CONSTRAINT KALMAN FILTER FOR INDOOR BLUETOOTH LOCALIZATION

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ABSTRACT

This paper studies sequential estimation of indoor localization based on fingerprints of received signal strength indicators (RSSI). Due to the lack of an analytic formula for the fingerprinting measurements, the Kalman filter can not be directly applied. By introducing a hidden variable to represent the unknown positioning coordinate, a state model is formulated and a constrained Kalman filter (CKF) is then derived within the Bayesian framework. The update of the state incorporates the prior information of the motion model and the statistical property of the hidden variable estimated from the RSSI measurements. The positioning accuracy of the proposed CKF method is evaluated in indoor field tests by a self-developed Bluetooth fingerprint positioning system. The conducted field tests demonstrate the effectiveness of the method in providing an accurate indoor positioning solution.

Index Terms— Kalman filter, fingerprinting, receiver signal strength indicator (RSSI), Bayesian estimation

1. INTRODUCTION

In the last several years, interest in indoor positioning has increased significantly because of various emerging applications. Due to the severe attenuation and scattering of radio frequency signals in the indoor circumstances, Global Navigation Satellite System (GNSS), as the most effective method for outdoor navigation, is unable to provide the desired performance or even unavailable indoors. One alternative for indoor navigation is to utilize signals of opportunity (SoOP) [1], which are not inherently intended for purposes other than navigation.

Recently, with the increased use of the wireless local area networks (WLAN) and Bluetooth technology, which offer the flexibility and mobility to the users, received signal strength indicators (RSSI) based fingerprinting has become a feasible technique for indoor positioning [2]. Instead of depending on accurate estimations of angle or distance to deduce the location with standard geometry, the basic idea behind the fingerprinting method is to match elements in a database to particular signal strength measurements in the area at hand [3]. The method operates in two phases: (1) *the training phase*: a RSSI radio map is created based on the reference points within the area of interest. The radio map implicitly characterizes the RSSI position relationship through the training measurements at the reference points with known coordinates. (2) *the online positioning phase*: the mobile device measures RSSI of the wireless transmitters and the positioning system uses the radio map to obtain a position estimate. A thorough summary and analysis for different factors that affect fingerprints is given in reference [4]. Different fingerprint positioning algorithms are compared within WLAN in [5]. These methods are basically static fingerprint positioning methods, which only compare the current RSSI measurements with the radio map to estimate the position. The positioning accuracy can be further improved by exploiting the measurements collected in time series. In this work, we consider the sequential estimation of indoor positioning with RSSI fingerprints. Due to the implicit relation between the state variables and the fingerprinting measurements, the direct application of Kalman filter is impractical in use. By introducing a hidden variable to represent the unknown position coordinates, a state model is then formulated and a constrained Kalman filter (CKF) is then derived within the Bayesian framework to recursively update the state estimation, which incorporates the prior information from the motion model and the statistical property estimated from the RSSI fingerprints. The method is further validated by a wireless localization scenario indoors using Bluetooth RSSI fingerprints.

The rest of the work is organized as follows: the system model and the problem of indoor positioning are formulated in Section II. Section III derives the modified Kalman filter for indoor fingerprint localization. The experimental tests and performance comparison of three algorithms are presented and discussed in Section IV. Section IV gives the conclusions.

2. SYSTEM DESCRIPTION

2.1. Motion model

Assume a mobile of interest moves on a two-dimensional Cartesian plane. The state at time instant t_k is defined as

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the vector $x_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$, where $[x_k, y_k]^T$ corresponds to the east and north coordinates of the mobile position; $[\dot{x}_k, \dot{y}_k]^T$ are the corresponding velocities. The mobile state with random acceleration can be modeled as [6, p. 267]:

$$x_{k+1} = \mathbf{F}_k x_k + w_k, \tag{1}$$

where the state transition matrix $\mathbf{F}_k = \begin{bmatrix} \mathbf{I}_2 & \Delta t_k \mathbf{I}_2 \\ 0 & \mathbf{I}_2 \end{bmatrix}$, with $\mathbf{I}_2$ the 2×2 matrix and Δt_k is the sampling period. The random process w_k is a white zero mean Gaussian noise, with covariance $\mathbf{Q}_k = \begin{bmatrix} \frac{\Delta t_k^4}{4}\mathbf{\Omega} & \frac{\Delta t_k^3}{2}\mathbf{\Omega} \\ \frac{\Delta t_k^3}{2}\mathbf{\Omega} & \Delta t_k^2 \mathbf{\Omega} \end{bmatrix}$ where $\mathbf{\Omega} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$.

2.2. Measurements

This subsection presents the measurements utilized in the training and the online positioning phases.

2.2.1. measurements to build a radio map (training phase)

In the training phase, the RSSI values of the radio signals transmitted by the access points (APs) are collected in the calibration points for a certain period of time and stored into a radio map. Denote the ith fingerprint as $\boldsymbol{RM}_i$ with the form: $\boldsymbol{RM}_i = (c_i, \{a_{i,j}\}), j \in \{1, \cdots, N\}$ where c_i is the coordinate of th ith calibration point and $\mathbf{a}_{ij}$ holds the l RSSI samples from the access point AP_j, i.e. $\mathbf{a}_{i,j} = \{a_{i,j}^1, a_{i,j}^2, \cdots, a_{i,j}^l\}$, N is the total number of APs. The set of all fingerprints is denoted as $\boldsymbol{RM} = \{\boldsymbol{RM}_1, \cdots, \boldsymbol{RM}_M\}$, where M is the total number of calibration points.

2.2.2. measurements for position estimation (online phase)

In the positioning phase, denote $z_{k,j}$ as the RSSI values measured from the jth AP at time epoch t_k and $z_k = [z_{k,1}, \cdots, z_{k,j}], j \in N$. Then, the measurement sequence to time k is $z_{1:k} \triangleq \{z_1, \cdots, z_k\}$.

2.3. Problem formulation

The problem of tracking the pedestrian indoors is to infer the mobile state x_k from the measurement sequence $z_{1:k}$ and the constructed radio map R. Within the Bayesian estimation framework, solving this problem corresponds to computing the posterior $p(x_k|z_{1:k}, R)$. By applying the Bayes' Rule, the posterior can be calculated as:

$$p(x_k|z_{1:k}, R) = \frac{p(z_k|x_k, R)p(x_k|z_{1:k-1}, R)}{p(z_k|z_{1:k-1}, R)}$$

Due to the complex electromagnetic environment indoors, it is not easy to give an explicit measurement function $z_k = h_k(x_k)$ within the whole positioning area. Thus, the likelihood $p(z_k|x_k, R)$ could not be exactly calculated.

An alternative approximate is to compute $p(z_k|R)$, which is based on the assumption that the whole area of interest is divided into M small cells and the RSSI distribution on the ith calibration point represents the distribution of all the points within the corresponding cell. However, $p(z_k|R)$ is the discrete probability distribution on the M coordinates of calibration points, based on which the mean and covariance of the position can be estimated, while $p(x_k|z_{1:k-1}, R)$ predicts of the position and velocity. Therefore, the posterior $p(x_k|z_{1:k}, R)$ relates to fuse two state estimations with different dimensions, which is not straightforward to update.

3. ALGORITHM DESCRIPTION

3.1. Bayesian static localization (BSL)

In fingerprinting localization, $p(y_k|z_k)$ is a discrete p.d.f. on the M coordinates of calibration points. According to the Bayes' rule,

$$p(y_k = c_i|z_k) = \frac{p(z_k|y_k = c_i)p(y_k = c_i)}{\sum_{i=1}^{M} p(z_k|y_k = c_i)p(y_k = c_i)} \tag{2}$$

For lack of the specific prior information on y_k, we set a uniform prior to $p(y_k)$ and then the posterior $p(y_k|z_k)$ in (2) is equivalent to the likelihood $p(z_k|y_k)$. By assuming that the measurements z_k from different AP_j are independent and a Gaussian approximation to the histogram of $\mathbf{a}_{i,j}$, the likelihood $p(z_k|y_k)$ can be expressed as

$$l_{k,i} = p(z_k|y_k = c_i) = \prod_{j=1}^{N} p(z_{k,j}|y_k = c_i) \tag{3}$$

where

$$p(z_{k,j}|y_k = c_i) = \begin{cases} \mathrm{N}(z_{k,j}; \bar{a}_{i,j}, \sigma_{i,j}^2) & \text{if } \text{AP}_j \text{ hearable} \\ l_0 & \text{if } \text{AP}_j \text{ unhearable} \end{cases}$$

Theoretically, $l_0 = 0$, however, in practice, considering the computation stability, it is set to a small value, e.g. 10^{-11}.

Suppose the number of the reference points is large enough, and a Gaussian p.d.f. can be approximated to the posterior, i.e. $p(y_k|z_k) \sim \mathrm{N}(\mu_k, \mathbf{\Sigma}_k)$, where the estimated mean μ_k and the covariance $\mathbf{\Sigma}_k$ are

$$\begin{aligned} \mu_k &= \Sigma_{i=1}^{M} \bar{l}_{k,i} c_i \\ \mathbf{\Sigma}_k &= \Sigma_{i=1}^{M} \bar{l}_{k,i} (\mu_k - c_i)(\mu_k - c_i)^T \end{aligned} \tag{4}$$

where $\bar{l}_{k,i} = l_{k,i} / \left(\Sigma_{i=1}^{M} l_{k,i}\right)$.

3.2. Sequential Bayesian state estimation

The problem of tracking a pedestrian indoors is to infer the mobile state x_k from the measurement sequence $z_{1:k}$. Within the Bayesian estimation framework, solving this problem corresponds to computing the posterior $p(x_k|z_{1:k})$. By applying the Bayes' Rule, the posterior can be calculated as:

$$p(x_k|z_{1:k}) \propto p(z_k|x_k)p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})$$

Due to the complex electromagnetic environment indoors, it is not easy to give an explicit measurement function $z_k = h_k(x_k)$ within the whole positioning area. Thus, the likelihood $p(z_k|x_k)$ can not be exactly calculated and the basic Kalman filter can not be directly applied.

Alternatively, in Section 3.1, by Bayesian estimation, the posterior p.d.f. $p(y_k|z_k)$ can be approximated to a Gaussian distribution, where $y_k \triangleq [x_k, y_k]^T$. Meanwhile, y_k has the explicitly linearized relationship with the state variable x_k. Therefore, the sequential estimation problem considered here amounts to making inference to the posterior $p(x_k|z_{1:k})$ based on the following state model

$$\begin{aligned} x_{k+1} &= \mathbf{F}_k x_k + w_k \\ y_k &= \mathbf{H}_k x_k + v_k \\ y_k|z_k &\sim \mathrm{N}(\mu_k, \mathbf{\Sigma}_k) \end{aligned} \tag{5}$$

where $\mathbf{H} = [\mathbf{I}_2, \mathbf{0}]$ and μ_k and $\mathbf{\Sigma}_k$ are the mean and covariance of a Gaussian distribution.

Consider the sequential estimation of (5) within the Bayesian framework. Given the measurements $z_{1:k}$, the posterior probability density function (p.d.f.) $p(x_k|z_{1:k})$ can be calculated by integrating out $y_{1:k}$, i.e.,

$$p(x_k|z_{1:k}) = \int p(x_k|y_{1:k})p(y_{1:k}|z_{1:k})\mathrm{d}y_{1:k} \tag{6}$$

Decompose the posterior $p(x_k|y_{1:k})$ by Bayes rule:

$$p(x_k|y_{1:k}) = \frac{p(y_k|x_k)p(x_k|y_{1:k-1})}{\int p(y_k|x_k)p(x_k|y_{1:k-1})\mathrm{d}x_k} \tag{7}$$

Since the statistical distribution of y_k is only decided by the current step of the observation z_k, the posterior $p(y_{1:k}|z_{1:k})$ can be decomposed as:

$$p(y_{1:k}|z_{1:k}) = p(y_k|z_k)p(y_{1:k-1}|z_{1:k-1}) \tag{8}$$

Then, by substituting (7) and (8) into (6), we get

$$\begin{aligned} p(x_k|z_{1:k}) \propto \int & p(y_k|x_k)p(x_k|y_{1:k-1}) \cdot \\ & p(y_k|z_k)p(y_{1:k-1}|z_{1:k-1})\mathrm{d}y_k \end{aligned} \tag{9}$$

For the linear Gaussian dynamic equation in (5), if at time t_k, $p(x_{k-1}|y_{1:k-1})$ conforms to a Gaussian distribution $\mathrm{N}(\hat{x}_{k-1}, \mathbf{P}_{k-1})$, then the density of the one step prediction $p(x_k|y_{1:k-1})$ is also Gaussian with the mean $\hat{x}_{k|k-1}$ and variance $\mathbf{P}_{k|k-1}$, where

$$\begin{aligned} \hat{x}_{k|k-1} &= \mathbf{F}_{k-1}\hat{x}_{k-1} \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \end{aligned} \tag{10}$$

and the likelihood

$$p(y_k|x_k) = \mathrm{N}(y_k; \mathbf{H}_k x_k, \mathbf{R}_k)$$

then, (9) can be rewritten as

$$\begin{aligned} & p(x_k|z_{1:k}) \\ & \propto \int c_1 e^{-\frac{1}{2}\left(\|y_k - \mathbf{H}_k x_k\|^2_{\mathbf{R}_k^{-1}} + \|x_k - \hat{x}_{k|k-1}\|^2_{\mathbf{P}_{k|k-1}^{-1}} + \|y_k - \mu_k\|^2_{\mathbf{\Sigma}_k^{-1}}\right)} \mathrm{d}y_k \end{aligned} \tag{11}$$

where $\|A - B\|_{\mathbf{C}^{-1}} \triangleq (A - B)^T\mathbf{C}^{-1}(A - B)$ and c_1 is constant.

By expanding and completing the square of x_k and y_k, and applying the matrix inverse lemma in the exponent index in (11), $p(x_k|z_{1:k})$ can be further simplified as

$$\begin{aligned} p(x_k|z_{1:k}) \propto & \int c_2 e^{-\frac{1}{2}\left(\|y_k - \hat{y}_k\|^2_{\mathbf{S}_k^{-1}} + \|x_k - \hat{x}_k\|^2_{\mathbf{P}_k^{-1}}\right)} \mathrm{d}y_k \\ = & c_3 e^{-\frac{1}{2}\|x_k - \hat{x}_k\|^2_{\mathbf{P}_k^{-1}}} \end{aligned} \tag{12}$$

where c_2 and c_3 are constant and

$$\hat{x}_k = \hat{x}_{k|k-1} + \mathbf{G}_k(\mu_k - \mathbf{H}_k\hat{x}_{k|k-1}) \tag{13a}$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{G}_k\mathbf{H}_k)\mathbf{P}_{k|k-1} \tag{13b}$$

$$\mathbf{G}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k + \mathbf{\Sigma}_k)^{-1} \tag{13c}$$

$$\mathbf{S}_k = \mathbf{R}_k(\mathbf{R}_k + \mathbf{\Sigma}_k)^{-1}\mathbf{\Sigma}_k \tag{13d}$$

$$\hat{y}_k = \mathbf{S}_k(\mathbf{R}_k^{-1}\mathbf{H}_k x_k + \mathbf{\Sigma}_k^{-1}\mu_k) \tag{13e}$$

Thus, the posterior update of x_k in (12) is conformed to a Gaussian distribution with the mean $\hat{x}_k$ (13a) and covariance $\mathbf{P}_k$ (13b). It is noticed that the posterior update (13a-13c) is similar with the basic Kalman filter [7]. However, the difference is that, $\hat{x}_k$ (13a) is updated by utilizing the mean μ_k and the Kalman gain $\mathbf{G}_k$ (13c) incorporates the covariance $\mathbf{\Sigma}_k$. It is because the explicit relation between the measurements z_k and the state x_k is not available, the update of the x_k can only use the information from y_k, i.e. $\{\mu_k, \mathbf{\Sigma}_k\}$ estimated in (4). The method is denoted as CKF and is summarized in Algorithm 1.

Algorithm 1: CKF for indoor fingerprint localization

Input: $\{\hat{x}_{k-1}, \mathbf{P}_{k-1}, z_k\}$
Output $\{\hat{x}_k, \mathbf{P}_k\}$

1. Predict mean $\hat{x}_{k|k-1}$ and covariance $\hat{\mathbf{P}}_{k|k-1}$ according to (10)
2. estimate mean μ_k and the covariance $\mathbf{\Sigma}_k$ according to (3)(4)
3. update $\hat{x}_k$ and $\mathbf{P}_k$ according to (13a)(13b)(13c)

4. EXPERIMENTAL TESTS

4.1. Test Platform

In this study, a Bluetooth RSS data collecting system is developed for indoor positioning. The system consists of a Bluetooth evaluation kit and a data collecting software. The basic function of the system is to scan the Bluetooth Access Points (APs) nearby, collect the RSS from the detected APs, and then send the measurements to the laptop via a serial port.

To evaluate the positioning accuracy of different algorithms, a reference trajectory, used as the ground truth, is obtained via NovAtel's high-accuracy SPAN system. SPAN technology can provide reliable, continuously available measurements including position, velocity, and attitude even through short periods of time when no GNSS satellites are available.

Indoor tests were carried out in an office corridor with typical structures of concrete, steel and glass for office premises. During the test, 13 Bluetooth access points are deployed in the corridor area [9]. RSSI measurements were collected by a self-developed Bluetooth data collecting system. A reference trajectory, used as the ground truth, is obtained by a NovAtel's high-accuracy GPS/INS SPAN positioning system including an HG1700 IMU (inertial measurement unit). The whole testing platform is described in detail in [8].

4.2. Field Results

Two tests were carried out in the scenario. In both tests, a tester walked along the corridors back and forth with the test cart. Test 1 lasts about 6 minutes, while with a relatively faster speed, test 2 only lasts 3 minutes. The sampling interval of the Bluetooth is set as $\Delta t \approx 9$ s, which guarantees the receiver has enough time to scan the surrounding APs. We compare the proposed CKF with the BSL (Section 3.1) and the point Kalman filter (PKF) [5]. The BSL method only uses the current RSSIs to estimate the posterior mean and covariance of the position (see (2)-(4)). The PKF further smooth the mean of the position estimated by BSL with a Kalman filter, in which the mean of the position μ_k is used as the direct observation of the mobile state x_k and a stationary motion model is applied to formulate the movement [5].

In our tests, the covariance of the process noise in the PKF is set as $\mathbf{Q}_{\text{PKF}} = (V_{\text{max}} \cdot \Delta t)^2 \cdot \mathbf{I}_2$, where V_{max} is the (empirical) maximum walking speed for indoor pedestrians and set to 2 m/s [9]. In the CKF, we set $\sigma_x = \sigma_y = 1/\Delta t$, which means that the changes in the velocity are in the order of 1 m/s in each direction within a sampling interval. The initial position of the CKF and the PKF is obtained from the first output of the BSL with the initial covariance set as $9 * \mathbf{I}_2$. The initial velocity for the CKF is 0 m/s with covariance $\mathbf{I}_2$.

Fig. 1 and 3 represent the estimated trajectories of 3 different algorithms in a North-East coordinate frame including also the SPAN reference track as the ground truth. Fig. 2 and 4 show the position error vs. time epoch of the three algorithms. Position errors are compared in Table I.

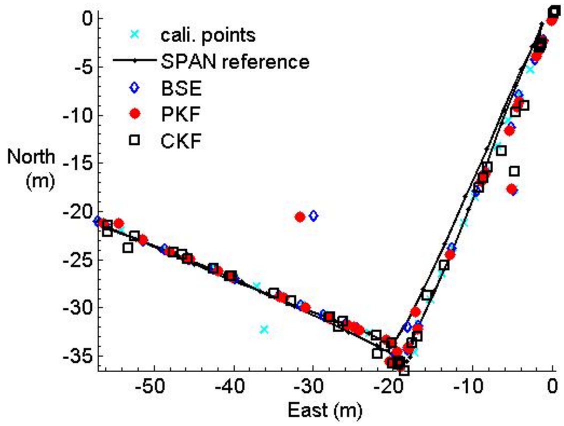

Fig. 1. Position estimation in Test 1

It can be observed from the results that the mean error of the BSL is about 5 m. The PKF smooths the positions obtained by the BSL. Based on the prior motion model, the estimation errors in PKF are reduced in several epochs, e.g. $k = 8$, 10, 13 in Fig. 3 and $k = 4$, 5, 7, 9 in Fig. 5. However, the improvement of the PKF is minor, only 0.1 m. The reason for this may lie in the fact that the $\mathbf{Q}_{\text{PKF}}$ is relatively large due to the long sampling interval of the Bluetooth inquiring. Thus, the prior information from the motion model has limited impact on the position estimation at each epoch. In comparison, the positioning error of the CKF is 4.0 m on the average, about 1 m improvement compared to BSL and 0.9 m compared to the PKF. Significant improvements can be observed at $k = 13$ in test 1 and $k = 4$ to 10 in test 2, where the large errors are detected and filtered out.

Thus, according to the test results, the proposed CKF effectively corrected the large outliers in position estimation and achieves the best position accuracy among the three algorithms. This is achieved by utilizing the statistical information (both mean and covariance considered) from the indirect observables and also with the prior information from the motion model. By comparison, the PKF only considers the mean information of the mobile state, while the BSL did not take the sequential measurements into accounts.

Table 1. Position Error Comparison

Stat.	Test 1			Test 2		
	BSL	PKF	CKF	BSL	PKF	CKF
mean (m)	5.0	4.9	4.1	5.2	5.1	4.0
std (m)	5.8	5.7	4.2	6.2	6.0	3.1
max (m)	27.2	26.5	21.3	22.6	21.8	11.3
95th (m)	12.7	12.9	8.5	18.5	18.3	8.9
min (m)	0.4	0.4	0.3	0.6	0.6	0.5

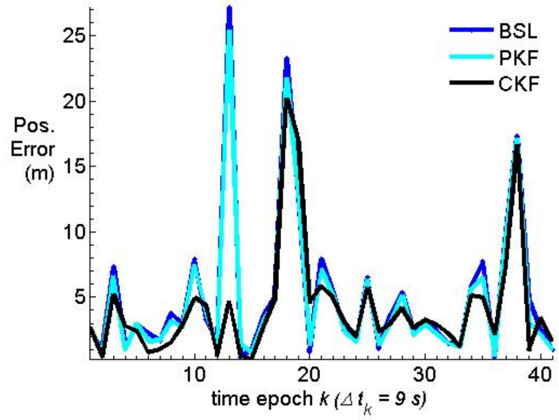

Fig. 2. Position error vs. time epoch in Test 1

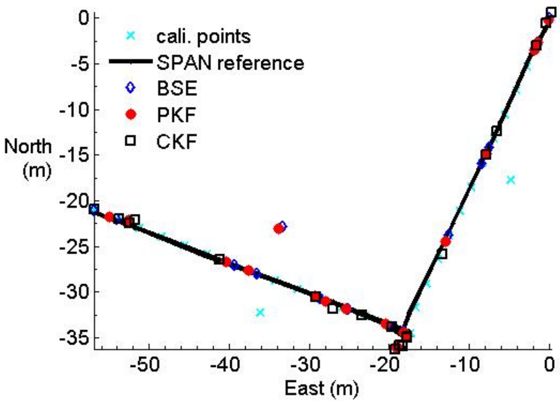

Fig. 3. Position estimation in Test2

5. CONCLUSIONS

This paper studied the sequential estimation of indoor positioning with RSSI fingerprints. The problem is formulated within the Bayesian framework and a modified Kalman filter is derived. The indoor field tests based on Bluetooth RSSI fingerprint positioning show the effectiveness of the method in providing a more accurate solution than the other two classic methods.

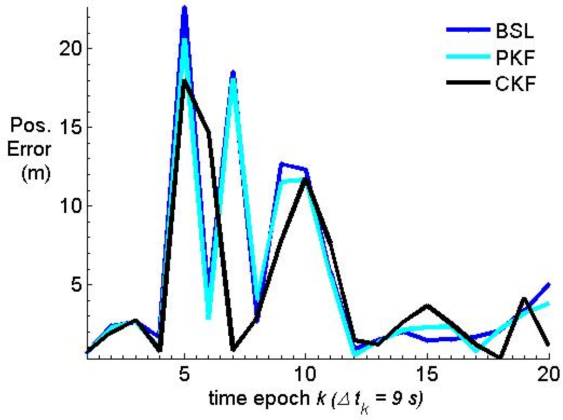

Fig. 4. Position error vs. time epoch in Test 2

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