MULTI-GROUP MULTICAST BEAMFORMING FOR SIMULTANEOUS WIRELESS INFORMATION AND POWER TRANSFER

Özlem Tuğfe Demir, T. Engin Tuncer

Electrical and Electronics Engineering Department, METU, Ankara, Turkey
{deozlem , etuncer}@metu.edu.tr

ABSTRACT

In this paper, simultaneous wireless information and power transfer (SWIPT) concept is introduced for multi group multicast beamforming. Each user has a single antenna and a power splitter which divides the radio frequency (RF) signal into two for both information decoding and energy harvesting. The aim is to minimize the total transmission power at the base station while satisfying both signal-to-interference-plus-noise-ratio (SINR) and harvested power constraints at each user. Unlike unicast and certain broadcast scenarios, semidefinite relaxation (SDR) is not tight and global optimum solution cannot be found for this problem. We propose an iterative algorithm where a convex optimization problem is solved at each iteration. Both perfect and imperfect channel state information (CSI) at the base station are considered. Simulation results show that the proposed solution is very close to the SDR lower bound and a few number of iterations are enough for the algorithm convergence.

Index Terms— Multicast beamforming, wireless power transfer, convex optimization, alternating minimization

1. INTRODUCTION

Radio frequency (RF) signals can be used not only for conveying information but also for transmitting the energy in modern wireless communication systems. Recently simultaneous wireless information and power transfer (SWIPT) became a promising research area especially for energy constrained wireless networks [1]. Mobile users and devices usually have limited battery and energy harvesting is a useful approach to improve the energy efficiency and battery duration [2].

The idea of SWIPT is first introduced in [1] and then considered for multi-user multi-input single-output (MISO) systems in [2], [3], etc. In these works, receivers are assumed to have a power splitting (PS) device by which the received signal is split into two streams with different powers, one for decoding information and the other for harvesting energy [2]. Other practical receivers use time switching (TS) for SWIPT [2]. In this paper, we consider PS scheme in accordance with the recent works in beamforming area [2], [3].

In multi-user MISO systems, transmit beamforming is an effective approach for increasing channel capacity and diversity [4]. In [2], unicast beamforming for SWIPT is considered and global optimum solution is found using the tightness of the SDR approach. In [3], single group multicast beamforming design for SWIPT systems is elaborated, but except for certain scenarios, the optimum solution is not guaranteed. In fact, even single group multicast beamforming problem is shown to be NP-hard [4]. In this paper, we consider a more general version of this problem, namely multi-group multicast beamforming for SWIPT systems. To the best of our knowledge, this is the first work which considers this joint problem.

There exists some efficient algorithms to solve multicast beamforming problem without SWIPT [5], [6]. The algorithm proposed in our previous work [5] uses exact penalty function to convert the single group multicast beamforming problem into an equivalent biconvex structure. This algorithm solves the max-min fair beamforming problem. In this paper, we modify this algorithm for quality of service (QoS) based multi-group multicast beamforming design in SWIPT systems. In QoS based beamforming problem for SWIPT, our aim is to minimize the total transmitted power from the base station while satisfying the SINR and harvested power constraints at each user. In addition, robust beamforming problem for imperfect channel state information (CSI) is formulated and solved with the proposed method. Simulations show that the proposed method is very effective and approaches to the SDR lower bound closely.

2. SYSTEM MODEL

Consider a wireless scenario comprising a base station equipped with M transmit antennas and N receivers. Each receiver has a single antenna. Assume that there are G multicast groups, $\{\mathcal{G}_1, ..., \mathcal{G}_G\}$, where $\mathcal{G}_k$ denotes the k^{th} multicast group of users. Each receiver listens to a single multicast, i.e., $\mathcal{G}_k \bigcap \mathcal{G}_l = \emptyset$. The signal transmitted from the antenna array is $\mathbf{x}(t) = \sum_{k=1}^{G} \mathbf{w}_k s_k(t)$ where $s_k(t)$ is the information signal for the users in $\mathcal{G}_k$ and $\mathbf{w}_k$ is the related $M \times 1$ complex beamformer weight vector. It is assumed that information signals $\{s_k(t)\}_{k=1}^{G}$ are mutually uncorrelated each with zero mean and unit variance, $\sigma_{s_k}^2 = 1$. In this case, the total transmitted power is $\sum_{k=1}^{G} \mathbf{w}_k^H \mathbf{w}_k$. The received signal at the i^{th} receiver is $y_i(t) = \mathbf{h}_i^H \mathbf{x}(t) + n_{A,i}(t)$ where $\mathbf{h}_i$ is the $M \times 1$

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conjugated complex channel vector for the i^{th} receiver and $n_{A,i}(t)$ is the additive zero mean Gaussian noise at the i^{th} receiver's antenna with variance $\sigma^2_{A,i}$. $n_{A,i}(t)$ is uncorrelated with the source signals.

The received signal at the i^{th} receiver is split into energy harvester (EH) and information decoder (ID) with the aid of a power splitter (PS) device. PS is assumed to be ideal without any induced noise. A portion of the signal power denoted by $0 < \rho_i < 1$ is transmitted to ID while the remaining $1 - \rho_i$ portion is fed into EH. The received signal at the information decoder of the i^{th} receiver can be expressed as,

$$y_{I,i}(t) = \sqrt{\rho_i}(\mathbf{h}_i^H \mathbf{x}(t) + n_{A,i}(t)) + n_{I,i}(t) \tag{1}$$

where $n_{I,i}(t)$ is the additional zero-mean Gaussian noise introduced by ID of the i^{th} receiver. $n_{I,i}(t)$ is independent of source signals and $n_{A,i}(t)$ and has a variance $\sigma^2_{I,i}$. Assuming that i^{th} receiver is in the k^{th} multicast group, $\mathcal{G}_k$, signal-to-interference-plus-noise ratio (SINR) for the i^{th} receiver is,

$$SINR_i = \frac{\rho_i |\mathbf{w}_k^H \mathbf{h}_i|^2}{\rho_i \sum_{l\neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \rho_i \sigma^2_{A,i} + \sigma^2_{I,i}} \tag{2}$$

The signal fed into the EH of i^{th} receiver can be expressed as,

$$y_{E,i}(t) = \sqrt{1-\rho_i}(\mathbf{h}_i^H \mathbf{x}(t) + n_{A,i}(t)) \tag{3}$$

Then, the power harvested by the EH of the i^{th} receiver is given as, $P_i = \xi_i(1-\rho_i)(\sum_{k=1}^G |\mathbf{w}_k^H \mathbf{h}_i|^2 + \sigma^2_{A,i})$ where $0 < \xi_i \leq 1$ is the energy conversion efficiency of the EH at the i^{th} receiver. Quality of service (QoS) multicast beamforming problem is to minimize the total transmitted power subject to receive-SINR constraint for each user, i.e.,

$$\min_{\{\mathbf{w}_k\}_{k=1}^G, \{\rho_i\}_{i=1}^N} \sum_{k=1}^G \mathbf{w}_k^H \mathbf{w}_k \tag{4.a}$$

$$s.t. \quad \frac{\rho_i \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k}{\rho_i \sum_{l\neq k} \mathbf{w}_l^H \mathbf{R}_i \mathbf{w}_l + \rho_i \sigma^2_{A,i} + \sigma^2_{I,i}} \geq \gamma_i, \tag{4.b}$$

$$\xi_i(1-\rho_i)(\sum_{k=1}^G \mathbf{w}_k^H \mathbf{R}_i \mathbf{w}_k + \sigma^2_{A,i}) \geq \mu_i \tag{4.c}$$

$$0 < \rho_i < 1, \quad \forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\} \tag{4.d}$$

where γ_i and μ_i are the SINR and harvested power thresholds respectively for the i^{th} receiver and $\mathbf{R}_i = \mathbf{h}_i \mathbf{h}_i^H$. The problem in (4) is not convex due to quadratic and coupled terms of $\mathbf{w}_k$'s and ρ_k's [2]. Let us define $\mathbf{w} = [\ \mathbf{w}_1^T\ \mathbf{w}_2^T\ ...\ \mathbf{w}_G^T\ ]^T$ and $\mathbf{W} = \mathbf{w}\mathbf{w}^H$. $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ shows the $(k,k)^{th}$ block of $\mathbf{W}$. The problem in (4) can be written as,

$$\min_{\mathbf{W}, \{\rho_i\}_{i=1}^N} Tr\{\mathbf{W}\} \tag{5.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i \mathbf{W}_k\} - \gamma_i \sum_{l\neq k} Tr\{\mathbf{R}_i \mathbf{W}_l\} \geq \frac{\gamma_i \sigma^2_{I,i}}{\rho_i} + \gamma_i \sigma^2_{A,i} \tag{5.b}$$

$$\sum_{k=1}^G Tr\{\mathbf{R}_i \mathbf{W}_k\} \geq \frac{\mu_i}{\xi_i(1-\rho_i)} - \sigma^2_{A,i} \tag{5.c}$$

$$0 < \rho_i < 1, \quad \forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\} \tag{5.d}$$

$$\mathbf{W} \succeq 0 \tag{5.e}$$

$$rank(\mathbf{W}) = 1 \tag{5.f}$$

The problem in (5) except rank condition in (5.f) is convex since $\frac{1}{\rho_i}$ and $\frac{1}{1-\rho_i}$ are convex functions of ρ_i in the domain $0 < \rho_i < 1$ [2]. The common technique to solve this type of problems is semidefinite relaxation (SDR) [4]. In SDR technique, the only nonconvex rank constraint is dropped and the relaxed version of the original problem is solved with efficient convex optimization algorithms. If $rank(\mathbf{W}^\star) = 1$ where $\mathbf{W}^\star$ is the solution of the relaxed problem, then $\mathbf{W}^\star$ is the optimum solution of the original problem. However, this case is shown to occur very rarely in multicasting problems [5]. In this paper, the following theorem is used to express rank constraint in a bilinear form.

Theorem 1: For $GM \times GM$ Hermitian symmetric, positive semidefinite matrices $\mathbf{W^I}$ and $\mathbf{W^{II}}$, $Tr\{\mathbf{W^I}\mathbf{W^{II}}\}$ is upper bounded by $Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\}$, i.e. $Tr\{\mathbf{W^I}\mathbf{W^{II}}\} \leq Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\}$. This upper bound is reached if and only if $\mathbf{W^I}$ and $\mathbf{W^{II}}$ are rank one matrices and $\mathbf{W^{II}} = \alpha \mathbf{W^I}$ where α is a positive scalar.

Proof: The proof of this theorem can be found in [5]. ■

Corollary 1: For two Hermitian symmetric, positive semidefinite matrices $\mathbf{W^I}$ and $\mathbf{W^{II}}$, $Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I}\mathbf{W^{II}}\} = 0$ condition implies rank one matrices, i.e., $\mathbf{W^{II}} = \frac{\lambda_1(\mathbf{W^{II}})}{\lambda_1(\mathbf{W^I})}\mathbf{W^I}$ where $\lambda_1(.)$ is the maximum eigenvalue.

The following theorem is presented to obtain an intermediate problem structure before the final form.

Theorem 2: The optimum solution of (5) and the following optimization problem in (6) are the same, namely $\mathbf{W^I_{opt}} = \mathbf{W^{II}_{opt}} = \mathbf{W_{opt}}$, $\rho_{i_{opt}}^I = \rho_{i_{opt}}^{II} = \rho_{i_{opt}}$, $i = 1, ..., N$ where $\{\mathbf{W_{opt}}, \{\rho_{i_{opt}}\}_{i=1}^N\}$ is the optimum solution of (5):

$$\min_{\mathbf{W^I}, \mathbf{W^{II}}, \{\rho_i\}_{i=1}^N} Tr\{\mathbf{W^I}\} + Tr\{\mathbf{W^{II}}\} \tag{6.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i \mathbf{W^I}_k\} - \gamma_i \sum_{l\neq k} Tr\{\mathbf{R}_i \mathbf{W^I}_l\} \geq \frac{\gamma_i \sigma^2_{I,i}}{\rho_i{}^I} + \gamma_i \sigma^2_{A,i} \tag{6.b}$$

$$Tr\{\mathbf{R}_i \mathbf{W^{II}}_k\} - \gamma_i \sum_{l\neq k} Tr\{\mathbf{R}_i \mathbf{W^{II}}_l\} \geq \frac{\gamma_i \sigma^2_{I,i}}{\rho_i{}^{II}} + \gamma_i \sigma^2_{A,i} \tag{6.c}$$

$$\sum_{k=1}^G Tr\{\mathbf{R}_i \mathbf{W^I}_k\} \geq \frac{\mu_i}{\xi_i(1-\rho_i{}^I)} - \sigma^2_{A,i} \tag{6.d}$$

$$\sum_{k=1}^G Tr\{\mathbf{R}_i \mathbf{W^{II}}_k\} \geq \frac{\mu_i}{\xi_i(1-\rho_i{}^{II})} - \sigma^2_{A,i} \tag{6.e}$$

$$0 < \rho_i < 1, \quad \forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\} \tag{6.f}$$

$$\mathbf{W^I} \succeq 0,\ \mathbf{W^{II}} \succeq 0 \tag{6.g}$$

$$Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I W^{II}}\} = 0 \tag{6.h}$$

Proof: $\mathbf{W^I_{opt}}$ and $\mathbf{W^{II}_{opt}}$ are rank one matrices due to (6.h), which is the condition in Corollary 1. Hence $\mathbf{W^I_{opt}} = \alpha\mathbf{W^{II}_{opt}}$ where α is a positive scalar by Theorem 1 and Corollary 1. Since $\{\mathbf{W^I}, \{\rho_i{}^I\}_{i=1}^N\}$ and $\{\mathbf{W^{II}}, \{\rho_i{}^{II}\}_{i=1}^N\}$ independently solve the same problem, $\mathbf{W^I_{opt}} = \mathbf{W^{II}_{opt}} = \mathbf{W_{opt}}$ and $\rho_{i\,opt}^{I} = \rho_{i\,opt}^{II} = \rho_{i\,opt}$, $i = 1, ..., N$.

In the above problem, (6.h) is still a nonconvex constraint. Fortunately this constraint can be moved into the objective function using exact penalty approach [7], [8]. This modification does not change the optimum solution of the problem. In the following theorem, the equivalency of the new form and (6) are established.

Theorem 3: The problem in (6) is equivalent to the problem in (7) for $\zeta > \zeta_0$ with ζ_0 being a finite positive value in the sense that any local minimum of the problem in (7) is also a local minimum of the problem in (6).

$$\min_{\mathbf{W^I},\mathbf{W^{II}},\{\rho_i\}_{i=1}^N} \quad Tr\{\mathbf{W^I}\} + Tr\{\mathbf{W^{II}}\}$$

$$+\zeta|Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I W^{II}}\}| \tag{7.a}$$

$$s.t. \quad \text{(6.b), (6.c), (6.d), (6.e), (6.f), (6.g)}$$

Proof: Assume that (6) is feasible, i.e. $Tr\{\mathbf{W^I_{opt}}\} + Tr\{\mathbf{W^{II}_{opt}}\} < \infty$. Constraints in (7) are all continuous functions. The feasible sets of (6) and (7) are both closed and bounded and hence they are compact due to the finite dimensional space. Therefore the objective function of (7) corresponds to an l_1 exact penalty function [7], [8]. Theorem 3 is valid by definition [7] and due to [8] (page 408). ■

Note that $|Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I W^{II}}\}| = Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I W^{II}}\}$ from Theorem 1. In this case, the final form of the optimization problem can be given as,

$$\min_{\mathbf{W^I},\mathbf{W^{II}},\{\rho_i\}_{i=1}^N} \quad Tr\{\mathbf{W^I}\} + Tr\{\mathbf{W^{II}}\}$$

$$+\zeta(Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\} - Tr\{\mathbf{W^I W^{II}}\}) \tag{8.a}$$

$$s.t. \quad \text{(6.b), (6.c), (6.d), (6.e), (6.f), (6.g)}$$

The problem in (8) is a biconvex problem and alternating minimization can be used to solve it [9]. Alternating minimization is implemented by using iterations where $\{\mathbf{W^{I,r}}, \mathbf{W^{II,r}}, \{\rho_i^{I,r}\}_{i=1}^N, \{\rho_i^{II,r}\}_{i=1}^N\}$ are the terms at the r^{th} iteration. At the r^{th} iteration, $\{\mathbf{W^{II,r}}, \{\rho_i^{II,r}\}_{i=1}^N\}$ are obtained by considering $\{\mathbf{W^{I,r-1}}, \{\rho_i^{I,r-1}\}_{i=1}^N\}$ as fixed terms. Then the fixed variables are alternated and $\{\mathbf{W^{I,r}}, \{\rho_i^{I,r}\}_{i=1}^N\}$ are obtained from (8) while $\{\mathbf{W^{II,r}}, \{\rho_i^{II,r}\}_{i=1}^N\}$ are kept as fixed.

The objective function in (8) is lower bounded by $Tr\{\mathbf{W^I_{opt}}\} + Tr\{\mathbf{W^{II}_{opt}}\}$ which can be found similar to [5]. Since a convex problem is solved at each iteration, the objective function improves at each iteration and the iterative approach is guaranteed to converge [9].

3. ALTERNATING MINIMIZATION ALGORITHM

In the previous parts, the problems in (5) and (8) are shown to be equivalent in the sense that they have the same optimum solutions. Furthermore, it is shown that (8) can be solved with alternating minimization. The convergence of this approach is guaranteed. However, there is no guarantee for the optimum solution after the convergence. The steps for the proposed approach can be presented as follows,

SWIPT with Multicast Beamforming (SMB)

Let $\lambda_1(\mathbf{W})$ be the maximum eigenvalue of the matrix $\mathbf{W}$.

Initialization: $r = 0$,

Set a proper ζ and solve the relaxed version of (5) by removing (5.f). Let $\{\hat{\mathbf{W}}, \{\hat{\rho}_i\}_{i=1}^N\}$ denote the solution. The singular value decomposition of each $\hat{\mathbf{W}}_k$ is calculated as $\hat{\mathbf{W}}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{U}_k^H$. The following initializations are done for simplicity, i.e., $\rho_i^{I,0} = \hat{\rho}_i$, $i = 1, ..., N$ and $\mathbf{W^{I,0}}_{k,l} = \mathbf{U}_k\sqrt{\mathbf{\Sigma}_k\mathbf{\Sigma}_l}\mathbf{U}_l$. Solve the problem in (8) for $\{\mathbf{W^{II,0}}, \{\rho_i^{II,0}\}_{i=1}^N\}$.

Iterations: r $\rightarrow$ r+1

1) Solve (8) for $\{\mathbf{W^{I,r}}, \{\rho_i^{I,r}\}_{i=1}^N\}$ while fixing $\{\mathbf{W^{II}}, \{\rho_i^{II}\}_{i=1}^N\}$ as $\{\mathbf{W^{II,r-1}}, \{\rho_i^{II,r-1}\}_{i=1}^N\}$.

2) If $rank(\mathbf{W^{I,r}}) = 1$ terminate and go to Step 7. If $\lambda_1(\mathbf{W^{I,r}}) \geq \lambda_1(\mathbf{W^{II,r-1}}) + \beta$ (improved solution), where β is a proper positive threshold value (Ex: $Tr\{\mathbf{W^{II,r-1}}\}/20$), keep the value of ζ same. Otherwise, increase ζ (Ex: $\zeta \rightarrow 2\zeta$)

4) Solve (8) for $\{\mathbf{W^{II,r}}, \{\rho_i^{II,r}\}_{i=1}^N\}$ while fixing $\{\mathbf{W^I}, \{\rho_i^I\}_{i=1}^N\}$ as $\{\mathbf{W^{I,r}}, \{\rho_i^{I,r}\}_{i=1}^N\}$.

5) If $rank(\mathbf{W^{II,r}}) = 1$ terminate and go to Step 7. If $\lambda_1(\mathbf{W^{II,r}}) \geq \lambda_1(\mathbf{W^{I,r}}) + \beta$ keep the value of ζ same. Otherwise, increase ζ.

6) Terminate if $r = r_0$ where r_0 is the maximum number of iterations.

7) Take the principal eigenvector of $\mathbf{W^{I,r}}$ or $\mathbf{W^{II,r}}$ depending on the termination. If the solution is not a rank one matrix, then scale its principal eigenvector appropriately.

4. ROBUST MULTICAST BEAMFORMING

In the previous sections, perfect CSI is assumed. In practice, perfect CSI may not be available and robust beamformer design is desired to account for the imprecise CSI. The aim of robust beamforming is to ensure that the received SINR and harvested energy does not degrade significantly for the channel errors in the uncertainty set [10]. Here, we consider the spherical uncertainty model which is one of the most common models in multicast beamforming. Using spherical uncertainty region, imperfect CSI can be handled by using an error ball radius, ϵ, where the channel vector error, $\mathbf{e}_i$, satisfies $\|\mathbf{e}_i\|_2 \leq \epsilon$. The robustness of the beamformer depends on the choice of ϵ. Hence a large ϵ generates a more robust solution with a larger performance loss compared to the perfect CSI. The problem in (5) should be modified to account for the imprecise channel information. Hence, given the nominal channel vectors, $\mathbf{h}_i$, and ϵ, the robust beamformer design problem can be given as,

$$\min_{\mathbf{W},\{\rho_i\}_{i=1}^N} \quad Tr\{\mathbf{W}\} \tag{9.a}$$

$$s.t. \quad \min_{\|\mathbf{e}_i\|_2\le\epsilon} Tr\{(\mathbf{h}_i+\mathbf{e}_i)(\mathbf{h}_i+\mathbf{e}_i)^H(\mathbf{W}_k-\gamma_i\sum_{l\neq k}\mathbf{W}_l)\}$$

$$\ge \frac{\gamma_i\sigma_{I,i}^2}{\rho_i}+\gamma_i\sigma_{A,i}^2 \tag{9.b}$$

$$\min_{\|\mathbf{e}_i\|_2\le\epsilon} Tr\{(\mathbf{h}_i+\mathbf{e}_i)(\mathbf{h}_i+\mathbf{e}_i)^H(\sum_{k=1}^{G}\mathbf{W}_k)\}$$

$$\ge \frac{\mu_i}{\xi_i(1-\rho_i)}-\sigma_{A,i}^2 \tag{9.c}$$

(5.d), (5.e), (5.f)

There are infinite number of constraints in (9.b) and (9.c) implicitly. In order to obtain finite number of constraints, a lower bound for the left hand sides of the inequalities in (9.b) and (9.c) should be found [10]. If we define $\hat{\mathbf{W}}_i = \mathbf{W}_k - \gamma_i \sum_{l\neq k}\mathbf{W}_l$, a lower bound for the term in (9.b) can be obtained as,

$$\min_{\|\mathbf{e}_i\|_2\le\epsilon} Tr\{(\mathbf{h}_i+\mathbf{e}_i)(\mathbf{h}_i+\mathbf{e}_i)^H\hat{\mathbf{W}}_i\} \ge Tr\{\mathbf{R}_i\hat{\mathbf{W}}_i\}$$

$$+\min_{\|\mathbf{e}_i\|_2\le\epsilon} 2\Re\{\mathbf{h}_i^H\hat{\mathbf{W}}_i\mathbf{e}_i\}+\min_{\|\mathbf{e}_i\|_2\le\epsilon}\mathbf{e}_i^H\hat{\mathbf{W}}_i\mathbf{e}_i$$

$$\ge Tr\{\mathbf{R}_i\hat{\mathbf{W}}_i\}+\min_{\|\mathbf{e}_i\|_2\le\epsilon} 2\Re\{\mathbf{h}_i^H\hat{\mathbf{W}}_i\mathbf{e}_i\}-\epsilon^2||\hat{\mathbf{W}}_i||_F$$

$$= Tr\{\mathbf{R}_i\hat{\mathbf{W}}_i\}-2\epsilon||\hat{\mathbf{W}}_i\mathbf{h}_i||_2-\epsilon^2||\hat{\mathbf{W}}_i||_F \tag{10}$$

where we used $\min_{\|\mathbf{e}_i\|_2\le\epsilon}\mathbf{e}_i^H\hat{\mathbf{W}}_i\mathbf{e}_i = \epsilon^2\lambda_{min}(\hat{\mathbf{W}}_i) \ge -\epsilon^2||\hat{\mathbf{W}}_i||_F$ and $\mathbf{e}_i = \frac{-\epsilon}{||\hat{\mathbf{W}}_i\mathbf{h}_i||_2}\hat{\mathbf{W}}_i\mathbf{h}_i$ in the last expression in (10), so instead of inequality, equality holds there. Similarly if we define $\tilde{\mathbf{W}} = \sum_{k=1}^{G}\mathbf{W}_k$, a lower bound for the term in (9.c) can be found as,

$$\min_{\|\mathbf{e}_i\|_2\le\epsilon} Tr\{(\mathbf{h}_i+\mathbf{e}_i)(\mathbf{h}_i+\mathbf{e}_i)^H\tilde{\mathbf{W}}\}$$

$$\ge Tr\{\mathbf{R}_i\tilde{\mathbf{W}}\}-2\epsilon||\tilde{\mathbf{W}}\mathbf{h}_i||_2 \tag{11}$$

where we used $\min_{\|\mathbf{e}_i\|_2\le\epsilon}\mathbf{e}_i^H\tilde{\mathbf{W}}\mathbf{e}_i \ge 0$. Using (10) and (11), the robust problem in (9) can be written as,

$$\min_{\mathbf{W},\{\rho_i\}_{i=1}^N} \quad Tr\{\mathbf{W}\} \tag{12.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i\hat{\mathbf{W}}_i\} \ge 2\epsilon||\hat{\mathbf{W}}_i\mathbf{h}_i||_2+\epsilon^2||\hat{\mathbf{W}}_i||_F$$

$$+\frac{\gamma_i\sigma_{I,i}^2}{\rho_i}+\gamma_i\sigma_{A,i}^2 \tag{12.b}$$

$$Tr\{\mathbf{R}_i\tilde{\mathbf{W}}\} \ge 2\epsilon||\tilde{\mathbf{W}}\mathbf{h}_i||_2+\frac{\mu_i}{\xi_i(1-\rho_i)}-\sigma_{A,i}^2 \tag{12.c}$$

(5.d), (5.e), (5.f)

The only nonconvex constraint in (12) is rank constraint in (5.f) since $||.||_2$ is a convex function over affine functions. Hence, SMB algorithm can be used to solve this problem.

5. SIMULATION RESULTS

In this part, the proposed method, SMB, is implemented with the convex programming solver CVX. Rayleigh fading channels with unit variances are considered. The total number of antennas is $M = 6$. There are $G = 2$ multicast groups each with 4 users, namely there are $N = 8$ users. SINR threshold, γ, harvested power threshold, μ, antenna noise variance, σ_A^2, and ID noise variance, σ_I^2, are same for each user, i.e., $\gamma = \gamma_i$, $\mu = \mu_i$ and $\sigma_A^2 = \sigma_{A,i}^2 = 0.01$, $\sigma_I^2 = \sigma_{I,i}^2 = 0.01$. The average of 100 random channel realizations is presented for each experiment. The initial value of ζ is taken as $\zeta = 0.001$. Proposed method returned rank=1 solution for all the experiments even though there is no guarantee for such an outcome.

Fig. 1 shows the transmission power for different SINR and harvested power thresholds. The red and blue lines denote the perfect CSI and robust case with $\epsilon = 0.01$ respectively for different harvested power thresholds, i.e. $\mu = 0$ dB and $\mu = -10$dB. The performance gap between the proposed method and the SDR lower bound is very small and the details are presented inside the small windows. The curves on the upper part belongs to a higher harvested power threshold, namely $\mu = 0$ dB. As the SINR threshold increases, transmission power increases as expected. The effect of energy harvesting diminishes at high SINR threshold naturally. In addition, the additional power to account for the imperfect CSI is relatively low demonstrating the effectiveness of the robust formulation. In Fig. 2, the average number of convex programming problems (CPP) solved for the proposed algorithm is presented for the same scenario as in Fig. 1. As the SINR threshold increases, the average number of CPP's decreases. It is seen that the proposed method requires small number of CPP's for convergence.

In Fig. 3, the proposed method is evaluated by considering the harvested energy threshold, μ. In this case, two different SINR threshold values, $\gamma = 10$ dB and $\gamma = 20$ dB are used respectively. Transmission power increases as the harvested power threshold increases. In this case, the effect of SINR is still seen for high μ values indicating the importance of SINR. The proposed method for both perfect and imperfect CSI performs very well and approaches to the SDR lower bound for all scenarios. In Fig. 4, the average number of CPP's solved for the proposed algorithm is presented for the same scenario as in Fig. 3. While different set of channels are used, small number of CPP's are required for convergence for this experiment as well.

6. CONCLUSION

In this paper, joint multi-group multicast beamforming and receive power splitting for SWIPT systems is considered. An equivalent biconvex formulation is obtained for both perfect CSI scenario and robust design. The equivalent problem is solved iteratively by using alternating minimization where a convex problem is solved at each iteration. This approach is guaranteed to converge and has been shown to give a near global optimum solution.

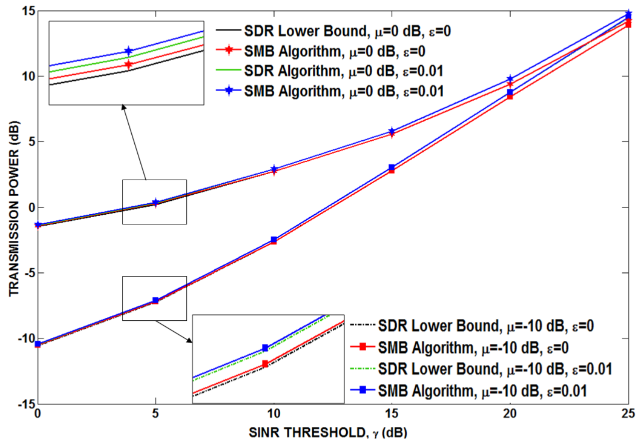

Fig. 1. Transmission power versus SINR threshold, γ

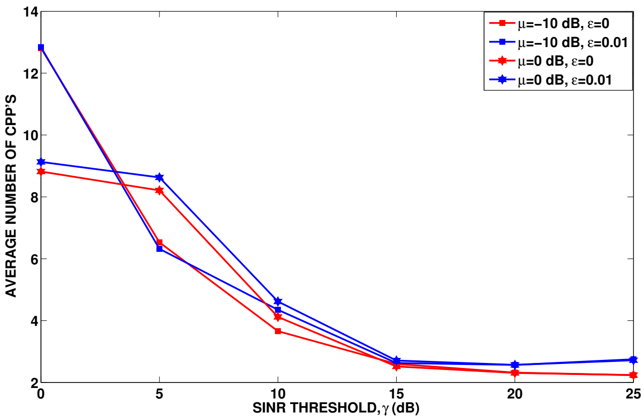

Fig. 2. Average number of convex programming problems (CPP) versus SINR threshold, γ

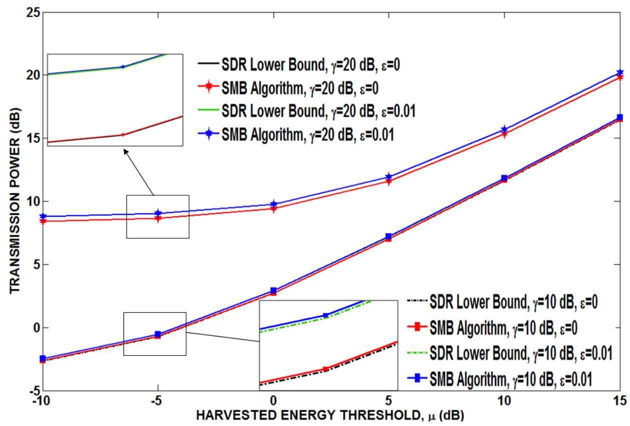

Fig. 3. Transmission power versus harvested power threshold, μ

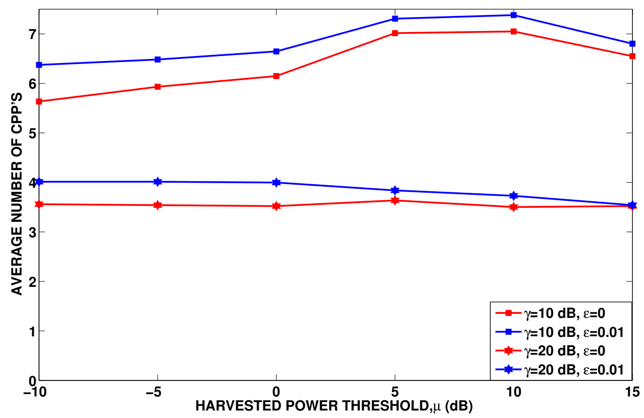

Fig. 4. Average number of convex programming problems (CPP) versus harvested power threshold, μ

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