ON THE CONVERGENCE, STEADY-STATE, AND TRACKING ANALYSIS OF THE SRLMMN ALGORITHM

Mohammed Mujahid Ulla Faiz and Azzedine Zerguine

Department of Electrical Engineering
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
E-mail: {mujahid, azzedine}@kfupm.edu.sa

ABSTRACT

In this work, a novel algorithm named sign regressor least mean mixed-norm (SRLMMN) algorithm is proposed as an alternative to the well-known least mean mixed-norm (LMMN) algorithm. The SRLMMN algorithm is a hybrid version of the sign regressor least mean square (SRLMS) and sign regressor least mean fourth (SRLMF) algorithms. Analytical expressions are derived to describe the convergence, steady-state, and tracking behavior of the proposed SRLMMN algorithm. To validate our theoretical findings, a system identification problem is considered for this purpose. It is shown that there is a very close correspondence between theory and simulation. Finally, it is also shown that the SRLMMN algorithm is robust enough in tracking the variations in the channel.

***Index Terms*—** LMS, LMF, LMMN, SRLMS, SRLMF, SRLMMN, sign regressor, mixed-norm, convergence, steady-state, tracking.

1. INTRODUCTION

The least mean mixed-norm (LMMN) algorithm [1] is a well-known member of the family of mixed-norm adaptive filtering algorithms, which combines the benefits of the well-established least mean square (LMS) [2] and least mean fourth (LMF) algorithms [3]. In [4], the LMMN algorithm was introduced for the first time in an adaptive echo cancellation problem where it has shown improved performance over the LMS algorithm in terms of convergence and misadjustment. In-depth convergence, steady-state, and tracking analysis of the LMMN algorithm can be found in [5]–[6].

Signed adaptive filters are extensively used for the processing and analysis of electrocardiogram (ECG) signals [7] as they are computationally less complex when compared to their unsigned counterparts. The proposed sign regressor least mean mixed-norm (SRLMMN) algorithm is a signed version of the LMMN algorithm, which is obtained by taking the signum function of the input data. The SRLMMN algorithm is a hybrid algorithm based on a combination of the sign regressor least mean square (SRLMS) [8]–[9] and sign regressor least mean fourth (SRLMF) [10] algorithms. The SRLMMN algorithm reduces to SRLMF and SRLMS algorithms when the mixing parameter becomes zero and one, respectively. In the present work, analytical expressions for the convergence, steady-state mean-square error (MSE), and tracking MSE of the SRLMMN algorithm are derived.

The rest of the paper is structured as follows. The weight update equation of the proposed algorithm is described in Section 2. The convergence, steady-state, and tracking analysis of the SRLMMN algorithm is carried out in Sections 3, 4, and 5, respectively. Simulation studies which confirm the theoretical findings are presented in Section 6, followed by conclusions in Section 7.

2. THE SRLMMN ALGORITHM

The weight update equation of the LMMN algorithm can be written as follows [9]:

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu\, \mathbf{u}_i^{\mathrm{T}} e_i[\delta + (1-\delta)e_i^2], \qquad 0 \le \delta \le 1, \qquad (1)$$

where $\mathbf{w}_i \in \mathbb{R}^{M\times 1}$ is the updated weight vector at iteration i, M is the length of adaptive filter, μ is the step-size, $\mathbf{u}_i \in \mathbb{R}^{1\times M}$ is the regressor vector, δ is the mixing parameter, and e_i is the estimation error given by

$$e_i = d_i - \mathbf{u}_i \mathbf{w}_{i-1}, \qquad (2)$$

where d_i is the desired value. The SRLMMN algorithm is obtained from the LMMN algorithm in (1) by replacing the regressor vector by its sign as shown below:

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu\, \mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i[\delta + (1-\delta)e_i^2], \qquad 0 \le \delta \le 1. \qquad (3)$$

3. CONVERGENCE ANALYSIS

To carry out the convergence analysis of the SRLMMN algorithm we rely on the assumptions mentioned in [11].
Subtracting both sides of (3) from the optimal weight vector $\mathbf{w}_i^o$ we get

$$\widetilde{\mathbf{w}}_i = \widetilde{\mathbf{w}}_{i-1} - \mu\, \delta\, \mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i - \mu\, (1-\delta)\mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i^3, \qquad (4)$$

where the weight error vector $\widetilde{\mathbf{w}}_i$ is given by

$$\widetilde{\mathbf{w}}_i = \mathbf{w}_i^o - \mathbf{w}_i. \tag{5}$$

Taking the expectation of both sides of (4) under the assumptions mentioned in [11] we obtain

$$\mathrm{E}[\widetilde{\mathbf{w}}_i] = \mathrm{E}[\widetilde{\mathbf{w}}_{i-1}] - \mu\,\delta\,\mathrm{E}\left[\mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i\right] - \mu\,(1-\delta)\mathrm{E}\left[\mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i^3\right]. \tag{6}$$

From [10], we have

$$\mathrm{E}\left[\mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i\right] = \sqrt{\frac{2}{\pi\sigma_u^2}}\mathbf{R}\mathrm{E}[\widetilde{\mathbf{w}}_{i-1}], \tag{7}$$

$$\mathrm{E}\left[\mathrm{sign}[\mathbf{u}_i]^{\mathrm{T}} e_i^3\right] = 3\sqrt{\frac{2}{\pi\sigma_u^2}}\sigma_e^2\mathbf{R}\mathrm{E}[\widetilde{\mathbf{w}}_{i-1}], \tag{8}$$

where σ_u^2 is the regressor variance, σ_e^2 is the estimation error variance, and $\mathbf{R} = \mathrm{E}[\mathbf{u}_i^{\mathrm{T}}\mathbf{u}_i]$ is the regressor autocorrelation matrix. Upon substituting (7) and (8) into (6), we have

$$\mathrm{E}[\widetilde{\mathbf{w}}_i] = \left[1 - \mu\sqrt{\frac{2}{\pi\sigma_u^2}}\mathbf{R}\left(\delta + 3(1-\delta)\sigma_e^2\right)\right]\mathrm{E}[\widetilde{\mathbf{w}}_{i-1}]. \tag{9}$$

From (9), it is easy to show that the mean behavior of the weight error vector, that is $\mathrm{E}[\widetilde{\mathbf{w}}_i]$, converges to the zero vector if the step-size μ is bounded by:

$$0 < \mu < \frac{\sqrt{2\pi\sigma_u^2}}{\lambda_{\max}\left(\delta + 3(1-\delta)\sigma_e^2\right)}, \tag{10}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{R}$. We can obtain the step-size bounds of the SRLMF and SRLMS algorithms from (10) by setting δ equal to 0 and 1, respectively, as shown below:

$$0 < \mu_{\mathrm{SRLMF}} < \frac{\sqrt{2\pi\sigma_u^2}}{3\lambda_{\max}\sigma_e^2}. \tag{11}$$

$$0 < \mu_{\mathrm{SRLMS}} < \frac{\sqrt{2\pi\sigma_u^2}}{\lambda_{\max}}. \tag{12}$$

Note that the step-size bound of the SRLMF algorithm in (11) is the same as that obtained by us in [10]. Equation (10) can also be rewritten in the following equivalent form:

$$0 < \mu < \delta\,\mu_{\mathrm{SRLMS}} + (1-\delta)\mu_{\mathrm{SRLMF}}. \tag{13}$$

4. STEADY-STATE ANALYSIS

To carry out the steady-state analysis of the SRLMMN algorithm we shall assume that the data $\{d_i, \mathbf{u}_i\}$ satisfy the assumptions of the stationary data model mentioned in [12].

For the adaptive filter of the form in (3), and for any data $\{d_i, \mathbf{u}_i\}$, assuming filter operation in steady-state, the following variance relation holds [9]:

$$\mu\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2 \mathrm{g}^2[e_i]\right] = 2\mathrm{E}\left[e_{a_i}\mathrm{g}[e_i]\right], \quad \text{as} \quad i \to \infty, \tag{14}$$

where

$$\mathrm{E}[\|\mathbf{u}_i\|_{\mathrm{H}}^2] = \mathrm{E}[\mathbf{u}_i\mathrm{H}[\mathbf{u}_i]\mathbf{u}_i^{\mathrm{T}}], \tag{15}$$

$$e_i = e_{a_i} + v_i, \tag{16}$$

with $\mathrm{g}[e_i]$ denoting some function of e_i, and $e_{a_i} = \mathbf{u}_i(\mathbf{w}^o - \mathbf{w}_{i-1})$ is the a priori estimation error. Then $\mathrm{g}[e_i]$ for the SRLMMN algorithm can be shown to be

$$\begin{aligned}\mathrm{g}[e_i] = {} & \delta(e_{a_i} + v_i) + \bar{\delta}\{e_{a_i}^3 + e_{a_i}v_i^2 + 2e_{a_i}^2 v_i + v_i e_{a_i}^2 + v_i^3 \\ & + 2e_{a_i}v_i^2\},\end{aligned} \tag{17}$$

where $\bar{\delta} = 1 - \delta$. By using the fact that e_{a_i} and v_i are independent, and by ignoring third and higher-order terms of e_{a_i}, we reach at the following expression for the term $\mathrm{E}\left[e_{a_i}\mathrm{g}[e_i]\right]$:

$$\mathrm{E}\left[e_{a_i}\mathrm{g}[e_i]\right] \approx (\delta + 3\bar{\delta}\sigma_v^2)\mathrm{E}[e_{a_i}^2]. \tag{18}$$

To evaluate the term $\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2 \mathrm{g}^2[e_i]\right]$, we start by noting that

$$\begin{aligned}\mathrm{g}^2[e_i] = {} & \delta^2[e_{a_i}^2 + v_i^2 + 2e_{a_i}v_i] + \bar{\delta}^2[e_{a_i}^6 + 6e_{a_i}^5 v_i + 6e_{a_i}v_i^5 \\ & + 15e_{a_i}^4 v_i^2 + 15e_{a_i}^2 v_i^4 + 20e_{a_i}^3 v_i^3 + v_i^6] + 2\delta\bar{\delta}[e_{a_i}^4 \\ & + 6e_{a_i}^2 v_i^2 + 4e_{a_i}^3 v_i + 4e_{a_i}v_i^3 + v_i^4].\end{aligned} \tag{19}$$

If we multiply $\mathrm{g}^2[e_i]$ by $\|\mathbf{u}_i\|_{\mathrm{H}}^2$ from the left, use the fact that v_i is independent of both $\mathbf{u}_i$ and e_{a_i}, and again ignoring third and higher-order terms of e_{a_i}, we obtain

$$\begin{aligned}\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2 \mathrm{g}^2[e_i]\right] \approx {} & (\delta^2 + 15\bar{\delta}^2\xi_v^4 + 12\delta\bar{\delta}\sigma_v^2)\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2 e_{a_i}^2\right] \\ & + (\delta^2\sigma_v^2 + \bar{\delta}^2\xi_v^6 + 2\delta\bar{\delta}\xi_v^4)\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2\right],\end{aligned} \tag{20}$$

where $\xi_v^4 = \mathrm{E}[v_i^4]$ and $\xi_v^6 = \mathrm{E}[v_i^6]$ are the fourth and sixth-order moments of the noise sequence v_i, respectively.
Substituting (18) and (20) into (14), and by using the separation principle [12], we get

$$\begin{aligned}\mu(\delta^2\sigma_v^2 + \bar{\delta}^2\xi_v^6 + 2\delta\bar{\delta}\xi_v^4)\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2\right] = {} & [2(\delta + 3\bar{\delta}\sigma_v^2) \\ & - \mu(\delta^2 + 15\bar{\delta}^2\xi_v^4 + 12\delta\bar{\delta}\sigma_v^2)\mathrm{E}\left[\|\mathbf{u}_i\|_{\mathrm{H}}^2\right]]\mathrm{E}[e_{a_i}^2].\end{aligned} \tag{21}$$

Ultimately, the expression for the steady-state MSE $\varphi = \mathrm{E}\left[e_i^2\right]$ of the SRLMMN algorithm can be shown to be

$$\varphi = \frac{\mu(\delta^2\sigma_v^2 + \bar{\delta}^2\xi_v^6 + 2\delta\bar{\delta}\xi_v^4)\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})}{2(\delta + 3\bar{\delta}\sigma_v^2) - \mu(\delta^2 + 15\bar{\delta}^2\xi_v^4 + 12\delta\bar{\delta}\sigma_v^2)\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2. \tag{22}$$

We can obtain the expressions for the steady-state MSE of the SRLMF and SRLMS algorithms from (22) by setting δ equal to 0 and 1, respectively, as shown below:

$$\varphi_{\mathrm{SRLMF}} = \frac{\mu\xi_v^6\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})}{6\sigma_v^2 - 15\mu\xi_v^4\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2. \tag{23}$$

$$\varphi_{\mathrm{SRLMS}} = \frac{\mu\sigma_v^2\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})}{2 - \mu\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2. \tag{24}$$

Note that the expression for the steady-state MSE of the SRLMF algorithm in (23) is the same as that obtained by us in [10]. Similarly, here Equation (22) can also be rewritten in the following equivalent form:

$$\varphi = \delta\, \varphi_{\text{SRLMS}} + (1-\delta)\varphi_{\text{SRLMF}}. \tag{25}$$

5. TRACKING ANALYSIS

To carry out the tracking analysis of the SRLMMN algorithm we shall assume that the data $\{d_i, \mathbf{u}_i\}$ satisfy the assumptions of the nonstationary data model mentioned in [11].

For the adaptive filter of the form in (3), and for any data $\{d_i, \mathbf{u}_i\}$, assuming filter operation in steady-state, the following variance relation holds [9]:

$$\mu \mathrm{E}\left[\|\mathbf{u}_i\|^2_{\mathrm{H}} g^2[e_i]\right] + \mu^{-1}\mathrm{Tr}(\mathbf{Q}) = 2\mathrm{E}\left[e_{a_i} g[e_i]\right], \quad \text{as } i \to \infty. \tag{26}$$

Tracking results can be obtained by inspection from the steady-state results in Section 4 as there are only minor differences. Therefore, by substituting (18) and (20) into (26) the expression for the tracking MSE φ' of the SRLMMN algorithm can be shown to be

$$\varphi' = \frac{\mu(\delta^2\sigma_v^2 + \bar{\delta}^2\xi_v^6 + 2\delta\bar{\delta}\xi_v^4)\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R}) + \mu^{-1}\mathrm{Tr}(\mathbf{Q})}{2(\delta + 3\bar{\delta}\sigma_v^2) - \mu(\delta^2 + 15\bar{\delta}^2\xi_v^4 + 12\delta\bar{\delta}\sigma_v^2)\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2. \tag{27}$$

We can obtain the expressions for the tracking MSE of the SRLMF and SRLMS algorithms from (27) by setting δ equal to 0 and 1, respectively, as shown below:

$$\varphi'_{\text{SRLMF}} = \frac{\mu\xi_v^6\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R}) + \mu^{-1}\mathrm{Tr}(\mathbf{Q})}{6\sigma_v^2 - 15\mu\xi_v^4\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2, \tag{28}$$

$$\varphi'_{\text{SRLMS}} = \frac{\mu\sigma_v^2\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R}) + \mu^{-1}\mathrm{Tr}(\mathbf{Q})}{2 - \mu\sqrt{\frac{2}{\pi\sigma_u^2}}\mathrm{Tr}(\mathbf{R})} + \sigma_v^2, \tag{29}$$

where $\mathbf{Q} = \mathrm{E}[\mathbf{q}_i\mathbf{q}_i^{\mathrm{T}}]$ is the autocorrelation matrix of the sequence $\mathbf{q}_i$ in the random walk model as described in the next section. Note that the expression for the tracking MSE of the SRLMF algorithm in (28) is the same as that obtained by us in [10]. Similarly, here Equation (27) can also be rewritten in the following equivalent form:

$$\varphi' = \delta\, \varphi'_{\text{SRLMS}} + (1-\delta)\varphi'_{\text{SRLMF}}. \tag{30}$$

6. SIMULATION RESULTS

In all the simulations, the problem of identification of an unknown system is considered with filter tap-length of $M = 10$. The tap-weight vector of the unknown system is considered to be stationary for both convergence and steady-state MSE of the SRLMMN algorithm and nonstationary for tracking MSE of the SRLMMN algorithm. The amount of nonstationarity added to the tap-weight vector is according to a random walk model, as described later in this section. The signal-to-noise ratio (SNR) is fixed at 30 dB (Figures 1–4), 20 dB (Figures 5–8), and 10 dB (Figures 9–10). The mixing parameter is fixed at 0.5 for Figures 1, 5, and 9. The step-size is fixed at 0.005 for Figure 2 and 0.01 for Figure 6. We have considered additive white Gaussian noise (AWGN) environment for Figures 1–8 and uniform noise environment for Figures 9–10.

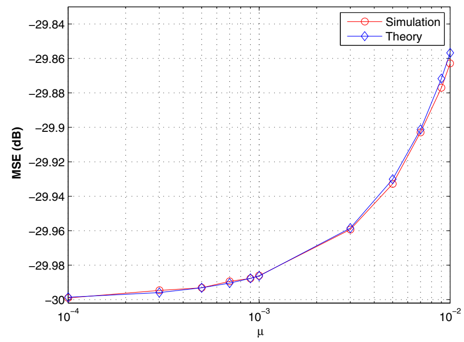

Fig. 1. Steady-state behavior of the SRLMMN algorithm versus μ for fixed δ ($\delta = 0.5$).

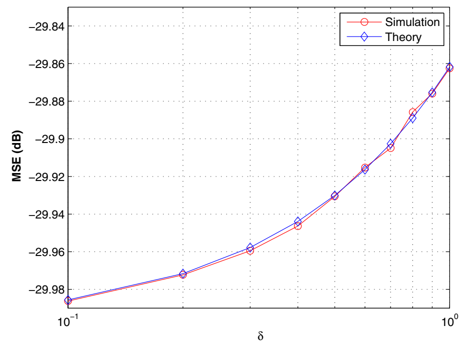

Fig. 2. Steady-state behavior of the SRLMMN algorithm versus δ for fixed μ ($\mu = 0.005$).

To demonstrate the tracking abilities of the SRLMMN algorithm, we have considered a random walk model as shown below:

$$\mathbf{w}_i^o = \mathbf{w}_{i-1}^o + \mathbf{q}_i, \tag{31}$$

where $\mathbf{q}_i$ is a zero mean Gaussian noise sequence with variance $\sigma_q^2 = 10^{-10}$.

In order to examine the steady-state and tracking behavior of the SRLMMN algorithm, we have performed simulations for four different cases: In the first case, steady-state/tracking MSE of the SRLMMN algorithm is plotted against the step-size for a fixed value of the mixing parameter, as shown in

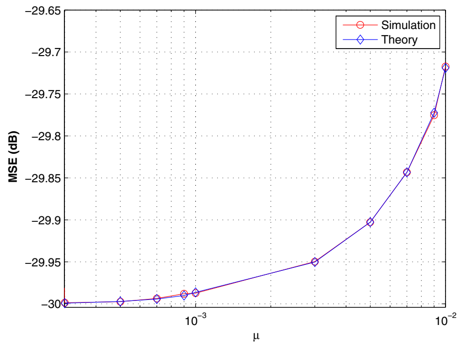

Fig. 3. Steady-state behavior of the SRLMMN algorithm versus μ for varying δ (δ varying from 0 to 1).

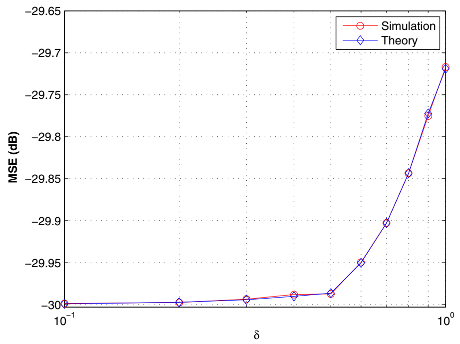

Fig. 4. Steady-state behavior of the SRLMMN algorithm versus δ for varying μ (μ varying from 10^{-4} to 10^{-2}).

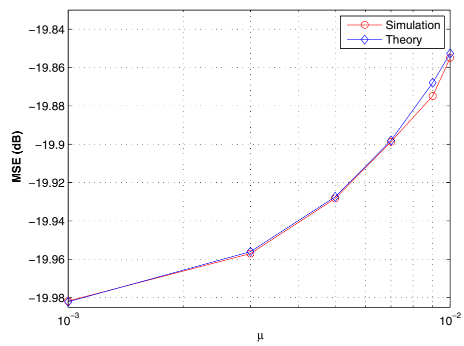

Fig. 5. Tracking behavior of the SRLMMN algorithm versus μ for fixed δ ($\delta = 0.5$).

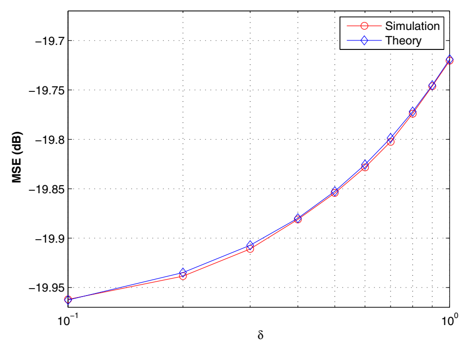

Fig. 6. Tracking behavior of the SRLMMN algorithm versus δ for fixed μ ($\mu = 0.01$).

Figures 1 and 5, respectively. In the second case, steady-state/tracking MSE of the SRLMMN algorithm is plotted against the mixing parameter for a fixed value of the step-size, as shown in Figures 2 and 6, respectively. In the third case, steady-state/tracking MSE of the SRLMMN algorithm is plotted against the step-size for varying values of the mixing parameter, as shown in Figures 3 and 7, respectively. Finally, in the fourth case, steady-state/tracking MSE of the SRLMMN algorithm is plotted against the mixing parameter for varying values of the step-size, as shown in Figures 4 and 8, respectively. In all these cases, an excellent match is observed between the theoretical and simulated results, as shown in Figures 1–8.

Moreover, in order to study the convergence aspects of the SRLMMN algorithm some results are presented in Figures 9–10. Figure 9 compares the convergence rate of the LMMN and SRLMMN algorithms. We can observe from Figure 9 that the learning curves of both the algorithms are almost identical. Finally, Figure 10 compares the convergence rate of the SRLMMN algorithm for various values of δ. Notice in Figure 10 that, the SRLMMN algorithm reduces to SRLMF and SRLMS algorithms when $\delta = 0$ and $\delta = 1$, respectively.

7. CONCLUSIONS

A new variant of the LMMN algorithm called the SRLMMN algorithm has been introduced and analyzed in this paper. It has been shown that the SRLMMN algorithm can achieve similar performance with respect to its adaptive counterpart but with reduced complexity. Furthermore, an excellent agreement is observed between the simulation and analytical results. Thus, a combination of the SRLMS and SRLMF algorithms has resulted in a hybrid algorithm, which is robust to variations in the channel statistics.

Acknowledgment: The authors would like to acknowledge the support provided by King Fahd University of Petroleum & Minerals.

8. REFERENCES

[1] J. A. Chambers, O. Tanrikulu, and A. G. Constantinides,“Least mean mixed-nom adaptive filtering,” *Electronics Letters*, vol. 30, no. 19, pp. 1574–1575, Sep. 1994.

[2] B. Widrow, J. M. McCool, M. G. Larimore, and C. R. Johnson, Jr., “Stationary and nonstationary learning characteristics of the LMS adaptive filter,” *IEEE Proc.*, vol. 64, no. 8, pp. 1151–1162, Aug. 1976.

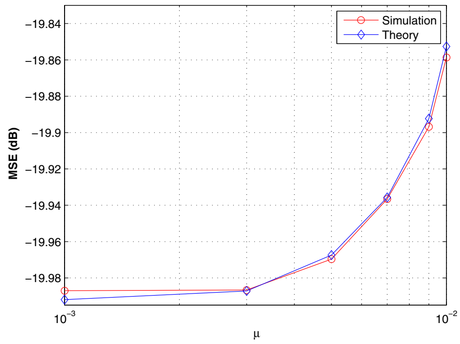

Fig. 7. Tracking behavior of the SRLMMN algorithm versus μ for varying δ (δ varying from 0 to 1).

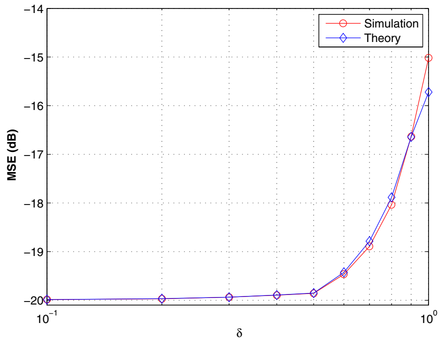

Fig. 8. Tracking behavior of the SRLMMN algorithm versus δ for varying μ (μ varying from 10^{-3} to 10^{-1}).

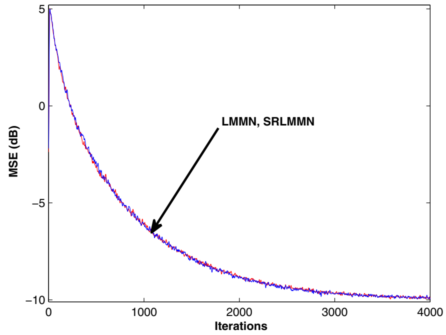

Fig. 9. Behavior of the LMMN and SRLMMN algorithms.

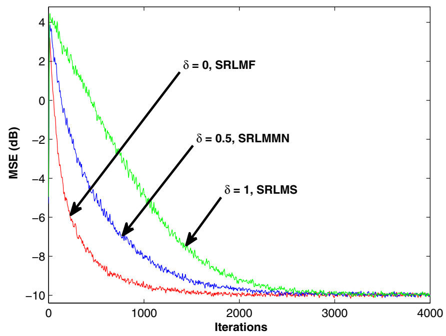

Fig. 10. Behavior of the SRLMMN algorithm for different values of δ.

[3] E. Walach and B. Widrow, "The least mean fourth (LMF) adaptive algorithm and its family," *IEEE Trans. Inf. Theory*, vol. 30, no. 2, pp. 275–283, Mar. 1984.

[4] A. Zerguine, C. F. N. Cowan, and M. Bettayeb, "Adaptive echo cancellation using least mean mixed-norm algorithm," *IEEE Trans. Signal Processing*, vol. 45, no. 5, pp. 1340-1343, May 1997.

[5] O. Tanrikulu and J. A. Chambers, "Convergence and steady-state properties of the least mean mixed-norm (LMMN) adaptive algorithm," *IEE Proc. Vision, Image and Signal Processing*, vol. 143, no. 3, pp. 137-142, June 1996.

[6] N. R. Yousef and A. H. Sayed, "Tracking analysis of the LMF and LMMN adaptive algorithms," *in the Conf. Record of the* 33rd *Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, USA, vol. 1, pp. 786-790, Oct. 1999.

[7] M. Z. U. Rahman, R. A. Shaik, and D. V. R. K. Reddy, "Efficient sign based normalized adaptive filtering techniques for cancelation of artifacts in ECG signals: Application to wireless biotelemetry," *Signal Processing*, vol. 91, no. 2, pp. 225–239, Feb. 2011.

[8] E. Eweda, "Analysis and design of a signed regressor LMS algorithm for stationary and nonstationary adaptive filtering with correlated Gaussian data," *IEEE Trans. Circuits Syst.*, vol. 37, no. 11, pp. 1367–1374, Nov. 1990.

[9] A. H. Sayed, "Fundamentals of Adaptive Filtering," *Wiley-IEEE Press*, 1st Ed., June 2003.

[10] M. M. U. Faiz, A. Zerguine, and A. Zidouri, "Analysis of the sign regressor least mean fourth adaptive algorithm," *EURASIP Journal on Advances in Signal Processing*, vol. 2011, pp. 1–12, Jan. 2011.

[11] M. M. U. Faiz and A. Zerguine, "Convergence and tracking analysis of the $\epsilon-$NSRLMF algorithm," *in Proc. of the* 38th *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, pp. 5657–5660, May 2013.

[12] M. M. U. Faiz and A. Zerguine, "Convergence analysis of the ϵ NSRLMMN algorithm," *in Proc. of the* 20th *European Signal Processing Conf. (EUSIPCO)*, Bucharest, Romania, pp. 235–239, Aug. 2012.