UNIVERSAL ALGORITHM FOR COMPRESSIVE SAMPLING

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ABSTRACT

In a standard compressive sampling (CS) setup, we develop a universal algorithm where multiple CS reconstruction algorithms participate and their outputs are fused to achieve a better reconstruction performance. The new method is called universal algorithm for CS (UACS) that is iterative in nature and has a restricted isometry property (RIP) based theoretical convergence guarantee. It is shown that if one participating algorithm in the design has a converging recurrence inequality relation then the UACS also holds a converging recurrence inequality relation over iterations. An example of the UACS is presented and studied through simulations for demonstrating its flexibility and performance improvement.

***Index Terms*—** Compressive sampling, greedy algorithms, iterative fusion, restricted isometry property.

1. INTRODUCTION

Compressed sensing (CS) refers to an under-sampling problem, where the sampled (or measured) data is assumed to be sparse in a domain. Typically CS considers an under-determined setup where a high-dimensional signal vector has to be reconstructed from a low-dimensional measurement vector. The measurement noise is considered additive. Design of CS reconstruction algorithms remains as a popular and challenging research area with active involvement of signal processing [1] and information theory [2] communities. In the 'design' arena, development of reconstruction algorithms with provable theoretical guarantees are more appreciated. A relevant theoretical question is: what is the quality of the reconstruction performance, and how is the performance related to the system and signal properties?

Almost all CS reconstruction algorithms can be categorized in three major types: convex optimization [3], Bayesian [4] and greedy pursuits [5–7]. A popular theoretical analysis tool is based on the restricted-isometry-property (RIP) of the CS measurement matrix. The RIP based analysis is recently carried out for convex optimization and greedy pursuits [6, 8]. However, there exists numerous CS reconstruction algorithms providing significantly better practical performance than many theoretically justified algorithms, but without a strong theoretical guarantee. Examples of such algorithms are look-ahead schemes [9, 10] and back tracking schemes [11] etc. In the absence of theoretical justifications, the example algorithms are ad-hoc in nature. The design of ad-hoc algorithms are solely motivated by engineering intuitions and success in applications.

Recent results have categorically shown that the performance of all CS reconstruction algorithms depends on many parameters, such as the level of under-sampling, sparsity level, measurement noise power, and the statistical distribution of non-zero elements of a sparse signal. There exists no algorithm that can be considered the best in all ranges of these parameters [9]. The design of algorithms with good performance is of utmost importance as there exists no best algorithm in a scenario with varying parameters. In this paper, we develop a fusion strategy where several algorithms can participate with provable theoretical guarantee. Surprisingly, all but one participating algorithm can be ad-hoc. *Our main contribution is to show that if one algorithm has a converging recurrence inequality relation among all the participating algorithms then the developed universal algorithm for CS (UACS) also has a converging recurrence inequality relation.* With the recurrence inequality relation, theoretical convergence of the quality of reconstruction over iterations can be analysed. The result implies that under certain conditions, the reconstruction is guaranteed to be perfect. Finally simulation results demonstrate better practical performance of an example UACS algorithm than the participating algorithms. The achievement of better practical performance supported by RIP based theoretical guarantee brightens future prospect of designing new ad-hoc, but powerful CS reconstruction algorithms.

1.1. CS setup, notations and preliminaries

The standard CS problem can be stated as follows, we acquire a T-sparse signal $\mathbf{x} \in \mathbb{R}^N$ via the linear measurements

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ is a matrix representing the sampling system, $\mathbf{y} \in \mathbb{R}^M$ represents a vector of measurements and $\mathbf{e} \in \mathbb{R}^M$ is additive noise representing measurement errors. A T-sparse signal vector consists of at most T non-zero scalar components. For the setup $T < M < N$ (under-determined

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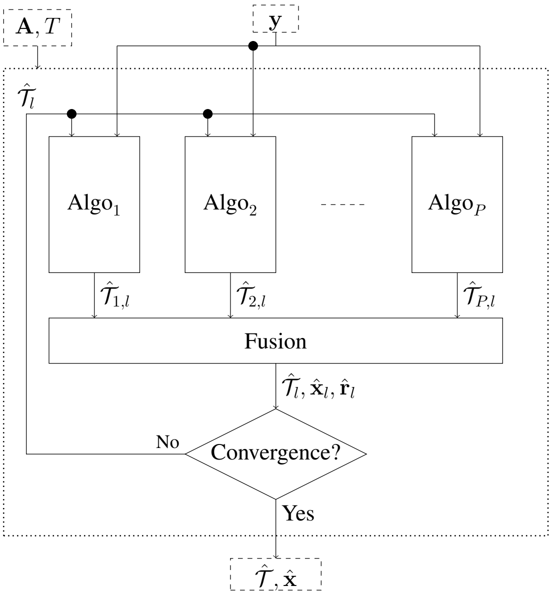

Fig. 1: Block diagram of UACS - Universal Algorithm for Compressive Sampling.

system of linear equations), the task is to reconstruct $\mathbf{x}$ from $\mathbf{y}$ as $\hat{\mathbf{x}}$. Without a-priori statistical knowledge of $\mathbf{x}$ and $\mathbf{e}$, the objective in CS is to strive for a reduced number of measurements (M) as well as achieving a good reconstruction quality. Note that, in practice, we may wish to acquire a signal $\mathbf{x}$ that is sparse in a known orthonormal basis and the concerned problem can be recast as (1).

We use calligraphic letters $\mathcal{T}, \mathcal{U}$ and $\mathcal{V}$ to denote sets that are sub-sets of $\Omega \triangleq \{1, 2, \ldots, N\}$. We use $|\mathcal{T}|$ and $\mathcal{T}^c$ to denote the cardinality and complement of the set $\mathcal{T}$, respectively. For the matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$, a sub-matrix $\mathbf{A}_{\mathcal{T}} \in \mathbb{R}^{M \times |\mathcal{T}|}$ consists of the columns of $\mathbf{A}$ indexed by $i \in \mathcal{T}$. Similarly, for $\mathbf{x} \in \mathbb{R}^N$, a sub-vector $\mathbf{x}_{\mathcal{T}} \in \mathbb{R}^{|\mathcal{T}|}$ is composed of the components of $\mathbf{x}$ indexed by $i \in \mathcal{T}$. Also we denote $(.)^t$ and $(.)^\dagger$ as transpose and pseudo-inverse, respectively. In this paper $\mathbf{A}_{\mathcal{T}}^\dagger \triangleq (\mathbf{A}_{\mathcal{T}})^\dagger$. We use $\|.\|$ to denote the standard ℓ_2 norm of a vector. Further $\|.\|_1$ and $\|.\|_0$ are used to denote ℓ_1 and ℓ_0 norms, respectively. For a sparse signal $\mathbf{x} = [x_1, x_2, \ldots, x_i, \ldots, x_N]^t$, the support-set $\mathcal{T}$ of $\mathbf{x}$ is defined as $\mathcal{T} = \{i : x_i \neq 0\}$. We use l and k to denote iteration counters for UACS and a participating algorithm, respectively. Then $\texttt{supp}(\mathbf{x}, \kappa) \triangleq \{$the set of indices corresponding to the κ largest amplitude components of $\mathbf{x}\}$.

Definition 1 (RIP: Restricted Isometry Property [12]) – *A matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$ satisfies the RIP with Restricted Isometry Constant (RIC) δ_T if*

$$(1 - \delta_T)\|\mathbf{x}\|^2 \leq \|\mathbf{A}\mathbf{x}\|^2 \leq (1 + \delta_T)\|\mathbf{x}\|^2 \quad (2)$$

holds for all vectors $\mathbf{x} \in \mathbb{R}^N$ such that $\|\mathbf{x}\|_0 \leq T$, and $0 \leq \delta_T < 1$.

Algorithm 1 Pseudo-code of UACS - Universal Algorithm for Compressive Sampling

Input: $\mathbf{y}, \mathbf{A}, T$
Initialization:
$l \leftarrow 0$ (l denotes iteration counter)
$\mathbf{r}_l \leftarrow \mathbf{y}$ (Residual at l'th iteration)
$\hat{\mathcal{T}}_l \leftarrow \emptyset$ (Support-set at l'th iteration)
$\hat{\mathbf{x}}_l \leftarrow \mathbf{0}$ (Sparse solution at l'th iteration)
Iteration:
repeat
$l \leftarrow l + 1$ (Iteration counter)
Several CS reconstruction algorithms (P algorithms):
$\hat{\mathcal{T}}_{1,l} \leftarrow \text{Algo}_1(\mathbf{y}, \mathbf{A}, T, \hat{\mathcal{T}}_{l-1})$
$\hat{\mathcal{T}}_{2,l} \leftarrow \text{Algo}_2(\mathbf{y}, \mathbf{A}, T, \hat{\mathcal{T}}_{l-1})$
$\vdots$
$\hat{\mathcal{T}}_{P,l} \leftarrow \text{Algo}_P(\mathbf{y}, \mathbf{A}, T, \hat{\mathcal{T}}_{l-1})$
Fusion:
$\tilde{\mathcal{U}}_l \leftarrow \hat{\mathcal{T}}_{1,l} \cup \hat{\mathcal{T}}_{2,l} \cup \ldots \cup \hat{\mathcal{T}}_{P,l}$
$\tilde{\mathbf{x}}_l$ such that $\tilde{\mathbf{x}}_{\tilde{\mathcal{U}}_l} \leftarrow \mathbf{A}_{\tilde{\mathcal{U}}_l}^\dagger \mathbf{y}$; $\tilde{\mathbf{x}}_{\tilde{\mathcal{U}}_l^c} \leftarrow \mathbf{0}$
$\hat{\mathcal{T}}_l \leftarrow \texttt{supp}(\tilde{\mathbf{x}}_l, T)$
$\hat{\mathbf{x}}_l$ such that $\hat{\mathbf{x}}_{\hat{\mathcal{T}}_l} \leftarrow \mathbf{A}_{\hat{\mathcal{T}}_l}^\dagger \mathbf{y}$; $\hat{\mathbf{x}}_{\hat{\mathcal{T}}_l^c} \leftarrow \mathbf{0}$
$\mathbf{r}_l \leftarrow \mathbf{y} - \mathbf{A}_{\hat{\mathcal{T}}_l}\hat{\mathbf{x}}_{\hat{\mathcal{T}}_l} = \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}_l$
until *stopping criterion*
Output: $\hat{\mathbf{x}} \leftarrow \hat{\mathbf{x}}_l$, $\hat{\mathcal{T}} \leftarrow \hat{\mathcal{T}}_l$, $\mathbf{r} \leftarrow \mathbf{r}_l$

2. UACS STRATEGY

Here we propose a UACS strategy, which is an iterative algorithm. A block diagram of the UACS is shown in Fig. 1, consisting of two main parts: (1) several (P) CS reconstruction algorithms and (2) fusion. Each participating algorithm can use knowledge of the support-set to improve its own estimation over iterations. The fusion is a least-squares based method that helps to estimate the support-set of $\mathbf{x}$. The pseudo-code of the UACS is described in **Algorithm 1**.

Assumption 1 *The sparsity level of $\mathbf{x}$ denoted by T is known a-priori and used as an input to UACS. Also, outputs of all participating algorithms are T-sparse.*

In **Algorithm 1**, l denotes the iteration counter and $\hat{\mathcal{T}}_{p,l}$ denotes the estimated support-set of the p'th participating algorithm. Note that the participating algorithms have an input that is the previous support-set estimate $\hat{\mathcal{T}}_{l-1}$, and then the fusion strategy helps to give a better estimate, $\hat{\mathcal{T}}_l$. The expectation is that the a-priori knowledge of $\hat{\mathcal{T}}_{l-1}$ will improve the estimate $\hat{\mathcal{T}}_{p,l}$. The fusion strategy is comprised of a union of estimated support-sets, and a least-squares based estimation and detection of support-set corresponding to the T largest amplitudes. A stopping criterion either based on non-decreasing residual norm or a fixed number of iterations can be employed. Now we state the main result as follows.

Main result: Without loss of generality, let the first algorithm among P participating algorithms in UACS satisfy a recurrence inequality that converges. Due to the converging recurrence inequality, we get a relation on the reconstruction performance quality for the first algorithm (Algo_1) over iterations, as follows

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c_{1,l}}\| \quad \leq \quad p_1\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l-1}}\| + p_2\|\mathbf{e}\|, \qquad (3)$$

where $p_1 < \frac{1-\delta_{(P+1)T}}{1+\delta_{(P+1)T}} < 1$ and p_2 are known constants, and l denotes the iteration count. If $K_1 = p_1\frac{1+\delta_{(P+1)T}}{1-\delta_{(P+1)T}} < 1$ and K_2 are known constants, then the UACS satisfies the following recurrence inequality

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l}}\| \quad \leq \quad K_1\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l-1}}\| + K_2\|\mathbf{e}\| \qquad (4)$$

that converges over iterations. Using the converging recurrence inequality, after $\lceil \log\left(\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|}\right)/\log(K_1)\rceil$ iterations (with the constraint $\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|} < 1$), the performance of the UACS is bounded by

$$\|\mathbf{x} - \hat{\mathbf{x}}\| \quad \leq \quad \frac{1}{1-\delta_{3T}}\left(2 + \frac{K_2}{1-K_1}\right)\|\mathbf{e}\|. \qquad (5)$$

It can be seen that if there is no measurement noise then we get perfect reconstruction. ∎

The proof of the main result is not shown here for brevity and will be detailed in a later extended manuscript.

3. EXTENDED SUBSPACE PURSUIT

In this section, we propose a general extension of the subspace pursuit (SP) algorithm of [6], referred to as the extended subspace pursuit algorithm (ESP). It can be shown that the ESP has a recurrence inequality that converges and hence can act as the first participating algorithm in UACS (that means ESP is Algo_1 in UACS). The pseudo-code of the ESP algorithm is shown in **Algorithm 2**. In the ESP, there are two extensions in iterations as follows. In the first step of iterations, we create a support-set of cardinality KT satisfying $(K+1)T \leq M$, and in the fifth step, we use the side information $\mathcal{T}_{si}$ by union. Note that $K = 1$ is SP [6] and $K = 2$ is CoSaMP [7]. The side information $\mathcal{T}_{si}$ provides a-priori knowledge of the support-set estimated by the UACS fusion. The recurrence inequality of the ESP is given as follows.

Proposition 1 (Recurrence inequality of ESP)

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c_k}\| \leq a_{esp}\|\mathbf{x}_{\hat{\mathcal{T}}^c_{k-1}}\| + b_{esp}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + c_{esp}\|\mathbf{e}\|,$$

where k denotes the (inner) iteration counter of ESP, and $a_{esp} = \frac{\delta_{(K+2)T}(1+\delta_{(K+2)T})^2}{(1-\delta_{(K+2)T})^4}, b_{esp} = \frac{1+\delta_{(K+2)T}}{2(1-\delta_{(K+2)T})},$ *and* $c_{esp} = \frac{4(1+\delta^2_{(K+2)T})}{(1-\delta_{(K+2)T})^3}$.

Algorithm 2 ESP - Extended Subspace Pursuit

Input: $\mathbf{y}$, $\mathbf{A}$, T, $\mathcal{T}_{si}$, K

Initialization:

1: $k \leftarrow 0$ (k denotes iteration counter)
2: $\mathbf{r}_k \leftarrow \mathbf{y}$ (Residual at k'th iteration)
3: $\hat{\mathcal{T}}_k \leftarrow \emptyset$ (Support-set at k'th iteration)
4: $\hat{\mathbf{x}}_k \leftarrow \mathbf{0}$ (Sparse solution at k'th iteration)

Iteration:

repeat

$k \leftarrow k+1$ (Iteration counter)
1: $\grave{\mathcal{T}}_k \leftarrow \mathtt{supp}(\mathbf{A}^*\mathbf{r}_{k-1}, KT)$
2: $\tilde{\mathcal{V}}_k \leftarrow \grave{\mathcal{T}}_k \cup \hat{\mathcal{T}}_{k-1}$
3: $\tilde{\mathbf{x}}_k$ such that $\tilde{\mathbf{x}}_{\tilde{\mathcal{V}}_k} \leftarrow \mathbf{A}^{\dagger}_{\tilde{\mathcal{V}}_k}\mathbf{y}$; $\tilde{\mathbf{x}}_{\tilde{\mathcal{V}}^c_k} \leftarrow \mathbf{0}$
4: $\acute{\mathcal{T}}_k \leftarrow \mathtt{supp}(\tilde{\mathbf{x}}_k, T)$
5: $\check{\mathcal{V}}_k \leftarrow \acute{\mathcal{T}}_k \cup \mathcal{T}_{si}$
6: $\check{\mathbf{x}}_k$ such that $\check{\mathbf{x}}_{\check{\mathcal{V}}_k} \leftarrow \mathbf{A}^{\dagger}_{\check{\mathcal{V}}_k}\mathbf{y}$; $\check{\mathbf{x}}_{\check{\mathcal{V}}^c_k} \leftarrow \mathbf{0}$
7: $\hat{\mathcal{T}}_k \leftarrow \mathtt{supp}(\check{\mathbf{x}}_k, T)$
8: $\hat{\mathbf{x}}_k$ such that $\hat{\mathbf{x}}_{\hat{\mathcal{T}}_k} \leftarrow \mathbf{A}^{\dagger}_{\hat{\mathcal{T}}_k}\mathbf{y}$; $\hat{\mathbf{x}}_{\hat{\mathcal{T}}^c_k} \leftarrow \mathbf{0}$
9: $\mathbf{r}_k \leftarrow \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}_k$

until *stopping criterion*

Output: $\hat{\mathbf{x}}_{ESP} \leftarrow \hat{\mathbf{x}}_k$, $\hat{\mathcal{T}}_{ESP} \leftarrow \hat{\mathcal{T}}_k$, $\mathbf{r} \leftarrow \mathbf{r}_k$

The proof is not shown here for brevity and will be detailed in a later extended manuscript.

Now, we can analyze the quality of the ESP reconstruction performance by the following theorem.

Theorem 1 *If $a_{esp} < 1$, then after $l^* = \lceil \log\left(\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|}\right)/\log(a_{esp})\rceil$ iterations (with the constraint $\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|} < 1$), the performance of the ESP algorithm converges and is bounded by*

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c_{esp}}\| \leq \frac{b_{esp}}{1-a_{esp}}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \frac{1-a_{esp}+c_{esp}}{1-a_{esp}}\|\mathbf{e}\|, \qquad (6)$$

Proof For sake of clarity, we drop the subscript 'esp'. From proposition 1, we can write,

$$\begin{aligned}\|\mathbf{x}_{\hat{\mathcal{T}}^c_k}\| &\leq a\|\mathbf{x}_{\hat{\mathcal{T}}^c_{k-1}}\| + \mathrm{b}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \mathrm{c}\|\mathbf{e}\| \\ &\leq a\left(a\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l-2}}\| + \mathrm{b}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \mathrm{c}\|\mathbf{e}\|\right) + \mathrm{b}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \mathrm{c}\|\mathbf{e}\|.\end{aligned}$$

At the end of l^* iterations, we get

$$\begin{aligned}\|\mathbf{x}_{\hat{\mathcal{T}}^c}\| &\leq a^{l^*}\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l-l^*}}\| + b\textstyle\sum_{i=0}^{l^*-1} a^i\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + c\sum_{i=0}^{l^*-1} a^i\|\mathbf{e}\| \\ &\overset{(a)}{\leq} a^{l^*}\|\mathbf{x}\| + \tfrac{b}{1-a}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \tfrac{c}{1-a}\|\mathbf{e}\| \\ &\overset{(b)}{=} a^{\lceil \log\left(\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|}\right)/\log(a)\rceil}\|\mathbf{x}\| + \tfrac{b}{1-a}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \tfrac{c}{1-a}\|\mathbf{e}\| \\ &= \tfrac{b}{1-a}\|\mathbf{x}_{\mathcal{T}^c_{si}}\| + \tfrac{1-a+c}{1-a}\|\mathbf{e}\|.\end{aligned}$$

In (a), we have used that $a \leq 1$ and $\|\mathbf{x}_{\hat{\mathcal{T}}^c_{l-l^*}}\| \leq \|\mathbf{x}\|$. In (b) we substituted the value of l^* from the assumption.

Compare (3) and (6) and note the necessity of theorem 1 for the main result of UACS.

4. SIMULATION RESULTS

In this section, we illustrate the performance of an example UACS algorithm where two CS algorithms participate: (1) ESP and (2) a modified orthogonal matching pursuit (OMP). The choice of OMP is due to its popularity and low complexity. The modified OMP is called extended OMP (EOMP) and is shown in **Algorithm 3**. Note that the EOMP can use the side information $\mathcal{T}_{si}$ for improvement in the estimation similar as the ESP. However we do not need theoretical justification of the EOMP for its use in UACS. At this point, we mention that any other off-the-shelf or new CS reconstruction algorithm (may be ad-hoc) could be used instead of EOMP as the second participating algorithm with appropriate modifications. The use of other CS algorithms, such as basis pursuit [3], are already verified and will be reported in an extended manuscript later.

We begin by describing the performance measure that we use to compare various algorithms and our simulation setup to evaluate them, as like [9]. In order to evaluate the average reconstruction accuracy of an algorithm, we use the following performance measure called signal-to-reconstruction-noise-ratio (SRNR), as

$$\text{SRNR} = \frac{\mathcal{E}\{\|\mathbf{x}\|_2^2\}}{\mathcal{E}\{\|\mathbf{x}-\hat{\mathbf{x}}\|_2^2\}}.$$

A higher SRNR means a better reconstruction in the mean square error sense.

We compute the SRNR for our example UACS algorithm and compare with standard SP and OMP as the reference algorithms. Noisy measurements are considered under various conditions described later. Let the ratio of the size of the sampling matrix, defined as fraction of measurements (α) be

$$\alpha = \frac{M}{N}.$$

This ratio measures the level of under-sampling. Our simulation setup is given below:

1. Given T and N, choose α and M such that M is an integer.
2. Construct a sensing matrix $\mathbf{A}$ of dimensions $M \times N$ with elements generated as independent Gaussian random variables distributed as $\mathcal{N}(0,1)$. Scale each column of $\mathbf{A}$ to unit norm.
3. Generate 100 realizations of the random sparse vector $\mathbf{x}$ of dimension N, such that $\|\mathbf{x}\|_0 = T$ with the non-zero indices picked uniformly over the set $\{1,2,\ldots,N\}$. The non-zero components in $\mathbf{x}$ are generated from either a Gaussian source, i.e, $\mathcal{N}(0,1)$ (referred to as Gaussian sparse vector), or set to ones (referred to as binary sparse vector)
4. For each realization, compute the noisy measurement as (1), where $\mathbf{e} \sim \mathcal{N}\left(0, \sigma_n^2 \mathbf{I}_M\right)$ is the measurement noise.

Algorithm 3 EOMP - Extended Orthogonal Matching Pursuit

Input: $\mathbf{y},\ \mathbf{A},\ T,\ \mathcal{T}_{si}$

Initialization:

$k \leftarrow 0$ (k denotes iteration counter)

$\mathbf{r}_k \leftarrow \mathbf{y}$ (Residual at k'th iteration)

$\hat{\mathcal{T}}_k \leftarrow \emptyset$ (Support-set at k'th iteration)

$\hat{\mathbf{x}}_k \leftarrow \mathbf{0}$ (Sparse solution at k'th iteration)

Iteration:

repeat

$k \leftarrow k+1$ (Iteration counter)

$\grave{\mathcal{T}}_k \leftarrow \texttt{supp}(\mathbf{A}^*\mathbf{r}_{k-1}, \min(k,T))$

$\tilde{\mathcal{V}}_k \leftarrow \grave{\mathcal{T}}_k \cup \hat{\mathcal{T}}_{k-1} \cup \mathcal{T}_{si}$

$\tilde{\mathbf{x}}_k$ such that $\tilde{\mathbf{x}}_{\tilde{\mathcal{V}}_k} \leftarrow \mathbf{A}^{\dagger}_{\tilde{\mathcal{V}}_k}\mathbf{y}$; $\tilde{\mathbf{x}}_{\tilde{\mathcal{V}}_k^c} \leftarrow \mathbf{0}$

$\hat{\mathcal{T}}_k \leftarrow \texttt{supp}(\tilde{\mathbf{x}}_k, \min(k,T))$

$\hat{\mathbf{x}}_k$ such that $\hat{\mathbf{x}}_{\hat{\mathcal{T}}_k} \leftarrow \mathbf{A}^{\dagger}_{\hat{\mathcal{T}}_k}\mathbf{y}$; $\hat{\mathbf{x}}_{\hat{\mathcal{T}}_k^c} \leftarrow \mathbf{0}$

$\mathbf{r}_k \leftarrow \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}_k$

until *stopping criterion*

Output: $\hat{\mathbf{x}} \leftarrow \hat{\mathbf{x}}_k,\ \hat{\mathcal{T}} \leftarrow \hat{\mathcal{T}}_k,\ \mathbf{r} \leftarrow \mathbf{r}_k$

5. Execute the UACS, OMP and SP algorithms.
6. Repeat steps 2-4 for 100 times. The performance measure (SRNR) is evaluated by averaging over $100 \times 100 = 10000$ realizations.

Next we define the signal-to-measurement-noise ratio (SMNR) as

$$\text{SMNR} = \frac{\mathcal{E}\{\|\mathbf{x}\|_2^2\}}{\mathcal{E}\{\|\mathbf{e}\|_2^2\}}.$$

Note that, $\mathcal{E}\{\|\mathbf{e}\|_2^2\} = \sigma_e^2 M$. We report the performance of the algorithms for the case with and without measurement noise, where we set SMNR to 20 dB in the former.

We run the simulation setup with $N = 500$ and $T = 20$. α is varied in steps from 0.1 to 0.22. We evaluate the SRNR as a function of the SMNR and α for the different algorithms for different data vectors. The SRNR performance is plotted in Fig. 2. It can be seen that the UACS algorithm provides performance improvement in the region of interest for both Gaussian and binary cases as compared to the reference OMP and SP algorithms. The improvement is more pronounced in the clean measurement scenario.

5. CONCLUSION

We developed a universal reconstruction algorithm for compressive sampling where virtually any CS reconstruction algorithm can participate subjected to appropriate modifications. We proved that under certain conditions, the universal algorithm converges. A case study of one example universal strategy is carried out by simulations and shown to provide better performance than reference algorithms.

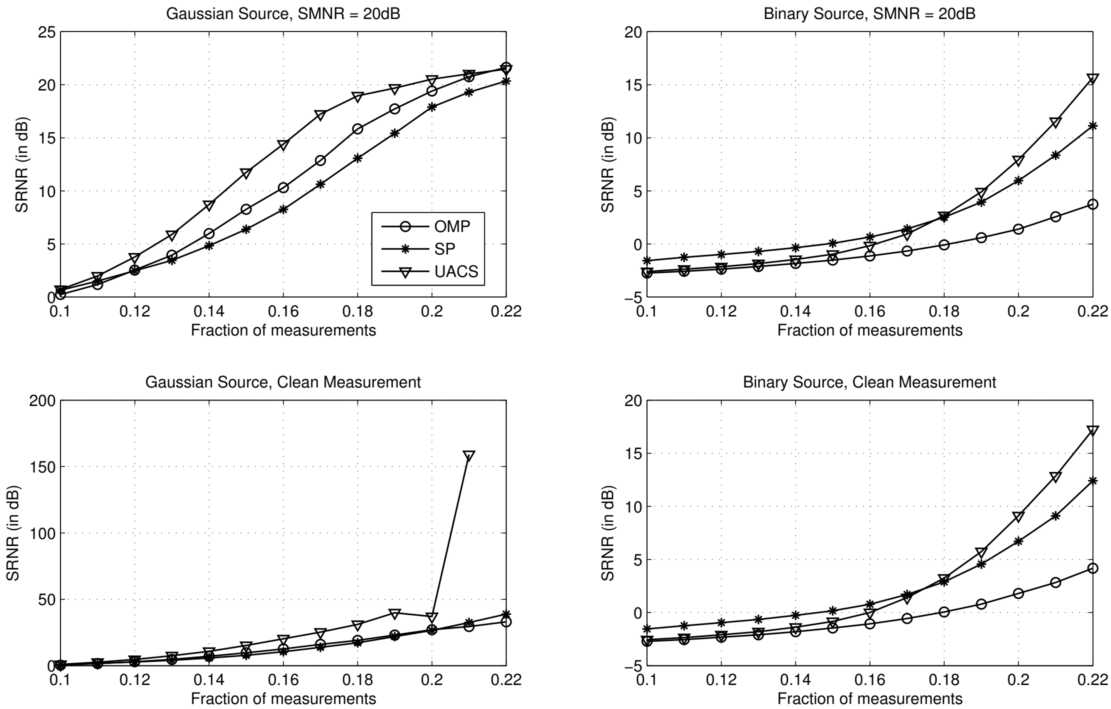

Fig. 2: SRNR performance of the example UACS where two CS reconstruction algorithms participate. The participating algorithms are ESP and EOMP. Performance results are shown for Gaussian and Binary sparse sources.

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