MIMO Systems Outage Capacity based on Minors Moments of Wishart Matrices

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***Abstract*—Complex Wishart matrices represent a class of random matrices exploited in a number of wireless communication problems. This paper analyzes the first and second order statistical moments of complex Wishart matrices' minors. This enables to derive new closed-form approximations for the outage capacity of multiple input multiple output (MIMO) systems operating in Rayleigh fading channels at any signal-to-noise ratio (SNR) regime and with any number of inputs and outputs. The derived expressions are compared with bounds known in the literature as well as with simulations. Results show the tightness of the proposed approximations to simulations for a broad range of MIMO settings.**

***Index Terms*—MIMO systems, outage capacity, Wishart matrix, fading channels, statistical characterization.**

I. Introduction

Recently, random matrix theory involving complex Wishart matrices has attracted the interest of researchers in wireless communications for their suitability to model the behavior of wireless systems [1]. Relevant applications of random matrix theory include MIMO and massive MIMO systems [2]–[4], which are key technologies for elevating the spectral efficiency in future wireless communications. The design of such systems require the characterization of the outage capacity. Although this subject has received a lot of interest in the last two decades, the derivation of a closed-form expression of the outage capacity serving for the design of MIMO systems in nonasymptotic regimes is still an open problem.

MIMO systems have been largely studied in the literature for high spectral efficiency communications [5]–[7]. Expressions for the maximum information rate and outage probability were derived in [8]. The capacity of MIMO systems was obtained in [9], [10] using an approach based on moment generating function. The ergodic capacity of Ricean-fading MIMO channels with rank-1 mean matrices was determined in [11] under the assumption that the channel is unknown at the transmitter and perfectly known at the receiver. Other typical approaches in the literature for evaluating the outage capacity of MIMO systems consist in determining the eigenvalues of a Wishart matrix [12] or in considering asymptotic regimes on the SNR and on the number of inputs and outputs [1], [13]. While the former approaches might result in cumbersome expressions; the latter might result in inaccurate expressions for cases of interest where a MIMO system employs a limited number of antennas and operate at moderate SNR.

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This paper proposes a different approach for evaluating the outage capacity of MIMO systems based on moments of the Wishart matrix minors. The proposed approach requires the extension of some theorems on minors of a real Wishart matrices to those of complex Wishart matrices, and the derivation of their expectation. This enables us to derive closed-form approximations for the outage capacity for any number of antennas and SNR values in Rayleigh fading channels. The resulting expression is compared with bounds known in the literature and with Monte Carlo simulations.

II. System Model

Consider a MIMO system composed of $n_{\rm T}$ transmitting antennas and $n_{\rm R}$ receiving antennas operating in a wireless channel described by a matrix $\boldsymbol{H}$ which is an $n_{\rm R} \times n_{\rm T}$ matrix. The elements of $\boldsymbol{H}$ are independent, identically distributed (IID) random variables (RVs) following a zero-mean complex Gaussian distribution (i.e., Rayleigh fading channel for each transmit-receive antenna pair). Let $n = \min\{n_{\rm T}, n_{\rm R}\}$ and $m = \max\{n_{\rm T}, n_{\rm R}\}$.

In an absence of channel state information at the transmitter side, the instantaneous Shannon capacity is given by [7]

$$C(D) = \log_2 D \tag{1}$$

where

$$D = \det\left(\boldsymbol{I}_n + \frac{\rho}{n_{\rm T}}\boldsymbol{W}\right) \tag{2}$$

in which $\boldsymbol{I}_n$ is the $n \times n$ identity matrix, ρ is the SNR per receiving antenna element averaged over small-scale fading, and

$$\boldsymbol{W} = \begin{cases} \boldsymbol{H}\boldsymbol{H}^\dagger & \text{for } n_{\rm R} \le n_{\rm T} \\ \boldsymbol{H}^\dagger\boldsymbol{H} & \text{for } n_{\rm T} \le n_{\rm R} \end{cases} \tag{3}$$

is an $n\times n$ complex Wishart matrix with m degrees of freedom and covariance $\boldsymbol{I}_n$, and with n eigenvalues that are elements of the column vector $\boldsymbol{\lambda}$ i.e. $\boldsymbol{\lambda}^T = [\lambda_1, \lambda_2, \ldots, \lambda_n]$.[1] Therefore, the instantaneous Shannon capacity (1) results in

$$C = \sum_{i=1}^{n} \log_2\left(1 + \frac{\rho}{n_{\rm T}}\lambda_i\right).$$

To characterize the statistics of the instantaneous capacity C, which depends on the randomness of the channels through the determinant D, or the eigenvalues $\boldsymbol{\lambda}$, the ergodic capacity

$$\mu_{\rm C} = \mathbb{E}\left\{C(D)\right\} \tag{4}$$

[1]Notations $\dagger$ and T indicate transpose-conjugate and transpose operations, respectively.

and the outage capacity $C^\star$ for a given target outage probability

$$P_{\text{out}} = \mathbb{P}\{C(D) \leq C^\star\} \tag{5}$$

can be determined.[2]

III. Ergodic and Outage Capacity Approximations

A tractable approximation for the outage capacity is obtained by considering the instantaneous capacity as a Gaussian distributed RV [14] with mean μ_C and variance σ_C^2. Thus, from (5) we have

$$C^\star \simeq \mu_C - \text{Inverfc}(2P_{\text{out}})\sqrt{2\sigma_C^2} \tag{6}$$

where Inverfc$(\cdot)$ is the inverse complementary error function.

Instead of determining μ_C and σ_C^2 from the statistics of $\boldsymbol{\lambda}$, consider an alternative approach based on the Taylor expansion of the instantaneous capacity $C(D)$ around $\mu_D = \mathbb{E}\{D\}$. This results in

$$\begin{aligned} C(D) &= C_k(D) + R_k(D) \\ C_k(D) &= \log_2 \mu_D + \sum_{i=1}^{k} \frac{(D-\mu_D)^i}{i!} \frac{d^i}{dD^i} \log_2 D|_{\mu_D} \\ &= \log_2 \mu_D + \log_2 e \sum_{i=1}^{k} \frac{(-1)^{i-1}(D-\mu_D)^i}{i\,\mu_D^i} . \end{aligned} \tag{7}$$

Here, $C_k(D)$ is the Taylor polynomial of order k approximating $C(D)$ and $R_k(D)$ is the corresponding residual, which can be written as

$$R_k(D) = \log_2 e \frac{(-1)^k (D-\mu_D)^{k+1}}{k\,\xi_D^{k+1}} \tag{8}$$

where ξ_D is a number between μ_D and D. Therefore, $C(D)$ can be approximated by $C_k(D)$ when the residual $R_k(D)$ is negligible. Unfortunately, $|R_k(D)|$ converges to 0 for k large only when $D < 2\mu_D$ (note from (2) that $D \geq 1$), whereas it diverges when $D > 2\mu_D$. Hence, such an approximation has to be handled carefully, since D is a random variable taking values in $[1,\infty)$. It is expected that the convergent behavior will be dominant when the values of D are mainly distributed around μ_D, i.e., when $\sigma_D^2 = \mathbb{E}\{(D-\mu_D)^2\}$ is much smaller than μ_D. Note also that $R_k(D)$ is negative when k is odd, whereas it is positive only when k is even and $D > \mu_D$.

We can use the approximated instantaneous capacity to evaluate the mean μ_C and the variance σ_C^2 as: $\mu_{C,k} = \mathbb{E}\{C_k(D)\}$ and $\sigma_{C,k}^2 = \mathbb{E}\{(C_k(D) - \mu_{C,k})^2\}$. The simplest approximations are those depending on μ_D and σ_D^2 only, i.e.

$$\mu_{C,1} = \log_2 \mu_D \geq \mu_C \tag{9}$$

$$\mu_{C,2} = \log_2 \mu_D - \frac{\log_2 e}{2} \frac{\sigma_D^2}{\mu_D^2} \tag{10}$$

$$\sigma_{C,1}^2 = (\log_2 e)^2 \frac{\sigma_D^2}{\mu_D^2} . \tag{11}$$

We also consider the following approximation[3]

$$\begin{aligned} \tilde{\sigma}_{C,2}^2 &= \mathbb{E}\{(C_1(D) - \mu_{C,2})^2\} \\ &= (\log_2 e)^2 \frac{\sigma_D^2}{\mu_D^2} - \frac{1}{4}(\log_2 e)^2 \frac{\sigma_D^4}{\mu_D^4} . \end{aligned} \tag{12}$$

Note that $\mu_{C,2}$, $\sigma_{C,1}^2$, and $\tilde{\sigma}_{C,2}^2$ depend on $\log_2 \mu_D$ and $\frac{\sigma_D^2}{\mu_D^2}$.

Hereafter, we determine the exact first and second moments of D, which will enable us to obtain closed form approximations of μ_C and σ_C^2. By applying the expansion presented in [16] and exploited in [8] to the determinant D in (2), we obtain

$$D = 1 + \sum_{i=1}^{n} \left(\frac{\rho}{n_{\text{T}}}\right)^i \sum_{\substack{\alpha_i \subseteq \{1,2,\ldots,n\} \\ |\alpha_i| = i}} D_{\alpha_i} \tag{13}$$

where $D_{\alpha_i} = \det(\boldsymbol{W}_{\alpha_i})$ and $\boldsymbol{W}_{\alpha_i}$ is an $i \times i$ matrix composed of the elements of $\boldsymbol{W}$ that are in the rows and the columns with indexes in the set $\alpha_i \subseteq \{1,2,\ldots,n\}$ with cardinality $|\alpha_i|$. Averaging the determinant and the squared determinant of $\boldsymbol{W}_{\alpha_i}$ over Rayleigh fading gives [8], [17]

$$\mathbb{E}\{D_{\alpha_i}\} = \frac{m!}{(m-i)!} \tag{14}$$

$$\mathbb{E}\{D_{\alpha_i}^2\} = \frac{m!(m+1)!}{(m-i)!(m-i+1)!} . \tag{15}$$

From (13) and (14), we have

$$\begin{aligned} \mu_D &= 1 + \sum_{i=1}^{n} \left(\frac{\rho}{n_{\text{T}}}\right)^i \binom{n}{i} \frac{m!}{(m-i)!} \\ &= 1 + \sum_{i=1}^{n} \left(\frac{\rho}{n_{\text{T}}}\right)^i \frac{n_{\text{R}}!\, n_{\text{T}}!}{i!\,(n_{\text{R}}-i)!\,(n_{\text{T}}-i)!} . \end{aligned} \tag{16}$$

To derive the variance σ_D^2, we first express it as

$$\begin{aligned} \sigma_D^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\rho}{n_{\text{T}}}\right)^{i+j} \sum_{\alpha_i} \sum_{\beta_j} \big(&\mathbb{E}\{D_{\alpha_i} D_{\beta_j}\} \\ &- \mathbb{E}\{D_{\alpha_i}\}\mathbb{E}\{D_{\beta_j}\}\big) \end{aligned} \tag{17}$$

where α_i and β_j are index sets both $\subseteq \{1,2,\ldots,n\}$ with cardinality i and j, respectively. The expectation $\mathbb{E}\{D_{\alpha_i} D_{\beta_j}\}$ in (17) is obtained in the following Lemma.

Lemma 1: Let $\boldsymbol{W}$ be an $n \times n$ complex Wishart matrix with m degrees of freedom, and let α_i and β_j be index sets both $\subseteq \{1,2,\ldots,n\}$ with cardinality i and j, respectively. The expectation $\mathbb{E}\{D_{\alpha_i} D_{\beta_j}\}$ is given by

$$\begin{aligned} &\mathbb{E}\{D_{\alpha_i} D_{\beta_j}\} = \mathbb{E}\{\det(\boldsymbol{W}_{\alpha_i})\det(\boldsymbol{W}_{\beta_j})\} \\ &= \mathbb{E}\{\det(\boldsymbol{W}_{\gamma_k})^2\}\,\mathbb{E}\left\{\det(\breve{\boldsymbol{W}}_{\alpha_i \setminus \gamma_k})\right\}\mathbb{E}\left\{\det(\breve{\boldsymbol{W}}_{\beta_j \setminus \gamma_k})\right\} \end{aligned} \tag{18}$$

where index set $\gamma_k \triangleq \alpha_i \cap \beta_j$ with cardinality $|\gamma_k| = k$, and $\breve{\boldsymbol{W}}$ is a $(n-k) \times (n-k)$ complex Wishart matrix with $m-k$ degrees of freedom.

Proof: For conciseness we only provide a sketch of the proof. In particular, we follow the method for evaluating the moments of minors for a real Wishart matrix presented in [18]

[2]Notations $\mathbb{E}\{\cdot\}$ and $\mathbb{P}\{\cdot\}$ denote the statistical expectation and the probability of the argument, respectively.

[3]In [15], an approximation of this form for the outage capacity of orthogonal space-time block codes was given as a function of the power covariance matrix of the channel.

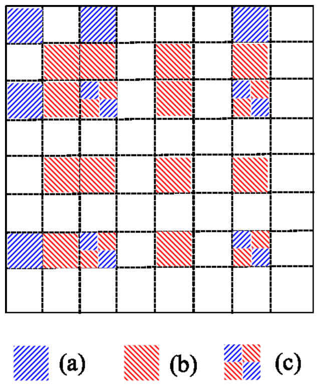


Fig. 1. Example of 8×8 matrix $\boldsymbol{W}$ and its partitions with $\alpha_i = \{1, 3, 7\}$ and $\beta_j = \{2, 3, 5, 7\}$: (a) elements of $\boldsymbol{W}_{\alpha_i}$; (b) elements of $\boldsymbol{W}_{\beta_j}$; and (c) elements of $\boldsymbol{W}_{\gamma_k}$.

Lemma 4.7 and [17] Theorem 3.2.10. Their proofs have been extended to complex Wishart matrices. □

An example of an 8×8 matrix $\boldsymbol{W}$ and its partitions with $\alpha_i = \{1, 3, 7\}$, $\beta_j = \{2, 3, 5, 7\}$, and γ_k is given in Fig. 1.

Corollary 1: For Rayleigh fading channels the expectation $\mathbb{E}\left\{D_{\alpha_i} D_{\beta_j}\right\}$ results in

$$\mathbb{E}\left\{D_{\alpha_i} D_{\beta_j}\right\} = \frac{m+1}{m-k+1} \mu_D(\alpha_i) \mu_D(\beta_j) \tag{19}$$

where $\mu_D(\alpha_i) \triangleq \mathbb{E}\left\{D_{\alpha_i}\right\}$ and $\mu_D(\beta_j) \triangleq \mathbb{E}\left\{D_{\beta_j}\right\}$.

Proof: From (14) and (15), the terms in (18) averaged over Rayleigh fading can be expressed as

$$\mathbb{E}\left\{\det(\boldsymbol{W}_{\gamma_k})^2\right\} = \frac{m!(m+1)!}{(m-k)!(m-k+1)!} \tag{20}$$

$$\mathbb{E}\left\{\det(\breve{\boldsymbol{W}}_{\alpha_i \setminus \gamma_k})\right\} = \frac{(m-k)!}{(m-i)!} \tag{21}$$

$$\mathbb{E}\left\{\det(\breve{\boldsymbol{W}}_{\beta_j \setminus \gamma_k})\right\} = \frac{(m-k)!}{(m-j)!}. \tag{22}$$

After some mathematical manipulations, $\mathbb{E}\left\{D_{\alpha_i} D_{\beta_j}\right\}$ in (18) can be written for Rayleigh fading channels as (19). □

The results above, together with the counting of the number of partitions α_i and β_j that have an overlapping set with cardinality k, enable us to express the variance of the determinant D of $\boldsymbol{W}$ as

$$\sigma_D^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\rho}{n_\mathrm{T}}\right)^{i+j} \binom{n}{i} \tag{23}$$
$$\sum_{k=\max\{0,i+j-n\}}^{\min\{i,j\}} \frac{k}{m-k+1} \binom{n-i}{j-k} \binom{i}{k} \mu_D(\alpha_i) \mu_D(\beta_j).$$

Since

$$\binom{n}{i}\binom{n-i}{j-k}\binom{i}{k} = \frac{n!}{k!(i-k)!(j-k)!(n-i-j+k)!}$$

the (23) becomes

$$\sigma_D^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\rho}{n_\mathrm{T}}\right)^{i+j} c_{i,j}\, \mu_D(\alpha_i) \mu_D(\beta_j) \tag{24}$$

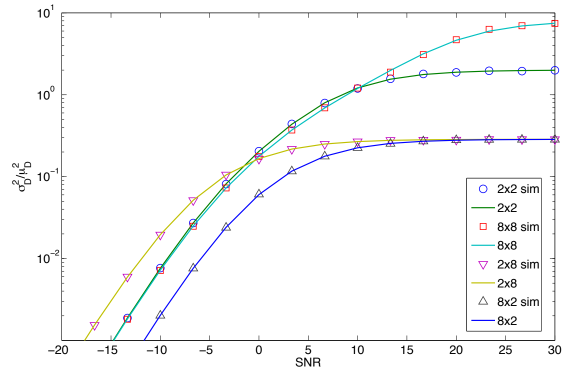


Fig. 2. σ_D^2/μ_D^2 as function of the SNR for different MIMO configurations.

where

$$c_{i,j} \triangleq \sum_{k=\max\{0,i+j-n\}}^{\min\{i,j\}} \frac{k}{m-k+1} \frac{(n-i-j+k+1)\ldots(n-1)n}{k!(i-k)!(j-k)!}. \tag{25}$$

By using (16) and (24) in (9)–(12) we obtain the closed-form approximations of the ergodic and outage capacity of MIMO systems operating in Rayleigh fading for various settings.

Fig. 2 shows σ_D^2/μ_D^2 as function of the SNR ρ for different MIMO settings. It can be appreciated the accuracy of (16) and (24) with respect to simulations. We can see that σ_D^2/μ_D^2 tends to zero for ρ approaching to zero and achieves its maximum constant value when ρ approaches infinity. As already pointed out, when σ_D^2/μ_D^2 is small, the approximations (9) and (12) are expected to be tight to simulations. Results of this kind can be used to identify for which sets of system parameters the approximations are reliable. The most critical region for the approximations is for high SNR when the MIMO is almost squared ($n_\mathrm{R} = n_\mathrm{T}$). This is confirmed by observing, from (16) and (24), that σ_D^2/μ_D^2 approaches $n/(m-n+1)$ for large ρ and approaches $nm\rho^2/n_\mathrm{T}^2$ for small ρ. Note that σ_D^2/μ_D^2 is small when ρ is tending to 0, or for any ρ when m is sufficiently large with respect to n, e.g. for the 8x2 and 2x8 MIMO settings in the figure.

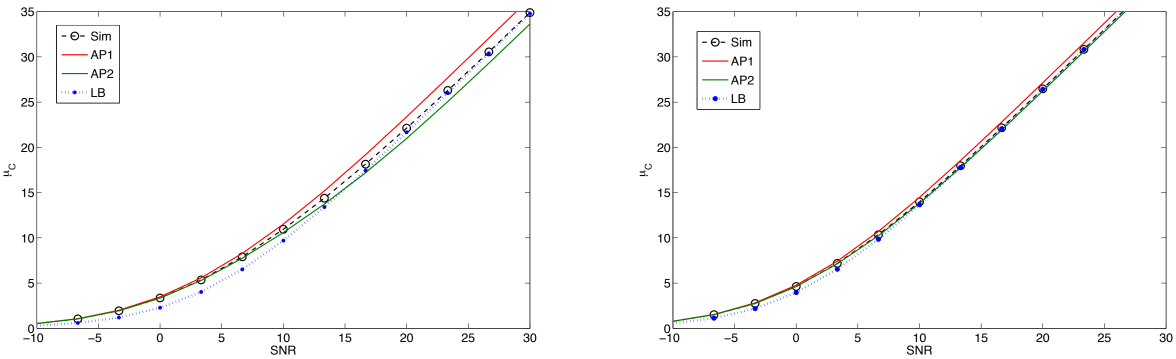
IV. Other Known Bounds

The upper bound $\mu_C^{(1)} = \log_2 \mu_D \geq \mu_C$ was already considered in [11], which also proposed a simple lower-bound for ergodic capacity in MIMO channels. This lower bound was derived by extending the asymptotic expression of μ_C for large SNR obtained in [8], which is given for Rayleigh fading channels by

$$\mu_{C,\mathrm{L}} = \sum_{j=1}^{n} \log_2\left(1 + \frac{\rho}{n_\mathrm{T}} e^{\psi(m-j+1)}\right) \tag{26}$$

where $\psi(i) = \sum_{p=1}^{i-1}\left(\frac{1}{p} - \gamma\right)$ and γ is the Euler number.

An upper bound for the variance of $C(D)$ is obtained by considering the asymptotic expression of σ_C^2 for large SNR as



(a) 4x4 MIMO

(b) 4x6 MIMO

Fig. 3. Ergodic capacity, μ_C, as function of SNR for a 4x6 MIMO system. AP1 refers to $\mu_{\mathrm{C},1}$, AP2 refers to $\mu_{\mathrm{C},2}$, LB refers to $\mu_{\mathrm{C},L}$.

derived in [8], given by

$$\sigma^2_{C,\mathrm{U}} = (\log_2 e)^2 \sum_{j=0}^{n-1} \dot{\psi}(m-j) \tag{27}$$

where $\dot{\psi}(i) = \dot{\psi}(i-1) - (i-1)^{-2}$ and $\dot{\psi}(1) = \pi^2/6$.

Remark: It is worth noting that asymptotic bounds can also be exploited to refine the approximations of μ_C and σ^2_C introduced in the previous Section, as follows:

$$\tilde{\mu}_{C,2} = \max\left\{\log_2 \mu_D - \frac{\log_2 e}{2}\frac{\sigma^2_D}{\mu^2_D}, \mu_{C,\mathrm{L}}\right\} \tag{28}$$

$$\tilde{\sigma}^2_{C,1} = \min\left\{(\log_2 e)^2 \frac{\sigma^2_D}{\mu^2_D}, \sigma^2_{C,\mathrm{U}}\right\}. \tag{29}$$

V. Results

In this Section we present results that illustrate the accuracy of the closed-form approximated evaluation of μ_C, σ^2_C and outage capacity $C^\star$ compared with simulation and known bounds. We plot these parameters as function of SNR, by considering two different MIMO configurations. The first setting refers to a 4x4 MIMO system representing the case where $n_\mathrm{R} = n_\mathrm{T}$, which is expected to be the most critical case for large SNR. The second setting refers to a 4x6 MIMO system representing a case where $m-n$ is a sufficiently large fraction of n, which is expected to lead to tight approximation.

We observe from Figures 3, 4, and 5 that the approximations are asymptotically tight for small SNRs. From the Figures 3(b), 4(b), and 5(b) we also note that first and second order approximations of μ_C and σ^2_C still work well, even for large SNRs. Figures 3(a), 4(a), and 5(a) highlight the limitations of the proposed approximations. We can see that at large SNRs when $m = n$ the values of $\tilde{\sigma}^2_{\mathrm{C},2}$ loose accuracy, and in general the second order approximations do not improve the first order ones. On the other hand, in this SNR region the upper bound for σ^2_C and the lower bound for μ_C, both asymptotically tight, can be easily exploited.

Remark: The outage capacity (Fig. 5) can be well approximated in all the SNR regions by the proposed closed-form expressions, with few limitations in the high SNR region when the MIMO is squared. In such particular case, good approximations are those of order 1.

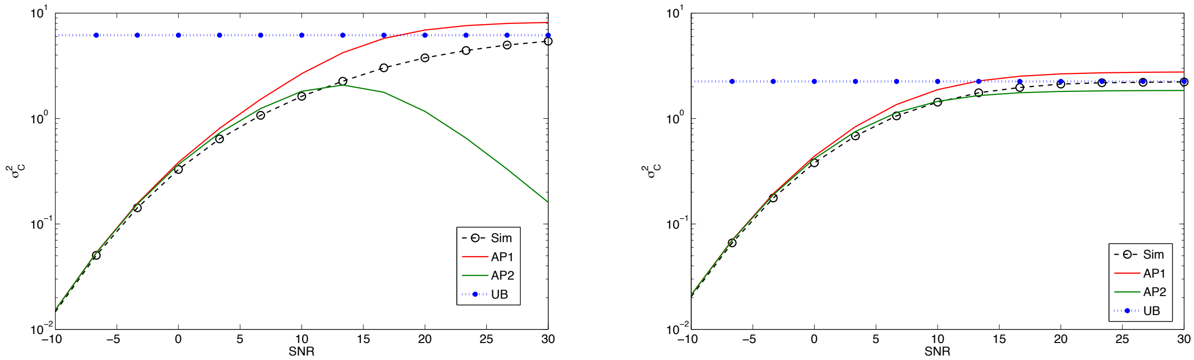
TABLE I
Normalized errors on μ_C and σ^2_C for different approximations and settings: $\delta_{\mathrm{M},1} = (\mu_{C,1} - \mu_C)/\mu_C$; $\delta_{\mathrm{M},2} = (\mu_{C,2} - \mu_C)/\mu_C$; $\delta_{\mathrm{V},1} = (\sigma^2_{C,1} - \sigma^2_C)/\sigma^2_C$; and $\delta_{\mathrm{V},2} = (\sigma^2_{C,2} - \sigma^2_C)/\sigma^2_C$.

n_T	n_R	μ_C	$\delta_{\mathrm{m},1}$	$\delta_{\mathrm{m},2}$	σ^2_C	$\delta_{\mathrm{v},1}$	$\delta_{\mathrm{v},2}$
1	1	9.14	0.091	0.012	3.29	-0.37	-0.53
2	2	17.75	0.066	-0.014	4.42	-0.063	-0.53
3	3	26.35	0.053	-0.028	5.13	-0.20	-0.69
4	4	34.95	0.043	-0.037	5.31	0.54	-0.97
6	6	52.01	0.035	-0.045	5.85	1.05	-1.90
1	6	12.43	0.0097	0.000040	0.39	-0.11	-0.15
2	6	22.58	0.012	-0.0011	0.84	-0.014	-0.11
3	6	31.61	0.014	-0,0032	1.4	0.11	-0.093
4	6	39.63	0.018	-0.0058	2.25	0,23	-0.18
16	20	150.93	0.0078	-0.00761	2.25	0,.82	-0.57

Finally, to gain more insights on the behavior of different approximations for μ_C and σ^2_C in the high SNR region we have collected in Table I the values of μ_C and σ^2_C together with an estimate of the the accuracy of the different approximations in terms of normalised errors for different MIMO configurations at $\rho = 30$ dB. It can observed that the second order approximation $\mu_{\mathrm{C},2}$ behaves as a lower bound of μ_C when $n > 1$. It generally improves $\mu_{\mathrm{C},1}$, with exception of cases where $m - n$ is zero or small and n is large. Also, the first order approximation $\sigma^2_{\mathrm{C},1}$ behaves as an upper bound of σ^2_C when $n > 2$. Moreover, the second order approximation $\tilde{\sigma}^2_{\mathrm{C},2}$ behaves as a lower bound of σ^2_C at large SNRs and improves $\sigma^2_{\mathrm{C},1}$ only when both $m - n$ and n are not small. By also looking at the results for σ^2_D/μ^2_D in Figure 2 we may conclude that in general the proposed closed-form approximations are tight when σ^2_D/μ^2_D is smaller than 1-2. When σ^2_D/μ^2_D is large, second order approximations are not useful, while first order approximation still works well but should be handled carefully.

VI. Final Remarks

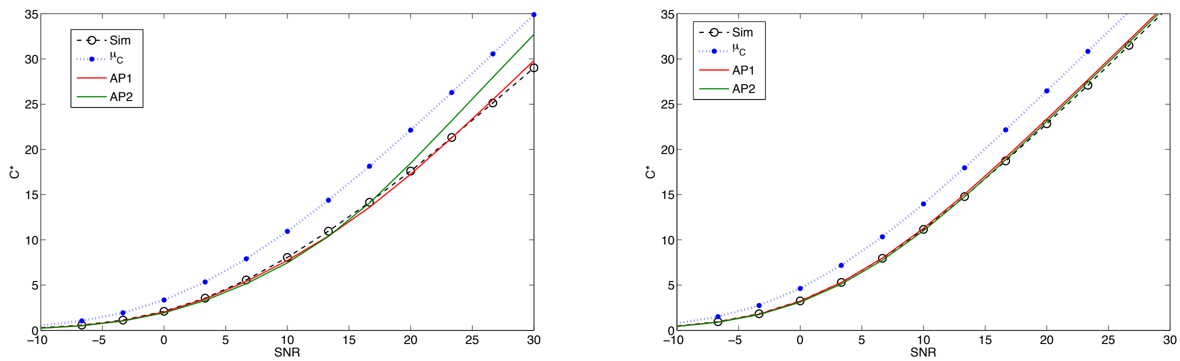
Building up on minors moments of complex Wishart matrices, closed-form approximated expressions for the outage capacity of MIMO systems operating in Rayleigh fading channels are derived. The methodology proposed not only



(a) 4x4 MIMO

(b) 4x6 MIMO

Fig. 4. Variance of capacity, σ^2_C, as function of SNR for a 4x6 MIMO system. AP1 refers to $\sigma^2_{\mathrm{C},1}$, AP2 refers to $\tilde{\sigma}^2_{\mathrm{C},2}$, UB refers to $\sigma^2_{\mathrm{C},U}$.



(a) 4x4 MIMO

(b) 4x6 MIMO

Fig. 5. Outage capacity, $C^\star$, as function of SNR for a 4x6 MIMO system with $P_{\text{out}} = 10^{-2}$. AP1 refers to the evaluation using $\mu_{\mathrm{C},1}$ and $\sigma^2_{\mathrm{C},1}$, AP2 refers to the evaluation using $\mu_{\mathrm{C},2}$ and $\tilde{\sigma}^2_{\mathrm{C},2}$.

provides closed-form expressions of the outage capacity, but it can also be extended to analyze MIMO networks with multiple sources. The approximations derived are compared with the capacity obtained through Monte Carlo simulations and that obtained using known bounds. Such comparison confirms the validity of the proposed methodology for a broad range of MIMO settings - i.e., for different SNR values and number of antennas - including massive MIMO.

REFERENCES

[1] A. M. Tulino and S. Verdú, "Random matrix theory and wireless communications," *Foundations and Trends in Communications and Information Theory*, vol. 1, no. 1, pp. 1–182, Jun. 2004.

[2] A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection—An overview," *IEEE Microw. Mag.*, vol. 5, no. 1, pp. 46–56, Mar. 2004.

[3] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[4] K. T. Truong and R. W. Heath, Jr., "Effects of channel aging in massive MIMO systems," *J. of Commun. and Netw.*, vol. 15, no. 4, pp. 338–351, Aug. 2013.

[5] J. H. Winters, "On the capacity of radio communication systems with diversity in Rayleigh fading environment," *IEEE J. Sel. Areas Commun.*, vol. 5, no. 5, pp. 871–878, Jun. 1987.

[6] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[7] İ. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[8] A. Grant, "Rayleigh fading multi-antenna channels," *EURASIP J. on Adv. in Signal Proc.*, vol. 2002, no. 3, pp. 316–329, 2002.

[9] H. Shin, M. Z. Win, and J. H. Lee, "Saddlepoint approximation to the outage capacity of MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 10, pp. 2679–2684, Oct. 2006.

[10] M. Kang and M.-S. Alouini, "Capacity of MIMO Rician channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 1, pp. 112–122, Jan. 2006.

[11] S. Jin, X. Gao, and X. You, "On the ergodic capacity of rank-1 Ricean-fading mimo channels," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 502–517, Feb. 2007.

[12] M. Chiani, M. Z. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh fading channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2363–2371, Oct. 2003, special issue on *The Academic and Industrial Embrace of Space-Time Methods*.

[13] A. Lozano, A. M. Tulino, and S. Verdú, "Multiple-antenna capacity in the low-power regime," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2527–2544, Oct. 2003.

[14] P. Smith and M. Shafi, "An approximate capacity distribution for MIMO systems," *IEEE Trans. Commun.*, vol. 52, no. 6, pp. 887–890, Jun. 2004.

[15] J. Perez, J. Ibanez, L. Vielva, and I. Santamaria, "Closed-form approximation for the outage capacity of orthogonal STBC," *IEEE Commun. Lett.*, vol. 9, no. 11, pp. 961–963, Nov. 2005.

[16] A. C. Aitken, *Determinants and Matrices*. Edinburgh: Oliver and Boyd, 1944.

[17] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*. New York: Wiley, 1982.

[18] M. Drton, H. Massam, and I. Olkin, "Moments of minors of Wishart matrices," *The Annals of Statistics*, vol. 36, pp. 2261–2283, May 2008.