathbf{Z}}^\star)\ \ 1\ ]$.

ii) Rotate the channel matrices such that $|Tr(\tilde{\mathbf{Z}}^\star\mathbf{G}_k\mathbf{H})|$ is real and positive, i.e., $\mathbf{G}_k \leftarrow \mathbf{G}_k e^{-j\angle Tr(\tilde{\mathbf{Z}}^\star\mathbf{G}_k\mathbf{H})}$, $k = 1, 2, ...N$. Solve the problem in (18) for $\mathbf{Z^{II,0}}$.

Iterations: r $\rightarrow$ r+1

1) Perform channel rotation, $\mathbf{G}_k \leftarrow \mathbf{G}_k e^{-j\angle Tr(\tilde{\mathbf{Z}}^{\mathbf{II,r-1}}\mathbf{G}_k\mathbf{H})}$, $k = 1, 2, ...N$. Solve (18) for $\mathbf{Z^{I,r}}$ while fixing $\mathbf{Z^{II}}$ as $\mathbf{Z^{II,r-1}}$.

2) If $rank(\mathbf{Z^{I,r}}) = 1$ go to Step 6. Otherwise if $\frac{\varsigma_1(\mathbf{Z^{I,r}})}{||\mathbf{Z^{I,r}}||_\star} \ge \frac{\varsigma_1(\mathbf{Z^{II,r-1}})}{||\mathbf{Z^{II,r-1}}||_\star} + \beta$ (improved solution), where β is a proper positive threshold value (Ex: $||\mathbf{Z^{I,r}}||_\star/20$), keep the value of μ same. Otherwise, increase μ (Ex: $\mu \rightarrow 2\mu$).

3) Perform channel rotation, $\mathbf{G}_k \leftarrow \mathbf{G}_k e^{-j\angle Tr(\tilde{\mathbf{Z}}^{\mathbf{I,r}}\mathbf{G}_k\mathbf{H})}$, $k = 1, 2, ...N$. Solve (18) for $\mathbf{Z^{II,r}}$ while fixing $\mathbf{Z^I}$ as $\mathbf{Z^{I,r}}$.

4) If $rank(\mathbf{Z^{II,r}}) = 1$ go to step 6. Otherwise if $\frac{\varsigma_1(\mathbf{Z^{II,r}})}{||\mathbf{Z^{II,r}}||_\star} \ge \frac{\varsigma_1(\mathbf{Z^{I,r}})}{||\mathbf{Z^{I,r}}||_\star} + \beta$ keep the value of μ same. Otherwise, increase μ.

5) Terminate if the maximum iteration number, r_0, is reached and go to step 6. Otherwise continue iterations.

6) Take the principal singular vectors of $\tilde{\mathbf{Z}}^{\mathbf{I,r}}$ or $\tilde{\mathbf{Z}}^{\mathbf{II,r}}$ (depending on the termination) as $\mathbf{b}^\star$ and $\mathbf{w}^\star$. If rank of the solution matrix is not one, then scale $\mathbf{b}^\star$ to satisfy the SNR constraints without violating the individual relay power constraints. If scaling is not possible, the algorithm has failed to give a feasible solution or the original problem is already infeasible.

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6. SIMULATION RESULTS

In this part, the proposed algorithm, AMACR, is implemented with the convex programming solver CVX. Rayleigh fading channels with unit variances are considered. SNR threshold, γ and noise variance, σ_k^2 are the same for each user, i.e., $\gamma = \gamma_k$, $\sigma_k^2 = 0.1$. The power limit of each relay is the same and taken as $p_i = 2$ W. The relay noise variance is $\sigma_v^2 = 0.1$. The average of 100 random channel realizations is presented for each experiment. The parameters of the AMACR algorithm are selected as $\beta = ||\mathbf{Z}||_\star/20$ and $r_0 = 25$. Initial value of μ is taken as $\mu = 1$.

Fig. 1 shows the total transmitted power for different SNR thresholds. There are $N = 8$ users. As the SNR threshold increases, transmitted power increases as expected. As the number of antennas at the base station and the relays increases, the transmitted power decreases. The increase in the number of relays is more effective for power saving.

Table 1 presents the average number of convex programming problems (CPP) solved for the proposed algorithm for the same scenario as in Fig. 1. As the SNR threshold increases, the average number of CPP's increases. On the other hand, increasing the number of the antennas and the relays result in less number of iterations. Overall, it is seen that the proposed method requires small number of CPP's for convergence.

In Fig. 2, total transmitted power is presented for different number of users and scenarios. The SNR threshold is selected as γ=10 dB. The transmission power increases almost linearly with the number of users. Furthermore, any increase in the number of antennas or relays decreases the transmitted power.

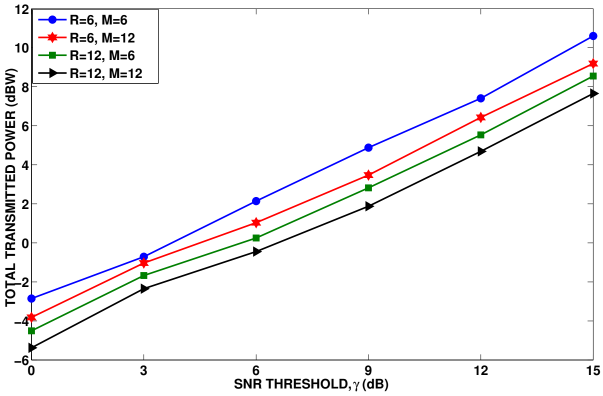

Fig. 1. Total transmitted power versus SNR threshold, γ.

7. CONCLUSION

Joint design of transmit and relay beamformer for single group multicast transmission is considered. Amplify-and-forward relaying protocol is used for distributed beamforming. After relaxation, an equivalent biconvex formulation is obtained by embedding the rank condition into the objective function with the aid of exact penalty method. The equivalent problem is solved iteratively by using alternating minimization and channel rotation. A convex problem is solved at each iteration. Hence the proposed algorithm is guaranteed to converge. Simulation results show that the increase in the number of relays is more effective than the increase in the number of antennas at the base station. The proposed algorithm is efficient and converges in small number of iterations.

Table 1. Average number of convex programming problems

γ (dB)	0	3	6	9	12	15
R=6, M=6	7.33	7.82	7.97	9.09	9.91	14.53
R=6, M=12	6.91	7.21	7.58	8.39	9.33	11.27
R=12, M=6	6.64	6.95	7.53	8.06	8.64	9.33
R=12, M=12	6.51	6.82	7.46	7.48	8.15	9.61

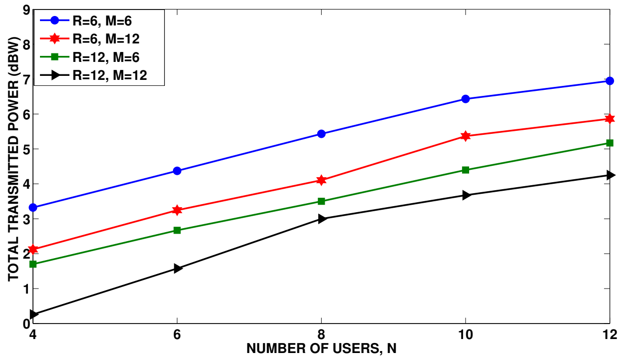

Fig. 2. Total transmitted power versus number of users, N.

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