INSTANTANEOUS FREQUENCY ESTIMATION FOR A SINUSOIDAL SIGNAL COMBINING DESA-2 AND NOTCH FILTER

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ABSTRACT

In this paper, we propose a new frequency estimation method for a sinusoidally modulated signal. The proposed method is constructed by three processing; the noise reduction, the frequency estimation, the updating of the noise reduction filter. In the noise reduction, we utilize an adaptive IIR bandpass filter which can extract only a specific frequency components. In the frequency estimation, we employ DESA-2, which is a non-linear operator with low computational complexity. The extraction frequency of the bandpass filter is effectively updated by integrating the estimation results of DESA-2 and the gradient estimation method. Through the estimation simulation for the noisy fluctuate sinusoidal signal, we show that the proposed method can estimate the instantaneous frequency accurately.

***Index Terms*—** Frequency Estimation, Sinusoidal Signal, Bandpass Filter

1. INTRODUCTION

An instantaneous frequency estimation for sinusoidal signals is an important issue in speech and audio signal processing, digital communication system, electric power system, and many other fields. Examples of applications are an active noise control for acoustic noise [1], a carrier frequency estimation for PSK [2], a fluctuation detection for power supply frequency [3]. Many applications requires the on-line frequency estimation and the ease of implementation for the small control devices.

The frequency estimation methods are broadly divided into the following three types; a frame-based estimation using the time-frequency domain analysis [4], a sample based recursive estimation using an adaptive filter [1, 3, 5, 6], and a sample based non-recursive estimation using the closed-form expression [7]. The first type utilizes the time-frequency distribution function, such as the short time FFT, wavelet transform, Wigner distribution function, and so on. This type is performed with high estimation accuracy even if under high noise environment, but it requires too large computational complexity to implement in small devices. In the point of the computational complexity, the sample based estimation methods are more useful. The second type is based on a gradient method which minimizes the square error between the input signal and the output signal of the filter. This type is implemented with low complexity and provides the robust estimation under high noise environment. However, it is difficult to track the non-stationary frequency because of slow convergence speed. The third type, which is a sample based non-recursive estimation, can provide high estimation accuracy under low noise environment despite very low complexity. This type is essentially close to the very low order adaptive filter. The estimation accuracy of this type deteriorates seriously along with increasing the noise level.

To improve the problems of the accuracy and complexity, we utilize the sample based estimation and develop a high accurate instantaneous frequency estimation for a sinusoidal signal combining the non-recursive one and the recursive one. The proposed method is based on the DESA-2 [7], which is the sample based non-recursive type. DESA-2 can be performed with comparatively high estimation accuracy in this type because of its non-linear structure. In order to improve the estimation accuracy of DESA-2, we introduce an adaptive IIR bandpass filter before DESA-2 in order to enhance the sinusoidal signal. The adaptive bandpass filter has a narrow band-pass characteristic at a center frequency (center frequency) [6]. In the proposed method, the updating of the center frequency is achieved by integrating DESA-2 and the gradient method. Through the simulation for the frequency estimation, we confirm that the proposed method can provide the high estimation accuracy under high noise environment compared with other conventional methods.

2. REVIEW OF CONVENTIONAL METHODS

In this chapter, we review the two conventional frequency estimation methods. First, we introduce DESA-2 which is the sample based estimation. Next, we also review an adaptive IIR bandpass filter.

2.1. Frequency Estimation Using DESA-2

DESA-2 [7] is a typical frequency estimation method which is easily implemented because of its low computational com-

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plexity. Now, we assume that the input signal consists of an unknown sinusoidal signal and an additive white Gaussian noise, i.e., the input signal is represented by

$$x(n) = A(n)\cos\big(\omega(n)n + \phi\big) + w(n), \tag{1}$$

where $A(n)$ is an amplitude, $\omega(n)$ is a sinusoidal frequency, ϕ is a phase. The $w(n)$ is an additive white Gaussian noise with 0 mean and σ^2 variance. In DESA-2, the frequency estimation is based on the energy tracking using a General Teager Energy Operator (GTEO) defined by

$$\psi_K[x(n)] = x(n)^2 - x(n-K)x(n+K), \tag{2}$$

where K is a integer parameter to adjust sensitivity for noise. Using GTEO, the sinusoidal frequency can be estimated by

$$\hat{\omega}(n) = \frac{1}{2}\cos^{-1}\left[1 - \frac{E[\psi_K[x(n-K) - x(n-K-2)]]}{2E[\psi_K[x(n-K+1)]]}\right], \tag{3}$$

where $E[\cdot]$ is an averaging operator for reduction of the noise effect. Let $\phi_n = E[y_n]$, then $E[y_n]$ is calculated by

$$\phi_n = \alpha\phi_{n-1} + (1-\alpha)y_n, \tag{4}$$

where α is an averaging parameter which takes $0 < \alpha < 1$. DESA-2 is performed with high accuracy for low noise input, even if the sinusoidal frequency is non-stationary. However, the estimation accuracy degrades seriously when the input includes the high level noise.

2.2. Frequency Estimation and Noise Reduction Using Adaptive Bandpass Filter

In order to improve the performance of DESA-2, we introduce a second order adaptive bandpass filter for the noise reduction before DESA-2 processing. The adaptive bandpass filter can estimate the unknown frequency and extract a narrow-band frequency component at a center frequency, simultaneously. The transfer function of the bandpass filter [5] is given by

$$H(z) = \frac{1-r}{2}\frac{1-z^{-2}}{1+a(n)z^{-1}+rz^{-2}}, \tag{5}$$

where r $(0 < r < 1)$ is a parameter which controls the passband width. Specifically, the bandwidth becomes narrow along with the increasing value of r toward 1. The parameter $a(n)$ determines the center frequency $\omega_p(n)$. The relation between $a(n)$ and $\omega_p(n)$ is given by

$$a(n) = -(1+r)\cos(\omega_p(n)). \tag{6}$$

Figure 2 shows the magnitude characteristics of the bandpass filter with $\omega_p(n) = \pi/2$, where we gives the different r as 0.8, 0.9, 0.99. As seen in figure 1, the extraction performance

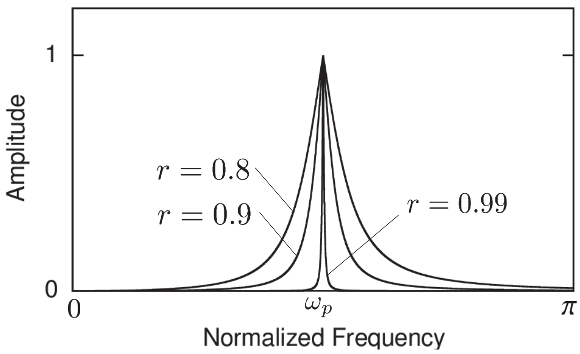

Fig. 1. Magnitude response of a bandpass filter.

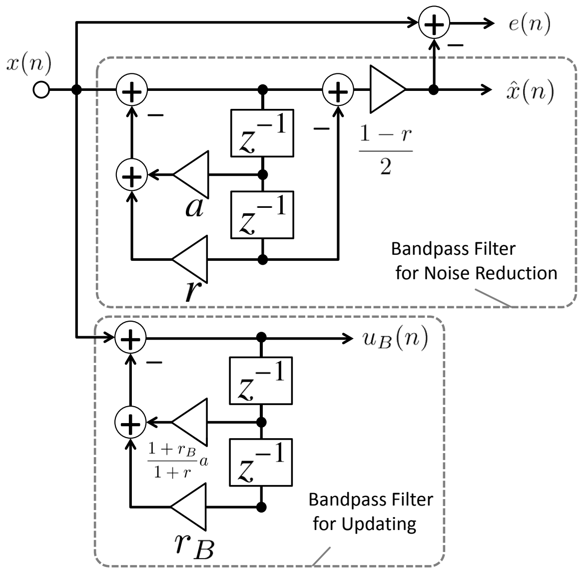

Fig. 2. Structure of a fast IIR bandpass filter.

of the bandpass filter is improved along with increasing the value of r toward to 1.

The center frequency can be adjusted to the unknown sinusoidal frequency using a gradient method. In the conventional gradient method, the convergence speed is significantly decreased when $r \approx 1$ because of the very long impulse response [5]. To reduce the effect of the impulse response, we develop the fast bandpass filter on the basis of the parallel composed notch filter [8]. The structure of the fast bandpass filter is shown in figure 2. It consists the two filters, where one is a very narrow bandpass filter for noise reduction and another is a competitively wide bandpass filter for updating. The $e(n)$ is the error signal defined by $e(n) = x(n) - \hat{x}(n)$. The $u_B(n)$ is the output signal of the bandpass filter calculated by

$$u_B(n) = x(n) - \frac{1+r_B}{1+r}a(n)u_B(n-1) - r_B u_B(n-2), \quad (7)$$

where r_B is a bandwidth parameter which satisfies $0 < r_B < r$. For updating of $a(n)$, we utilize the monotonically increasing algorithm [6] represented by

$$a(n+1) = a(n) + \mu\frac{\text{sgn}[e(n)u_B(n-1)]E[e(n)^2]}{E[u_B(n-1)^2] + E[e(n)^2]}, \quad (8)$$

where μ is a step size parameter and $\text{sgn}[\cdot]$ is a signum function. Although the algorithm is performed with comparatively fast convergence, the tracking performance for the non-stationary frequency is still insufficient and thus the estimation delay occurs. In the next chapter, we introduce the proposed system combining DESA-2 and the bandpass filter.

3. PROPOSED FREQUENCY ESTIMATION METHOD

We construct the proposed method by three processing, which are noise reduction using the adaptive bandpass filter, frequency estimation using DESA-2, and updating of the center frequency parameter $a(n)$. In the proposed method, DESA-2 section can provide extremely high estimation accuracy when the center frequency $\omega_p(n)$ is close to the sinusoidal frequency $\omega(n)$, since the noise reduction is performed adequately. Conversely, DESA-2 section provides very low estimation accuracy when $\omega_p(n)$ is far from $\omega(n)$. Considering those properties, we update $a(n)$ by (8) when $\omega_p(n)$ is far from $\omega(n)$, and also update it by (6) with $\omega_p(n) = \hat{\omega}(n)$ when $\omega_p(n)$ is close to $\omega(n)$. The distance between $\omega_p(n)$ and $\omega(n)$ can be evaluated by the amount of the updating tern of (8), since it becomes small along with decreasing the distance. Namely, the improved updating expression of $a(n)$ is defined by

$$a(n+1) = a(n) + \gamma(n), \quad (9)$$

$$\gamma(n) = \begin{cases} c_1(n), & |c_1(n)| > |c_2(n)| \\ c_2(n), & |c_1(n)| \le |c_2(n)| \end{cases}, \quad (10)$$

$$c_1(n) = \mu_1\frac{\text{sgn}[e(n)u_B(n-1)]E[e(n)^2]}{E[u_B(n-1)^2] + E[e(n)^2]}, \quad (11)$$

$$c_2(n) = \mu_2\big(a(n) - (1+r)\cos(\hat{\omega}(n))\big). \quad (12)$$

Here, $c_1(n)$ and $c_2(n)$ are the updating terms obtained by the gradient algorithm and DESA-2, respectively. The μ_1 and μ_2 are the step size parameters. The procedure of the proposed method is shown in figure 3.

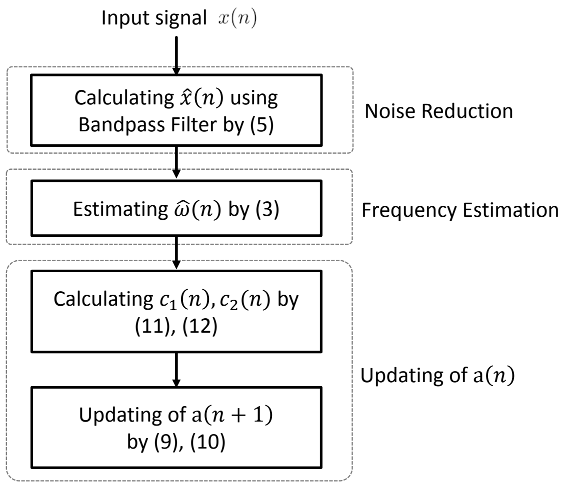

Fig. 3. Block diagram of the procedure of the proposed method.

4. SIMULATION

In this section, we confirm the convergence performance of the proposed frequency estimation method through several computational simulations. In the following simulations, the sinusoidal signal included in the input signal is defined by

$$s(n) = \cos\big(\omega(n)n + \pi/3\big), \quad (13)$$

$$\omega(n) = \begin{cases} \frac{\pi}{20} + \sin\left(\frac{\pi n}{1600}\right), & 0 \le n < 10^5 \\ \frac{3\pi}{40} + \sin\left(\frac{3\pi n}{4000}\right), & 10^5 \le n < 2\times 10^5 \end{cases}. \quad (14)$$

Obviously, the frequency $\omega(n)$ fluctuates. For evaluation, we compare the performances of the conventional DESA-2, the conventional adaptive bandpass filter, and the proposed method. The parameter settings of the proposed method are followings; the parameter $K = 6$, the averaging parameter $\alpha = 0.95$, the bandwidth parameter $r = 0.99$, $r_B = 0.95$, and the step size parameter $\mu_1 = 0.1$, $\mu_2 = 0.01$. These parameters are given experientially, but it is a future work to investigate how to set the optimal values. The parameter settings of the conventional method are the same as the proposed method.

First, figure 4 shows the behaviors of the estimation frequency under 30dB SNR, where we assume low noise environment. The each result is the average value of 100 trials. As seen in figure 4, the estimation frequency of DESA-2 tracks the true frequency, but it deeply vibrates around the true frequency because of noise effect. The adaptive bandpass filter estimates the frequency with low noise influence, but the estimation delay occurs for the frequency fluctuation. Figure 5 shows MSE curve for each method, which is derived by 100 trials average. It can be seen that the proposed method has lower MSE value. Compared with the adaptive bandpass filter, the proposed method provides faster convergence between $0 - 2000$-th iteration. Thus, it is revealed that the proposed method can provide fast and accurate estimation.

Next, figure 6 shows the behaviors of the estimation fre-

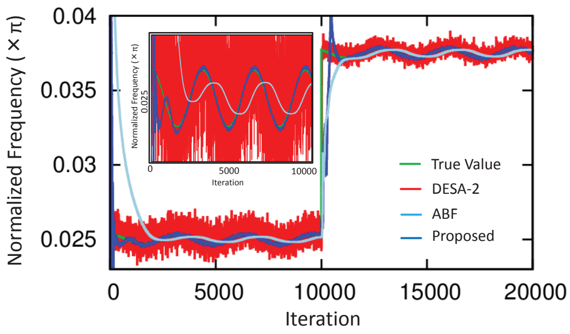

Fig. 4. Behaviors of the true frequency (green line) and the estimation frequency for the DESA-2 (red line), the adaptive bandpass filter (sky blue line), and proposed method (blue line) with 30dB SNR.

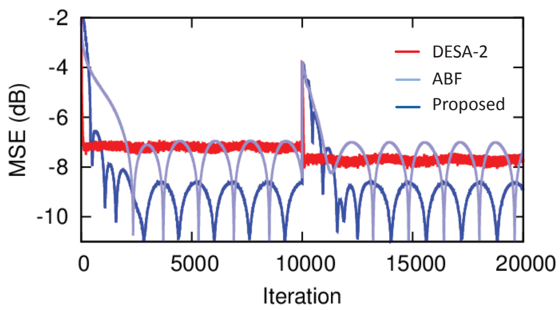

Fig. 5. MSE for the DESA-2 (red line), the adaptive bandpass filter (sky blue line), and proposed method (blue line) with 30dB SNR.

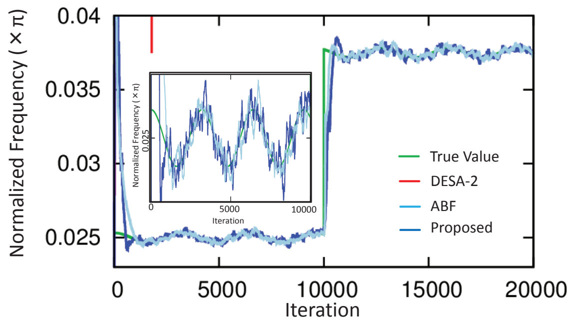

Fig. 6. Behaviors of the true frequency (green line) and the estimation frequency for the DESA-2 (red line), the adaptive bandpass filter (sky blue line), and proposed method (blue line) with 0dB SNR.

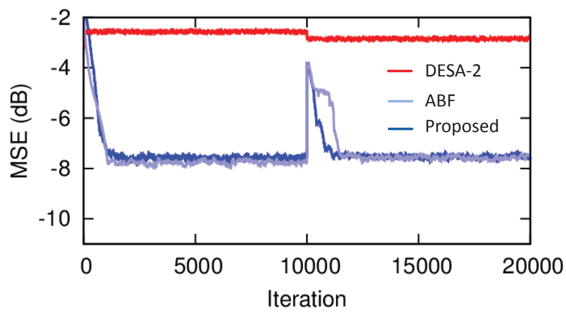

Fig. 7. MSE for the DESA-2 (red line), the adaptive bandpass filter (sky blue line), and proposed method (blue line) with 0dB SNR.

quency under 0dB SNR, where we assume high noise environment. The each result is also the average value of 100 trials. Here, DESA-2 almost estimates over 0.04π and thus it does not appear in this figure except around 1600-th iteration. As seen in figure 6, both the adaptive bandpass filter and the proposed method tracks the variation of the true frequency, although the both estimations are corrupted by noise. The adaptive bandpass filter shows an interesting property that the estimation delay is decreased compared with the case of 30dB SNR. The property appears when the correlation of the input is weakened by the non-correlated noise, and it often increases the estimation accuracy. Note that the very high level noise, of cause, decreases the estimation accuracy. Figure 7 shows MSE curve for each method with 0dB SNR, which is derived by 100 trials average. From this figure, it can be seen that the bandpass filter and the proposed method exhibit the almost same behaviors, except for the interval of $10000 - 12000$-th iteration.

Finally, figure 8 shows the MSE with different SNR, where the value of MSE is derived by averaging the results of 100 trials every 1dB. It is seen from this figure that the proposed method is performed with higher or equivalent estimation accuracy in -10dB - 38dB SNR. Specifically, the proposed method provides the 5.0dB improvement in MSE compared with the DESA-2 in 0dB SNR, and the 1.7dB improvement in MSE compared with the adaptive bandpass filter in 38dB SNR. In the very high SNR over 38dB, the proposed method provides lower accuracy compared with conventional DESA-2, since the effect of the long impulse response of the bandpass filter strongly appears compared with the effect of noise under the very low noise environment. The development of reduction for the effect of the impulse response is a future work. Also future works include that development of the parameter optimization.

5. CONCLUSION

In this paper, we have proposed a new frequency estimation for a single sinusoidal signal combining DESA-2 and an adaptive bandpass filter. Several computational simulations have shown that the proposed method is efficient in high noise environment with -10dB - 38dB SNR. In future works, we

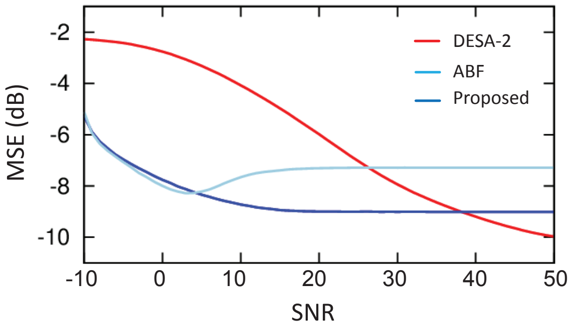

Fig. 8. MSE for the DESA-2 (red line), the adaptive bandpass filter (sky blue line), and proposed method (blue line) with different SNR.

derive the convergence condition of the step size parameter μ_1 and μ_2. We also develop the reduction method for the effect of the long impulse response of the bandpass filter.

REFERENCES

[1] T. Wang, W.S.Gan, and S.M. Kuo, "New feedback active noise control system with improved performance," in *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*. IEEE, 2014, pp. 6662–6666.

[2] L. Zhang and J. Wang, "A fast blind carrier frequency estimation algorithm for digitally modulated signals," in *Wireless Communications Networking and Mobile Computing (WiCOM), 2010 6th International Conference on*. IEEE, 2010, pp. 1–3.

[3] P.K. Dashand and S. Hasan, "A fast recursive algorithm for the estimation of frequency, amplitude, and phase of noisy sinusoid," *Industrial Electronics, IEEE Transactions on*, vol. 58, no. 10, pp. 4847–4856, 2011.

[4] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal. i. fundamentals," *Proceedings of the IEEE*, vol. 80, no. 4, pp. 520–538, 1992.

[5] Yosuke Sugiura, Arata Kawamura, and Youji Iiguni, "Performance analysis of an inverse notch filter and its application to f0 estimation," *Circuits and Systems*, vol. 4, no. 01, pp. 117, 2013.

[6] Y. Sugiura, "A fast and accurate adaptive notch filter using a monotonically increasing gradient," in *Signal Processing Conference (EUSIPCO), 2013 Proceedings of the 22nd European*. IEEE, 2014, pp. 1756–1760.

[7] P. Maragos, J.F. Kaiser, and T.F. Quatieri, "Energy separation in signal modulations with application to speech analysis," *Signal Processing, IEEE Transactions on*, vol. 41, no. 10, pp. 3024–3051, 1993.

[8] A. Kawamura, J. OKELLO, M. KOBAYASHI, and Y. FUKUI, "Parallel composition based adaptive notch filter: Performance and analysis," *IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences*, vol. 87, no. 7, pp. 1747–1755, 2004.