ROOM IMPULSE RESPONSE ESTIMATION USING PERFECT SEQUENCES FOR LEGENDRE NONLINEAR FILTERS

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ABSTRACT

The paper proposes a novel method for room impulse response estimation that is robust towards nonlinearities affecting the power amplifier or the loudspeaker of the measurement system. The method is based on measurements of the first order kernel of the Legendre nonlinear filter modeling the acoustic path. In the proposed approach, the first order kernel is efficiently estimated with the cross-correlation method using perfect periodic sequences for Legendre filters. Perfect sequences with period suitable for room impulse response identification are also developed within the paper. Simulation results in a realistic scenario illustrate the effectiveness and robustness towards nonlinearities of the proposed approach.

***Index Terms*—** Room impulse response, Legendre nonlinear filters, perfect periodic sequences, cross-correlation method

1. INTRODUCTION

Room impulse response (RIR) estimation is an important tool in audio processing. It is used for analyzing and characterizing the room response (measuring parameters like reverberation time, early decay time, definition, clarity, center time, etc.), spatial sound rendering, virtual audio, room response equalization, active noise control, room geometry inference, and many other applications. Different techniques have been proposed for RIR estimation. The most popular ones have been the maximal length sequence (MLS), the time-stretched pulse, the time delay spectrometry [1], the perfect periodic sequences (PPSs) for linear filters [2], and the exponential sweep technique [3]. Recently, perfect exponential sweeps have also been proposed [4]. While the room acoustic path can be considered as linear system, the power amplifier and the loudspeaker (PAL) system used to reproduce the test signals are often affected by nonlinearities, caused by the high reproduction levels used to contrast noise. As a result many of the RIR measurement techniques suffer from the nonlinearities. From this point of view, a remarkable technique is that based on exponential sweeps, which is immune to nonlinearities if the acoustic path can be modeled as a memoryless nonlinearity followed by a linear filter [3], [5]. Unfortunately, memoryless nonlinearities rarely occur and also the exponential sweep technique has been proved vulnerable to nonlinearities [6], [7].

In this paper, a novel technique for RIR estimation that is robust towards the nonlinearities that may affect the PAL system is proposed. To account for these nonlinearities the PAL system is modeled as a Legendre nonlinear filter [8], [9]. Legendre filters are polynomial filters, whose basis functions are product of Legendre polynomials of the input signal samples, and they have a first order kernel which is a linear filter. As the well known Volterra filters, Legendre filters can arbitrarily well approximate any discrete-time, causal, time-invariant, finite-memory, continuous nonlinear system. In contrast to Volterra filters, their basis functions are orthogonal for white uniform input signals in $[-1, +1]$. It was also shown in [9], [10] that PPSs can also be developed for these filters. PPSs are periodic sequences that guarantee the orthogonality of the basis functions on a finite period. Using a PPS as input signal, a Legendre filter can be efficiently identified with the cross-correlation method, i.e., computing the cross-correlation between each basis function and the system output. It is shown in the paper that the RIR can be recovered measuring with PPSs the first order kernel of the acoustic path. Unfortunately, the sequences developed in [9], [10] cannot be used for this purpose, because their period increases geometrically with the length of the filter. Even for low lengths the period can be prohibitively large. Nevertheless, since only the first order kernel has to be measured in this application, the constraints for the development of the PPS can be reduced. In this way, PPSs suitable for RIR estimation, whose period increases linearly with the RIR length, are derived. It is interesting to note that the proposed approach for RIR estimation can be implemented with the same measurement operations and computations of the methods for linear filters based on MLSs and PPSs. Only the input signal and the interpretation of the measured quantities change.

The paper is organized as follows. In Section 2, Legendre filters are reviewed. In Section 3, the proposed RIR measurement method is described. In Section 4, simplified PPSs for Legendre filters are developed. Simulation results illustrating the effectiveness and robustness towards nonlinearities of the

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proposed approach are presented in Section 5. Conclusions follow in Section 6.

2. LEGENDRE FILTERS

Legendre filters introduced in [8], [9] can arbitrarily well approximate the input-output relationship of any discrete-time, time-invariant, finite-memory, causal, continuous, nonlinear systems given by

$$y(n) = f[x(n), x(n-1), \dots, x(n-N+1)], \quad (1)$$

where f is a real continuous function and $x(n)$ belongs to $[-1, +1]$. They are polynomial filters whose basis functions are product of Legendre polynomials. The basis functions of orders 1, 2, and 3, memory length N, and diagonal number D are reported in Table 1. The diagonal number D is defined as the maximum time difference between the input samples involved in each basis functions. When $D = N - 1$, a complete Legendre filter is obtained, but it has been shown that a simplified filter having $D \ll N - 1$ is often sufficient to accurately model many real systems [9]. In Table 1, $\text{leg}_k(\xi)$ is the k-th order Legendre polynomial satisfying the recursion

$$\text{leg}_{k+1}(\xi) = \frac{2k+1}{k+1}\xi\text{leg}_k(\xi) - \frac{k}{k+1}\text{leg}_{k-1}(\xi), \quad (2)$$

with $\text{leg}_0(\xi) = 1$, $\text{leg}_1(\xi) = \xi$. The basis function of order 0 is the constant 1, which is neglected in the following.

A Legendre filter of order 3, memory N, and diagonal number D is a linear combination of the basis functions of Table 1. The filter can be implemented in the form of a filter bank as follows

$$\hat{y}(n) = h_1(n) * x(n) + \sum_{i=0}^{D} h_{2,i}(n) * b_{2i}(n) + + \sum_{i=0}^{D}\sum_{j=i}^{D} h_{3,i,j}(n) * b_{3,i,j}(n) \quad (3)$$

where $*$ indicates convolution, $b_{2,i}(n)$ and $b_{3,i,j}(n)$ are the zero-lag basis functions of 2-nd and 3-rd order. Specifically, $b_{2,0}(n) = \text{leg}_2[x(n)]$, $b_{2,i}(n) = x(n)x(n-i)$ with $i = 1...D$, $b_{3,0,0}(n) = \text{leg}_3[x(n)]$, $b_{3,0,j}(n) = \text{leg}_2[x(n)]x(n-j)$ with $j = 1...D$, $b_{3,i,i}(n) = x(n)\text{leg}_2[x(n-i)]$ with $i = 1...D$, and $b_{3,i,j}(n) = x(n)x(n-i)x(n-j)$ with $i = 1...D-1$ and $j = i+1...D$.

$h_1(n)$ is the first order kernel, i.e., a sequence of length N collecting the coefficients of the linear terms $x(n-i)$. Using the same naming convention of Volterra filters, $h_{2,i}(n)$ with $i = 0...D$ are the diagonals of the second order kernel, and are sequences of length $N-i$. Similarly, $h_{3,i,j}(n)$ with $i = 0...D$ and $j = i...D$ are the diagonals of the third order kernel with $N - j$ elements.

The first order kernel should not be confused with the impulse response of the nonlinear filter, which is

$$\hat{h}(n) = \lim_{A \longrightarrow 0} \frac{\hat{y}[A\delta(n)]}{A}$$

with $\hat{y}[A\delta(n)]$ the filter response to a pulse sequence of amplitude A. Indeed, also the basis functions $\text{leg}_{2k+1}[x(n)]$, with $k \geq 1$, include a linear term that contributes to the impulse response.

Table 1. Basis functions of Legendre filters.

Order 1: $x(n), x(n-1), \dots, x(n-N+1)$
Order 2: $\text{leg}_2[x(n)], \text{leg}_2[x(n-1)], \dots, \text{leg}_2[x(n-N+1)]$, $x(n)x(n-1), ..., x(n-N+2)x(n-N+1)$, $\vdots$ $x(n)x(n-D), ..., x(n-N+D+1)x(n-N+1)$.
Order 3: $\text{leg}_3[x(n)], \text{leg}_3[x(n-1)], \dots, \text{leg}_3[x(n-N+1)]$, $\text{leg}_2[x(n)]x(n-1), ...$ $..., \text{leg}_2[x(n-N+2)]x(n-N+1)$, $\vdots$ $x(n)x(n-1)x(n-2), ...$ $..., x(n-N+3)x(n-N+2)x(n-N+1)$, $\vdots$ $x(n)x(n-D+1)x(n-D), ...$ $..., x(n-N+D+1)x(n-N+2)x(n-N+1)$,

It was shown in [9], [10] that the Legendre filters admit PPSs, i.e., periodic sequences that guarantee the orthogonality of the basis functions over a period. Let us indicate with $< \cdot >_L$ the time average over the period L of the PPS. With a PPS input signal, for any couple of distinct basis functions $b_i(n)$ and $b_j(n)$, $< b_i(n)b_j(n) >_L = 0$ and the coefficients of the Legendre filter can be efficiently computed with the cross-correlation method. For example, the m-th coefficient of the first order kernel, is given by

$$h_1(m) = \frac{< \hat{y}(n)x(n-m) >_L}{< x^2(n) >_L}. \quad (4)$$

The expression in (4) is equal to that used for identifying the impulse response of linear systems with MLSs or PPSs, only the periodic sequence changes.

3. ROOM IMPULSE RESPONSE MEASUREMENT

Let us consider the RIR measurement system of Figure 1, which is composed of a power amplifier, a loudspeaker, the room acoustic path, and a microphone. The room acoustic path can be considered a linear system and the objective is to measure its impulse response $h_R(n)$, which has length M.

First, it is assumed that the power amplifier and the loudspeaker are linear systems. In this case, $h_1(n)$ is the impulse response of the PAL system, which has length N. The impulse response of the microphone can be accounted within the PAL system. Thus, the entire measurement system has impulse response $h_T(n) = h_1(n) * h_R(n)$ with length $N_T = M + N - 1$. In the classical approach for RIR estimation, $h_1(n)$ is measured in an anechoic chamber, $h_T(n)$ in the specific room to be characterized, and the RIR is estimated from

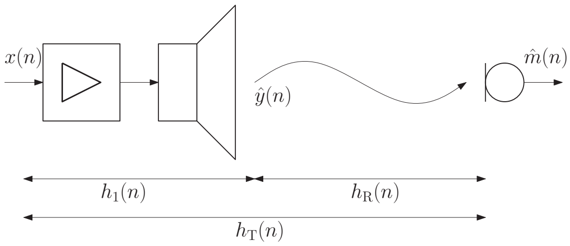

Fig. 1. The measurement system.

$h_{\rm T}(n)$ with the equalization of the equipment [11]. For example, assuming the loudspeaker response is omnidirectional in the frequency range of operation, the equalization can be performed as in [11] with the Kirkeby algorithm, as follows:

$$h_{\rm R}(n) = {\rm IFFT}\left[\frac{{\rm FFT}[h_{\rm T}(n)] \cdot {\rm FFT}[h_1(n)]^*}{{\rm FFT}[h_1(n)] \cdot {\rm FFT}[h_1(n)]^* + \epsilon(\omega)}\right], \tag{5}$$

where ${\rm FFT}[\cdot]$ and ${\rm IFFT}[\cdot]$ are the direct and inverse FFT operators, respectively, and the operations are performed at the single frequencies. $\epsilon(\omega)$ is a frequency-dependent regularization parameter.

Very often the RIR is estimated by measuring only $h_{\rm T}(n)$ and in that case the measure is affected by, i.e., convolved with, the impulse response of the PAL system.

Assume now the power amplifier and the loudspeaker are affected by nonlinearities and that the input-output relationship of the PAL system can be represented with a Legendre filter of order K, memory N, and diagonal number D. For $K = 3$, the input-output relationship is given by (3). In this case, $h_1(n)$ is the first order kernel of the Legendre filter and has length N. The input-output relationship of the entire measurement system is given by

$$\begin{aligned}\hat{m}(n) &= h_{\rm R}(n) * \hat{y}(n) = h_{\rm R}(n) * h_1(n) * x(n)\\ &+ \sum_{i=0}^{D} h_{\rm R}(n) * h_{2,i}(n) * b_{2i}(n)\\ &+ \sum_{i=0}^{D}\sum_{j=i}^{D} h_{\rm R}(n) * h_{3,i,j}(n) * b_{3,i,j}(n). \end{aligned} \tag{6}$$

This is still the input-output relationship of a Legendre filter having order $K = 3$, memory $N_T = M + N - 1$, and diagonal number D. The first order kernel of the system in (6) is $h_{\rm T}(n) = h_{\rm R}(n) * h_1(n)$. $h_1(n)$ can be measured in an anechoic chamber applying a suitable PPS for Legendre filters and using (4). $h_{\rm T}(n)$ can be measured in the specific room to be characterized with the same approach. Note that, since the filters in (3) and (6) depend on the amplitude range of the input signal, the PPS amplitude (i.e., the reproduction volume) should be the same in both measurements. Eventually, $h_{\rm R}(n)$ can be estimated with the Kirkeby algorithm as in (5). Thanks to the orthogonality of the basis functions for a PPS input signal, the measurements of $h_1(n)$ and $h_{\rm T}(n)$ are not influenced by the nonlinear kernels, i.e., by $h_{2,i}(n)$ and $h_{3,i,j}(n)$ for all i, j. Thus, the RIR measurement is not affected by the nonlinearities of the power amplifier or the loudspeaker.

When the RIR is directly estimated with $h_{\rm T}(n)$, the measure is affected by, i.e., convolved with, the first order kernel of the PAL system, whose spectrum however is not much different from the frequency response of the PAL system.

When the acoustic path has length or nonlinearity order larger than those considered in the PPS, as discussed in [9] the measurements of $h_1(n)$ and $h_{\rm R}(n)$ are affected by an aliasing error. Nevertheless, since the PPS is similar to a white uniform noise in $[-1, +1]$, for which all Legendre basis functions are orthogonal, the aliasing errors are often negligible.

4. PPS FOR RIR ESTIMATION

PPSs suitable for the RIR estimation need to be developed. In [9], [10], PPSs appropriate for the identification of Legendre filters of order K, memory N, and diagonal number $D = N - 1$ were developed by considering a periodic sequence with L unknown variables and imposing the orthogonality of all the basis functions over the period, i.e.,

$$< b_i(n) b_j(n) >_L = 0 \tag{7}$$

for any couple of distinct basis functions $b_i(n)$, $b_j(n)$. For sufficiently large period L, the system is under-determined and may have infinite solutions. Indeed, a solution for (7) has always been found [9]. The system in (7) has number of equations and variables that increases geometrically with the memory N and exponentially with the order K. The period of the PPSs obtained solving (7) increases geometrically with the order N and even for low values of N can be prohibitively large. Nevertheless, the system in (7) imposes more constraints than needed for RIR estimation. Indeed, in our case only the first order kernel of the systems in (3) and (6) has to be estimated. Consider the system in (6) of length N_T. A PPS suitable for the first order kernel estimation must guarantee the orthogonality of the basis functions $x(n)$, ..., $x(n - N_T + 1)$ with any other basis functions involved in the input-output relationship in (6). Such a PPS can be found by imposing the conditions in (8), (9), and (10):

$$< x(n) b_i(n) >_L = 0 \tag{8}$$

for all basis functions $b_i(n)$ in (6) with $b_i(n) \neq x(n)$;

$$< b_{2,i}(n) x(n-k) >_L = 0 \tag{9}$$

for all $i = 0...D$ and $k = 1...N_T - 1$;

$$< b_{3,i,j}(n) x(n-k) >_L = 0 \tag{10}$$

for all $i = 0...D$, $j = i...D$, and $k = 1...N_T - 1$.
The number of equations in the system (8)-(10) increases exponentially with the order K, geometrically with the number of diagonals D, but linearly with the RIR length N_T. By considering a sufficiently large period L, the system (8)-(10) is under-determined and can be solved with any algorithm for nonlinear equation systems. We found particularly efficient the Newton-Raphson method, implemented as in [12, ch.

Table 2. PPSs for RIR estimation.

Sequence	N_T	K	D	L	$\log_2(L)$
1	8192	3	0	131060	17
2	8192	3	1	262132	18
3	8192	5	0	262132	18
4	8192	3	2	524276	19
5	8192	5	1	1244656	20.2
6	8192	3	3	2359264	21.2

9.7], starting from a random uniform distribution in $[-1, +1]$ of the variables.

The number of equations and variables can be further reduced imposing a specific structures to the PPS. For example, as shown in [9] the number of equations and variables can almost be halved by exploiting symmetry (for each N-sample sub-sequence there is a symmetric sub-sequence), oddness (for each N-sample sub-sequence there is the negated one), oddness-1 (for each N sample sub-sequence there is a subsequence formed by alternatively negating the samples), ..., oddness-2^P (for each N sample sub-sequence there is a subsequence formed by alternatively negating blocks of 2^P samples). The reduction in the number of equations is obtained at the expense of a longer period of the resulting PPS, but it is often determinant to be able to solve the system in (8)-(10). Indeed, the Newton-Raphson algorithm has memory and processing time requirements that grow with the cube of the number of equations.

Using these conditions, PPSs suitable for the RIR have been developed. Table 2 summarizes the characteristics of the PPSs used for the simulations of Section 5. The first four sequences exploit symmetry, oddness, oddness-1; Sequence 5 exploits oddness, oddness-1, -2, -4, and Sequence 6 exploits oddness, oddness-1, -2, -4, -8. The sequences can be downloaded from www.units.it/ipl/res_PSeqs.htm.

5. SIMULATIONS RESULTS

In this section the performance of the proposed method is assessed and compared with that of the MLS technique, the exponential sweep approach, and the perfect sweep technique, simulating the measurement system of Fig. 1. Specifically, the power amplifier has been emulated with a real nonlinear device, i.e., a Behringer Tube Ultragain MIC 100 vacuum tube pre-amplifier. The input signals have been applied to the pre-amplifier and the corresponding outputs have been recorded at 44.1 kHz sampling frequency. Acting on the gain control of the pre-amplifier, different levels of nonlinear distortions can be generated. In particular, 14 different settings, with increasing level of nonlinear distortion, have been considered in the experiments. Table 3 shows the second (Dist_2), third (Dist_3), and total (Dist_T) harmonic distortions at the different settings measured with a 1 kHz sinusoidal signal with the maximum amplitude used in the experiments. The nonlinear distortions considered in the simulations have much higher values than those found in any real measurement system, but they have been purposely chosen so high in order to stress the differences between the competing methods. The output of the pre-amplifier has been applied to the loudspeaker model, a linear system having the impulse response of a real loudspeaker measured in anechoic chamber. The room impulse response has also been simulated with a previously measured real room impulse response. The memory length N_T is around 8000 samples.

The evaluation has been performed in terms of log-spectral distance (LSD) in the loudspeaker passband $[\omega_1, \omega_2]$, which is defined as

$$\mathrm{LSD} = \sqrt{\frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} 10 \log_{10} \frac{|H_{\mathrm{R}}(e^{j\omega})|^2}{|\hat{H}_{\mathrm{R}}(e^{j\omega})|^2} \mathrm{d}\omega}, \qquad (11)$$

where $|H_{\mathrm{R}}(e^{j\omega})|$ is the actual room magnitude response, and $|\hat{H}_{\mathrm{R}}(e^{j\omega})|$ is the estimated room magnitude response.

Fig. 2 shows the results obtained in terms of LSD as a function of the increasing level of nonlinear distortion, considering different sequence orders for each of the aforementioned methods. The order of a sequence of period/length L is $\log_2(L)$. In Fig. 2, for low nonlinear distortion all the methods show low values of LSD. However increasing the distortion level, each method has a different behaviour. In particular, for low orders the MLS estimation is affected by the well-known problem of nonlinear “spikes,” while it is not affected by nonlinearities for orders greater than 17. The exponential sweep shows in general good results (apart from order 19) but degrades its performance at very high distortion levels. The exponential sweep improves its robustness towards nonlinearities increasing the order, i.e., period, of the sequence. The proposed technique shows good results in all considered cases, and for a diagonal number $D > 0$, i.e., for sequences 2, 4, 5, and 6, appears to be almost immune to nonlinearities, even at high distortion levels. It can be observed that, for the same range of orders, the envelop of the curves in Fig. 2-(d) is much tighter than that of Fig. 2-(b) and (c), indicating a more robust behaviour in these experimental conditions.

6. CONCLUSIONS

In this work, a novel method for room impulse response estimation robust towards nonlinearities affecting the power amplifier or the loudspeaker of the measurement system has been presented. The method is based on measurements of the first order kernel of the Legendre filter modeling the acoustic path, taking advantage of perfect periodic sequences and cross-correlation method. The effectiveness of the proposed approach has been presented considering simulation results in a realistic scenario and comparing it with well-known techniques of the state of the art. In particular, different levels of nonlinear distortion have been considered, confirming the robustness of the proposed approach that is always capable to achieve good results in terms of log-spectral distance.

Table 3. Second (Dist_2), third (Dist_3), and total (Dist_T) harmonic distortion in percent at different settings.

Setting	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dist_2	2.6	3.1	4.0	4.9	5.5	6.3	7.2	7.8	4.7	0.9	7.8	18.2	36.3	38.8
Dist_3	0.5	0.7	1.0	1.4	2.0	2.9	4.0	6.1	12.0	18.4	22.5	22.9	9.9	6.7
Dist_T	3.3	4.0	5.2	6.5	7.8	9.8	12.6	16.5	22.9	27.3	45.9	77.0	149	161

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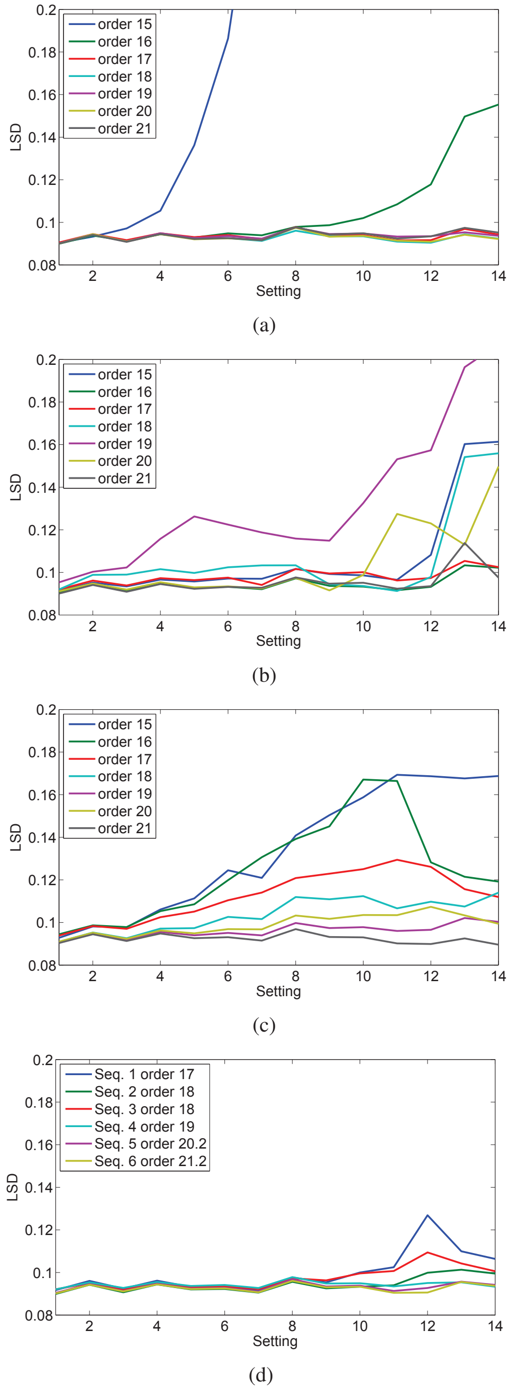

Fig. 2. Log-spectral distance at the different settings for (a) MLS, (b) exponential sweep, (c) perfect sweep, and (d) proposed perfect sequence.