ONLINE ESTIMATION OF A TIME-VARYING DELAY BASED ON A UNIVARIATE CROSS-AMBIGUITY FUNCTION ANALYSIS

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ABSTRACT

This paper presents a method to estimate a linearly time-varying delay between two continuous signals. The joint estimation of the time delay and Doppler shift by analyzing the cross-ambiguity function is state of the art, however, this method has high computational demands as it relies on a bivariate search. It is shown that, by using previous estimation results to initialize the analysis, a similar result can already be obtained with a univariate search, as is then sufficient to search for the variation of the delay since the last measurement. Perfect tracking is obtained with proper initialization, and for the case of incorrect initialization, convergence can be guaranteed and even influenced with a tuning parameter. A theoretical analysis and a numerical example illustrate the performance of the proposed method.

Index Terms— time-varying delay estimation, Doppler shift, wideband cross-ambiguity function.

1. INTRODUCTION

The estimation of the delay between two time-dependent signals is of high practical relevance. It is a key technique used, for instance, in speed measurement applications, in time-of-flight sensing systems, or in the analysis of some branches of natural sciences.

The fundamental method for the delay estimation between two time signals is the cross-correlation [1]. An analysis with a correlation algorithm and a maximum search leads to a reliable estimate of the delay, the result is quasi unaffected by measurement noise or variations in the signal amplitude. However, it is based on the three assumptions of constant delay, stationary process, and long observation interval.

In the practice, a time-varying delay is a problem. Standard methods can only handle quasi-constant delays [2]. The variation of the delay leads to a Doppler shift that affects the frequency components of the delayed signal, creating a frequency bias between both signals [3].

Fig. 1 illustrates the problem of time varying delays: In the section where the delay is increasing, the frequency of the delayed signal becomes lower. Also, the duration of the considered waveform becomes longer. Therefore, a delay estimation has to take into account the Doppler shift to prevent a mismatch in the cross-correlation.

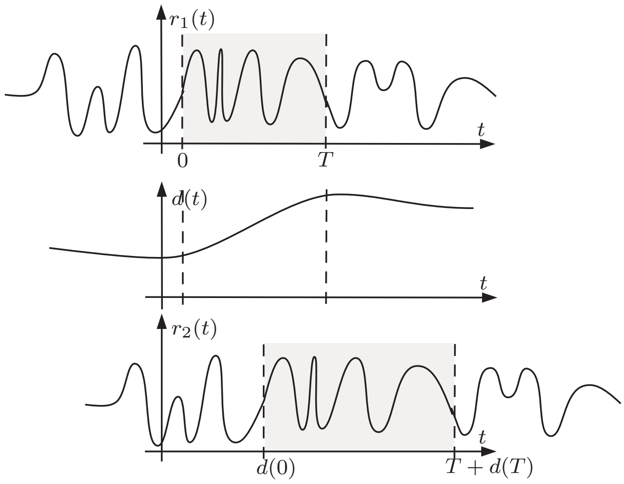

Fig. 1. A waveform subject to a non-constant delay.

Several techniques have already been proposed to determine time-varying delays. The maximum-likelihood approach analyses the probability that a hypothetical delay d caused the received signals. In [3], a parameter estimation method is applied, although a constant delay is searched, it is converging for time-varying delays. A design of the 'exact' maximum-likelihood estimator for time-varying delays $d(t)$ exists [4], however, the solution of a set of difficult differential equations is required. So far, due to a difficult implementation and a higher noise sensitivity, only a handful of application-specific implementations have been reported [5, 6].

The most widespread methods today are based on the cross-ambiguity function (CAF: frequency shift) [7] or the wideband cross-ambiguity function (WBCAF: time axis scale) [8, 9]. The delayed signal is analyzed jointly for two constant parameters, the delay and the Doppler shift. Thereby, a linearly varying time delay is assumed. It can be considered as an extension of the cross-correlation function. Generally, both parameters are determined with a bivariate (two-dimensional) search.

While the results are quite good and the method is sufficiently simple to reach a wide number of practical applica-

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tions, the computational requirements are high. The reasons are the two-dimensional full-enumeration search and the need for multirate signal processing.

Later extensions with Wavelets [10] and with efficient numerical searches [11] managed to reduce the computational demands, however, also led to an increased implementation complexity. In multi-path sensing, where the received signal is a sum of several delayed signals, more application-specific solutions to the computational problem have been proposed. The search was split into a two-step procedure under the assumption of small frequency offsets [12], and an a-priori known emitted waveform has been imposed [13].

In this paper, the WBCAF is analyzed based on a univariate (one-dimensional) search. The application area is ”online” delay estimation, where two waveforms are sequenced into intervals and successively analyzed for their delay. Here, the estimated delay of the previous interval can be used to initialize the search. It is then sufficient to search for the variation of the delay since the last measurement.

2. METHOD: UNIVARIATE WBCAF ANALYSIS

The observed data consists of two waveforms, the second being delayed,

$$r_1(t) = s(t), \qquad t \in [0;T], \tag{1}$$

$$r_2(t) = s(t - d(t)), \quad t \in [0;T]. \tag{2}$$

The waveforms can be subject to additive (uncorrelated) noise and may have a different magnitude, not modeled for conciseness. The analysis is performed sequentially on intervals of length T. For a concise notation, the analysis is assumed to start at $t = 0$. The delay is assumed positive $d(t) \geq 0$.

2.1. Delay model and Doppler shift

The delay is modeled to account for the rate of change of the delay d_Δ, via

$$d(t) = d_0 + d_\Delta t, \quad t \in [0;T]. \tag{3}$$

As in previous works on the WBCAF, the time delay and the Doppler shift are assumed as constant parameters [8, 9]. The parameter d_0 is the constant part of the delay and also the starting point of the delay $d(t = 0) = d_0$. With the additional parameter d_Δ, the second waveform reads as

$$r_2(t) = s(t - d_0 - d_\Delta t) \tag{4}$$

$$= s((1 - d_\Delta)t - d_0) \tag{5}$$

$$= s(\alpha t - d_0). \tag{6}$$

To simplify the notation, the parameter

$$\alpha = 1 - d_\Delta, \tag{7}$$

which represents the scaling of the time axis caused by the Doppler shift, is introduced. A practical restriction is $-\frac{d_0}{T} \leq d_\Delta < 1$, as $d(t) \geq 0$ and $\alpha > 0$.

2.2. Interval section

For a constant delay d, the signals $r_1(t)$ and $r_2(t)$ are considered over a window $t \in [0, T]$ and $t \in [d, T + d]$, respectively, such that the waveform (referring to $s(t)$) is identical [2].

For a time-varying delay $d(t)$, however, as indicated in Fig. 1, the window duration and the spectrum of $r_2(t)$ are affected. For an increasing delay, the delayed waveform is uniformly stretched and the interval becomes longer.

The signal $r_1(t)$ is considered over $[0;T]$. Following the model (2), (3), the same waveform is found in $r_2(t)$ over the interval $\left[\frac{d_0}{\alpha}; \frac{d_0+T}{\alpha}\right]$.

The scaling of the time axis through the Doppler shift has an influence on the sampling points. Consequently, multirate signal processing is required for the analysis of $r_2(t)$ [11].

2.3. Cross-ambiguity function

For each possible delay $\hat{d}_0$ and delay variation rate $\hat{d}_\Delta$, the Doppler-correction is applied to $r_2(t)$, the signal is resampled and the interval selected. The probability of a given $\hat{d}_0$ and $\hat{\alpha}$ between $r_1(t)$ and $r_2(t)$ is obtained via the wideband cross-ambiguity function

$$r_1 * r_2(\hat{d}_0, \hat{\alpha}) = \int_0^T r_1(\tau) r_2\left(\frac{\tau + \hat{d}_0}{\hat{\alpha}}\right) \mathrm{d}\tau. \tag{8}$$

A maximum search over the results should then lead to the delay variation rate as defined in (3).

2.4. One-dimensional search and convergence

Assuming that the delay has already been determined for the interval $[-T;0]$ on $r_1(t)$, the initial value of $d(t = 0) = d_0$ is already known from this previous measurement. By using this knowledge, the only unknown in the delay (3) is the delay variation rate d_Δ.

A theoretical study in section 3 will lead to more insight to this idea. In the ideal case where the initialization of d_0 is correct, the WBCAF (8) will lead to the correct results for d_Δ too. However, if the initial delay d_0 has an offset to the real value, which is possible due to noise, a low-resolution search, or approximations in the multirate signal processing, convergence may not be satisfactory. Therefore, the one-dimensional search is not solely performed over d_Δ, but instead, the initial delay d_0 is also adjusted by

$$\hat{d}_0 = \tilde{d}_0 + k \cdot T \cdot \hat{d}_\Delta. \tag{9}$$

Here, $\hat{d}_0$ and $\hat{d}_\Delta$ denote the search parameters and $\tilde{d}_0$ the initialization value. While this adaptation law leads to a small error, it improves convergence in the presence of initialization errors. In the case of a constant delay, it has no influence as $d_\Delta = 0$. The parameter k should be positive and within the range

$$0 \leq k \leq 1. \tag{10}$$

To initialize the search for the following interval, thus for $[T;2T]$, the predicted delay $d(T) = \hat{d}_0 + \hat{d}_\Delta \cdot T$ will be applied, which includes the correction (9).

3. CONVERGENCE ANALYSIS

The analysis in this section shall further explain the principle of the proposed delay estimation method, and demonstrate its convergence. As notation, the values d_0 and d_Δ denote the real values, $\hat{d}_0$ and $\hat{d}_\Delta$ denote the values assumed in the search, and $\tilde{d}_0$ denotes the initialization value.

Four cases can be separated:

case 1	$\hat{d}_0 = d_0$	$\alpha = 1$
case 2	$\hat{d}_0 = d_0$	$\alpha \neq 1$
case 3	$\hat{d}_0 \neq d_0$	$\alpha = 1$
case 4	$\hat{d}_0 \neq d_0$	$\alpha \neq 1$

As long as $\hat{d}_0 = d_0$, meaning, if the initial value of the delay is properly known, the reduction to a univariate search is not a restriction. A two-dimensional search over the WBCAF would have its maximum on the line d_0 anyway. This is valid if the delay did not change (case 1) as well as for a variation since the previous interval (case 2).

However, the main question is the result for a non-correctly assumed d_0, as the interval of $r_2(t)$ will be sectioned wrong. The following section therefore discusses the cases 3 and 4.

During the implementation of various WBCAF algorithms, it was found that, in general, the maximum of the WBCAF is not just located in one point. Instead, in the $(\hat{d}_0, \hat{d}_\Delta)$ pane, a maximum appears that is distributed over a line. This means that, if the error on $\hat{d}_0$ is not too high, a clear maximum of the WBCAF can still be recognized in a univariate search. This maximum will, however, be smaller than the autocorrelation value of $r_1(t)$, and inherit an error on $\hat{d}_\Delta$.

3.1. Analysis of the WBCAF

The shifted and time-scaled signal $r_2(t)$ in the WBCAF (8), following (2), reads as

$$r_2\left(\frac{\tau + \hat{d}_0}{\hat{\alpha}}\right) = s\left(\frac{\alpha}{\hat{\alpha}}\tau + (\frac{\alpha}{\hat{\alpha}}\hat{d}_0 - d_0)\right). \tag{11}$$

To simplify the notation, the two parameters $q = \frac{\alpha}{\hat{\alpha}}$ and $p = \frac{\alpha}{\hat{\alpha}}\hat{d}_0 - d_0$ are introduced. Further, the Fourier transform of $s(t)$ is denoted as $S(\omega)$ and $\hat{S}(\omega)$ for $t \in [0;T]$ and for $t \in [p; qT + p]$, respectively.

The WBCAF can be analyzed, analogously to the cross-correlation theorem, in the Fourier-space:

$$r_1 * r_2(\hat{d}_0, \hat{\alpha})$$

$$= \int_0^T r_1(\tau) r_2(\frac{\tau + \hat{d}_0}{\hat{\alpha}})\,\mathrm{d}\tau \tag{12}$$

$$= \int_0^T s(\tau) s(qt + p)\,\mathrm{d}\tau \tag{13}$$

$$= \int_0^T \left(\int_{-\infty}^{+\infty} \bar{S}(\omega) e^{+2\pi j \omega \tau} \mathrm{d}\omega \right) \cdot \left(\int_{-\infty}^{+\infty} \hat{S}(\omega') e^{-2\pi j \omega' (qt+p)} \mathrm{d}\omega' \right) \mathrm{d}\tau \tag{14}$$

$$= \int_0^T \iint_{-\infty}^{+\infty} \bar{S}(\omega)\hat{S}(\omega') e^{+2\pi j(\omega\tau - \omega' q \tau - \omega' p)} \mathrm{d}\omega' \mathrm{d}\omega \mathrm{d}\tau \tag{15}$$

$$= \iint_{-\infty}^{+\infty} \bar{S}(\omega)\hat{S}(\omega') e^{-2\pi j \omega' p} \cdot \left(\int_0^T e^{+2\pi j(\omega\tau - \omega' q\tau)} \mathrm{d}\tau \right) \mathrm{d}\omega' \mathrm{d}\omega \tag{16}$$

$$= \iint_{-\infty}^{+\infty} \bar{S}(\omega)\hat{S}(\omega') e^{-2\pi j \omega' p} \delta(q\omega' - \omega) \mathrm{d}\omega' \mathrm{d}\omega \tag{17}$$

$$= \int_{-\infty}^{\infty} \bar{S}(\omega)\hat{S}\left(\frac{\omega}{q}\right) e^{-2\pi j \omega \frac{p}{q}} \mathrm{d}\omega. \tag{18}$$

Some simlarity between $S(\omega)$ and $\hat{S}\left(\frac{\omega}{q}\right)$ can be assumed. In the interval $[0;T]$, the signal can be assumed as stationary, i.e., its magnitude spectrum is not changing if the interval is slightly shifted. The cross-correlation theorem can be completed to obtain

$$r_1 * r_2(\hat{d}_0, \hat{\alpha}) = \mathrm{F}\{|S(\omega)|^2\}\left(\frac{p}{q}\right) \tag{19}$$

$$= \mathrm{F}\{|S(\omega)|^2\}\left(\hat{d}_0 - \frac{\hat{\alpha}}{\alpha} d_0\right). \tag{20}$$

This result indicates that the maximum of the WBCAF is on the line

$$\hat{d}_0 = \frac{\hat{\alpha}}{\alpha} d_0. \tag{21}$$

This implies that, in any univariate search over the WBCAF, the errors on $\hat{d}_0$ and $\hat{\alpha}$ are connected via $\frac{\hat{d}_0}{d_0} = \frac{\hat{\alpha}}{\alpha}$.

3.2. Result with non-correct initialization d_0

The initialization error on d_0 is defined as as ϵ,

$$\tilde{d}_0 = d_0 + \epsilon. \tag{22}$$

With (21), the impact of limiting the search on one dimension (by imposing $\hat{d}_0 = \tilde{d}_0$) on the error of the Doppler scaling follows as

$$\hat{\alpha} = \alpha + \frac{\alpha}{d_0}\epsilon, \tag{23}$$

and the delay variation rate as

$$\hat{d}_\Delta = d_\Delta - \frac{\epsilon}{d_0}(1 - d_\Delta). \tag{24}$$

This implies that, if the initial delay $\tilde{d}_0$ is too high ($\epsilon > 0$), the estimated delay variation will be smaller than the real value ($\hat{d}_\Delta < d_\Delta$). Consequently, the initialization error for the following measurement will become smaller. The opposite is happening for a negative ϵ. This implies that the direct application of the univariate search for $\hat{\alpha}$ is generally converging, however, the result may not be satisfactory.

To improve this, the univariate search is not done along $\hat{d}_0 = \tilde{d}_0$, but along the line defined by (9). To analyze the impact, the WBCAF maximum (21) and the search line (9) are solved to find

$$\hat{\alpha} = \frac{\alpha}{d_0}(d_0 + \epsilon + k \cdot T \cdot \hat{d}_\Delta). \tag{25}$$

The resulting delay variation rate $\hat{d}_\Delta$ is

$$\hat{d}_\Delta = \frac{d_\Delta - \frac{\epsilon}{d_0}(1 - d_\Delta)}{1 + \frac{kT}{d_0}(1 - d_\Delta)}. \tag{26}$$

Compared to (24), the influence of a wrong initialization ϵ to an error on the estimated delay variation rate $\hat{d}_\Delta$ will be affected. Consequently, the initialization value for the following measurement will be affected, such that convergence can be properly adjusted.

In the practice, for k, a value of $k = 0.2$ has shown good results. Small values have only a small influence, whereas high values may lead to oscillations in the estimation results. A proper tuning of k is a tradeoff between the handling of the initialization error in a given application and the inherited estimation error caused by (9).

4. NUMERICAL EXAMPLE

This constructed example, shown in Fig. 2, considers a continuously estimated delay subject to some time variation. This is a typical problem, for instance, in industrial speed estimation applications. A bat chirp serves as an example signal. Although it is a strongly periodic signal, the frequency components are varying with time. The signal can be considered as non-stationary.

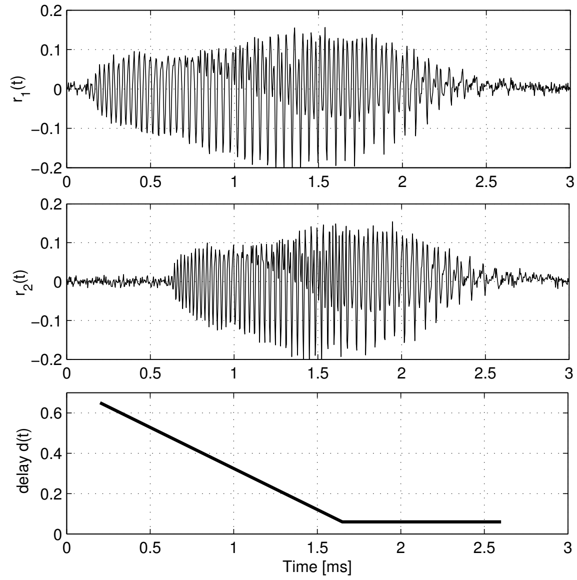


Fig. 2. Top & middle: signals $r_1(t)$ and $r_2(t)$. Bottom: original delay $d(t)$.

The signal is analyzed in intervals of 0.2 ms successively. Noise with a magnitude 0.01 (5% of the max. magnitude) is added to the signals. In the bottom plot on Fig. 2, the original delay trajectory $d(t)$ is shown. It is constantly decreasing until 1.6 ms and then remains constant. The results of four different methods are shown in Fig. 3.

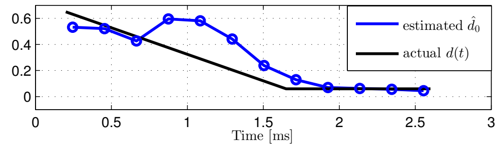
In Fig. 3 (a), the results of cross-correlation as in [1] are shown. Due to a frequency bias between both waveforms, there is a considerable delay mismatch. During the constant delay phase, the results become very reliable as the search algorithm has few degrees of freedom.

In Fig. 3 (b), the results of the bivariate (two-dimensional) WBCAF as in [8] are shown. The results are good for both parameters $\hat{d}_0$ and $\hat{d}_\Delta$. The only disadvantage of this method are the high computational demands.

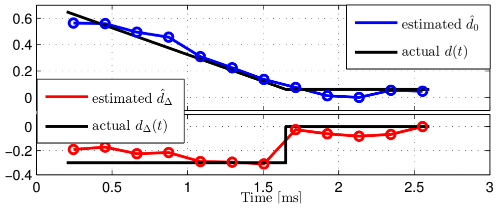
In Fig. 3 (c), the results of the univariate WBCAF are shown. The search is only done for $\hat{d}_\Delta$, assuming that the initialization value of $\tilde{d}_0$ is correct. The estimation of $\hat{d}_0$ has a lag during the first phase, but converges during the constant phase. The estimation of the delay variation rate $\hat{d}_\Delta$ is not usable.

In Fig. 3 (d), the results of the univariate WBCAF with the improved convergence with eq. (9) are shown. A parameter $k = 0.2$ has been used. Here, the estimation of $\hat{d}_0$ is of the same quality as with the bivariate WBCAF analysis. Again, the estimation of $\hat{d}_\Delta$ is not usable, however, this value could be derived from $\hat{d}_0$.

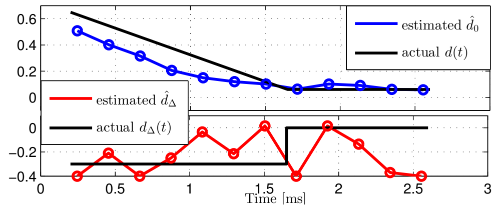
The noise sensitivity of the proposed method was found to be slightly higher than for the bivariate WBCAF. High noise results in a poor quality of $\hat{d}_\Delta$ and can lead to an estimation instability when analyzing strongly periodic signals.



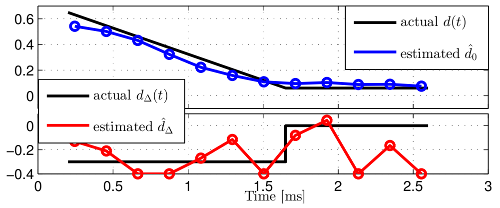
(a) Conventional cross-correlation



(b) Two-dimensional WBCAF analysis



(c) Univariate WBCAF analysis searching only for $\hat{d}_\Delta$



(d) Univariate WBCAF anal. with improved convergence

Fig. 3. Results of the numerical example.

5. CONCLUSIONS

This paper presents a method to estimate a linearly time-varying delay between two continuous signals. The joint estimation of the time delay and Doppler shift with a cross-ambiguity function is state of the art, however, this method has high computational demands.

As a remedy, a univariate search has been proposed. It assumes a sequential delay estimation, such that existing knowledge is available for an initialization. With a correct initialization and if the linear delay model fits, perfect tracking is obtained. In any other case, reliable convergence is obtained. The rate of convergence can be influenced with a tuning parameter, as was shown in theory as well as in a numerical example.

Some limitations apply to the results as a consequence of the univariate search. The compensation of initialization errors is a tradeoff, leading to a lower precision of the results, especially of the delay variation rate d_Δ.

Nevertheless, the estimated delay was found to have a similar quality as with the 2D-WBCAF method. The computational efficiency is good, and the implementation is not more complicated than the widespread 2D-WBCAF methods.

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