MULTI-DIMENSIONAL CONTINUOUS PHASE MODULATION IN UPLINK OF MIMO SYSTEMS

Mohammad A. Sedaghat[†], *Ralf R. Müller*[†*]

[†] Norwegian University of Science and Technology, Trondheim, Norway
[*] Friedrich-Alexander Universität Erlangen-Nürnberg, Erlangen, Germany

ABSTRACT

Phase Modulation on the Hypersphere (PMH) is considered in which the instantaneous sum power is constant. It is shown that for an i.i.d. Gaussian channel, the capacity achieving input distribution is approximately uniform on a hypersphere when the number of receive antennas is much larger than the number of transmit antennas. Moreover, in the case that channel state information is not available at the transmitter, it is proven that the capacity achieving input distribution is exactly uniform on a hypersphere. Mutual information between input and output of PMH with discrete constellation for an i.i.d. Gaussian channel is evaluated numerically. Furthermore, a spherical spectral shaping method for PMH is proposed to have Continuous Phase Modulation on the Hypersphere (CPMH). In CPMH, the continuous time signal has a constant instantaneous sum power. It is shown that using a spherical low pass filter in spherical domain followed by a Cartesian filter results in very good spectral properties.

Index Terms— Phase modulation, multiple-input multiple-output (MIMO) systems, peak-to-average power ratio (PAPR), single-RF transmitters, continuous phase modulation (CPM), spherical filtering.

1. INTRODUCTION

Load modulated single-RF Multiple-Input Multiple-Output (MIMO) transmitters have been proposed recently in [1] as an efficient implementation method of MIMO transmitters. In [1] and [2], load modulated massive MIMO transmitters have been shown to have high power efficiency and allow for a compact implementation in massive MIMO base stations. In load modulated MIMO transmitters, a central power amplifier is used for all antennas in contrast to the standard implementation method which uses one amplifier per antenna. The power efficiency of the central power amplifier in load modulated MIMO transmitters is affected by the Peak to Average Sum Power Ratio (PASPR). Due to a large number of antennas at massive MIMO base stations, by using load modulated MIMO transmitters, high Peak to Average Power Ratio (PAPR) signals such as Orthogonal Frequency-Division Multiplexing (OFDM) signals can be transmitted using an efficient power amplifier with low back-off [1]. This is, in fact, because of the PASPR decaying with the number of antennas [1].

Using load modulated MIMO transmitters in user terminals does not reduce PASPR as much as in base stations. This is due to the low number of antenna elements in user terminals.

In [3], a novel modulation technique called Phase Modulation on the Hypersphere (PMH) has been proposed to use in load modulated single-RF MIMO transmitters with low to moderate number of antennas in order to have better power efficiency[1]. In PMH, the sum power is fixed, i.e., $\boldsymbol{x}^\dagger \boldsymbol{x} =$ constant; therefore, the central power amplifier in load modulated single-RF MIMO transmitters requires no back-off. In fact, the signal at the central power amplifier has a PASPR of 0dB. PMH can be considered as a generalized form of classical phase modulation for multi-antenna applications. Note that PMH is a different approach than the method proposed in [4] for downlink of massive MIMO systems.

In [3], the sum capacity of PMH in an identity channel[2] has been derived and it has been shown that the capacity is achieved by a signal distributed uniformly on the surface of a hyperball. Discrete constellation PMH has been also proposed in [3] as a set of points distributed uniformly on a hypersphere. Moreover, in [3], the performance of discrete constellation PMH has been evaluated by using some known bounds on spherical codes.

In this paper, we investigate PMH in Gaussian i.i.d. channels. It is shown that in two scenarios, the capacity is achieved by the uniformly distributed signal on a hypersphere: 1) when the number of receive antennas is much larger than the number of transmit antennas, 2) when Channel State Information (CSI) is only available at the receiver. Note that both of these assumptions are valid in a massive MIMO uplink channel. We also evaluate the mutual information between the input

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[1]Note that the new modulation is called Multi-dimensional Phase Modulation in [3]; however, in the current paper we change the name to Phase Modulation on the Hypersphere for better intuition.

[2]By identity channel we refer to the channel $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{n}$.

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and the output of a discrete constellation PMH in Gaussian i.i.d. channels, numerically.

Furthermore, in this paper, we introduce Continuous Phase Modulation on the Hypersphere (CPMH) which has constant instantaneous sum power in continuous time domain, i.e., $\boldsymbol{x}(t)^\dagger \boldsymbol{x}(t) =$ constant. CPMH is constructed by applying a pulse shaping filter to PMH signals. In this paper, a novel spherical pulse shaping filter is proposed using the spherical filtering introduced in [5]. The spherical pulse shaping filter does not affect the peak to average ratio of the sum power. It is shown that to get an appropriate spectrum, we need a Cartesian pulse shaping filter after the spherical pulse shaping filter. Note that the spectral shaping method proposed in this paper can be also used in CPM per antenna proposed in [4].

The rest of this paper is organized as follows: in Section 2, capacity analysis of PMH in a point to point MIMO system with large number of receive antennas is presented. Section 3 discusses PMH capacity with no CSI at the transmitter. Spectral shaping is presented in Section 4. Section 5 shows the numerical results and finally Section 6 concludes the paper.

2. CHANNEL CAPACITY OF PMH IN A POINT TO POINT MIMO CHANNEL WITH LARGE NUMBER OF RECEIVE ANTENNAS

In a massive MIMO system, the aggregate number of all active user antennas is much smaller than the number of base station antennas. In this section, a massive MIMO uplink channel with only one active user is considered, i.e., the interference by the other users is neglected. Assume that the base station and the user are equipped with N and M antennas, respectively, with $N \gg M$ (e.g., $M = 2$ and $N = 100$). Consider the discrete channel model

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x}$ is the input, $\boldsymbol{y}$ is the output, $\boldsymbol{H}$ is the channel matrix and $\boldsymbol{n} \in \mathcal{N}(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$ is the noise vector. The transmitter sends a PMH signal with fixed sum power. Let

$$\boldsymbol{x}^\dagger \boldsymbol{x} = M, \tag{2}$$

without loss of generality. The elements of the channel matrix are assumed to be i.i.d. Gaussian and perfectly known at both transmitter and receiver. The capacity in such a system is described as

$$C = \max_{f(\boldsymbol{x}),\ \boldsymbol{x}^\dagger\boldsymbol{x}=M} \mathrm{I}(\boldsymbol{x};\boldsymbol{y}), \tag{3}$$

where $f(\boldsymbol{x})$ is the probability density function (pdf) of input signal. By using the singular value decomposition of the channel matrix, we have

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\dagger \boldsymbol{x} + \boldsymbol{n}, \tag{4}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices and $\boldsymbol{\Sigma}$ is an $N \times M$ matrix with at most M nonzero diagonal elements. To simplify the problem, we consider an equivalent channel

$$\boldsymbol{y}_p = \boldsymbol{\Sigma}\boldsymbol{x}_p + \boldsymbol{n}_p, \tag{5}$$

where $\boldsymbol{x}_p = \boldsymbol{V}^\dagger \boldsymbol{x}$, $\boldsymbol{n}_p = \boldsymbol{U}^\dagger \boldsymbol{n}$ and $\boldsymbol{y}_p = \boldsymbol{U}^\dagger \boldsymbol{y}$. Note that $\boldsymbol{n}_p$ has the same statistics as $\boldsymbol{n}$ and $\boldsymbol{x}_p^\dagger \boldsymbol{x}_p = M$; therefore, the channel described in (5) is equivalent to the channel in (4).

Let $\boldsymbol{\Sigma}_u$ be the matrix containing the first M rows of $\boldsymbol{\Sigma}$. It can be shown that the capacity of channel (5) is equal to the capacity of

$$\boldsymbol{y}_{p,u} = \boldsymbol{\Sigma}_u \boldsymbol{x}_p + \boldsymbol{n}_{p,u}, \tag{6}$$

where $\boldsymbol{y}_{p,u}$ and $\boldsymbol{n}_{p,u}$ consist of the first M elements of $\boldsymbol{y}_p$ and $\boldsymbol{n}_p$, respectively.

The nonzero singular values of $\boldsymbol{H}$ are the square roots of the eigenvalues of $\boldsymbol{H}^\dagger \boldsymbol{H}$. Furthermore, for N/M growing large, all the eigenvalues of $\boldsymbol{H}^\dagger \boldsymbol{H}$ converge to the same values. Thus, the matrix $\boldsymbol{\Sigma}_u$ converges to a diagonal matrix with identical diagonal elements. Using this asymptotics and considering the findings in [3], one can conclude that the optimum input distribution is approximately the uniform distribution on the hypersphere $\boldsymbol{x}^\dagger \boldsymbol{x} = M$.

Note that, the exact capacity achieving input distribution is not known in this case. Furthermore, it is not even clear whether the exact capacity achieving input distribution in a known channel is continuous or discrete. However, the analysis in this section shows that if $\frac{N}{M} \gg 1$, uniform distribution on a hypersphere is a good approximation for the capacity achieving distribution.

3. CHANNEL CAPACITY IN A POINT TO POINT MIMO SYSTEM WITH NO CSI AT TRANSMITTER

In this section, channel capacity of PMH in a MIMO system is discussed when there is no CSI at the transmitter and perfect CSI at the receiver. This is, in fact, the common scenario considered in massive MIMO uplinks. In massive MIMO systems, the users send pilots and the base stations estimate the channel, hence there is no CSI at transmitter.

Let's consider the channel model in (1). Furthermore, let the elements of $\boldsymbol{H}$ be i.i.d. Gaussian. The mutual information between the transmitter and the receiver is calculated as [6]

$$\begin{aligned} \mathrm{I}(\boldsymbol{x};(\boldsymbol{y},\boldsymbol{H})) &= \mathrm{I}(\boldsymbol{x};\boldsymbol{H}) + \mathrm{I}(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}) = \mathrm{I}(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}) \\ &= \mathop{\mathsf{E}}_{\boldsymbol{H}_0} \{\mathrm{I}(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}=\boldsymbol{H}_0)\} \end{aligned} \tag{7}$$

To calculate the capacity achieving input distribution, we consider

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{A}^\dagger \boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{8}$$

with $\boldsymbol{A}$ being a unitary matrix. It can be shown that $\boldsymbol{H}\boldsymbol{A}^\dagger$ and $\boldsymbol{H}$ have the same distribution when the channel coefficients are i.i.d. Gaussian [6]. Therefore, in the case of no CSI at the transmitter, an equivalent channel is

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}. \tag{9}$$

This actually shows that $\boldsymbol{x}_{\text{opt}}$ and $\boldsymbol{A}\boldsymbol{x}_{\text{opt}}$ have the same distribution, where $\boldsymbol{x}_{\text{opt}}$ denotes the capacity achieving input. Furthermore, $\boldsymbol{A}$ can be any arbitrary unitary matrix. Therefore, $\boldsymbol{x}_{\text{opt}}$ has to be distributed uniformly on the hypersphere $\boldsymbol{x}^\dagger \boldsymbol{x} = M$.

In [3], PMH with discrete constellation has been proposed. It is a more feasible modulation to use in practice. The constellation is made of uniformly distributed points on a hypersphere. To have an estimate of the mutual information of PMH with uniformly distributed inputs on a hypersphere, we consider discrete constellation PMH. The mutual information between the input and the output in this case is presented in the numerical results section.

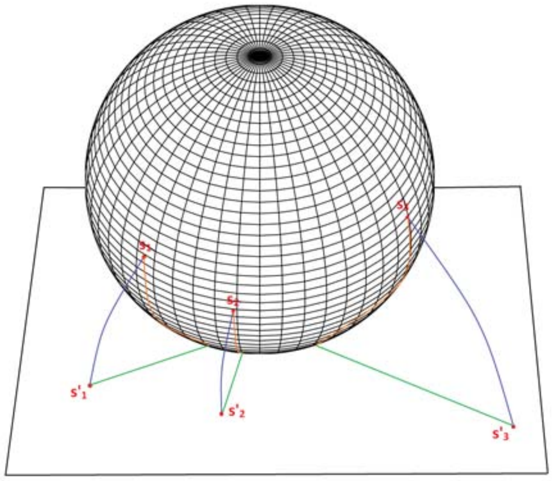

Fig. 1. Averaging in spherical coordinate system.

4. SPECTRAL SHAPING IN CPMH

In this section, we present a novel spectral shaping method to construct the continuous time modulation CPMH. In most of wireless communication systems, pulse shaping is done using a low-pass filter in digital domain. For instance, Root-Raised-Cosine (RRC) filters are used widely in communication systems. Nevertheless, this type of pulse shaping increases PAPR. In PMH, constellation points are on the surface of a hyperball and therefore, it meets PASPR $= 0\text{dB}$ in discrete domain. However, if we use an RRC filter, the PASPR increases.

To design a pulse shaping method for CPMH, we need a method to find a continuous signal on a hypersphere which passes through all the constellation points while is has good spectral properties. Note that a geodesic path does not do a good job due to corners.

In the classical modulation techniques in MIMO systems, pulse shaping is done for every antenna individually. However, for CPMH we need to do a joint pulse shaping. In the single-antenna case, CPMH is identical to the classical CPM modulation. Therefore, one can employ any low-pass filter to the phase component of the complex plane, e.g., a Gaussian filter in GMSK. However, in the case of several antennas, pulse shaping is not straightforward and applying a low-pass filter to the various phase components in a spherical coordinate system results in a wide spectrum. This is in fact due to the multiplications of sine and cosine functions when converting from spherical to Cartesian coordinates. One may use interpolation methods on the hypersphere, e.g., [7], as pulse shaping. However, we look for a general solution of filtering on hypersphere.

In [5], spherical filtering has been introduced by Buss et al. which in summary is as following: consider a filter with impulse response f_n. In a Cartesian coordinate system, filtering a data stream x_n can be done by the convolution $\sum_i f_{n-i} x_i$ which is a weighted average with coefficients f_{n-i}. Now, to apply the filter in the spherical domain, the key point is the relation between taking average in spherical and Cartesian coordinate systems. Note that in [5], it is shown that for spherical filtering, the filter coefficients should meet

$$f_i \geq 0 \quad \text{and} \quad \sum_i f_i = 1. \tag{10}$$

Let $\boldsymbol{s}_i$ be some constellation points on a hypersphere and also $\boldsymbol{s}_{\text{av}}$ be the average of the points on the hypersphere. Fig. 1 shows an example of a three dimensional case in which $\boldsymbol{s}_{\text{av}}$ is at the south pole of the sphere. Moreover, let the point $\acute{\boldsymbol{s}}_i$ be obtained by projecting the points $\boldsymbol{s}_i$ on the tangent plane at the point $\boldsymbol{s}_{\text{av}}$, where the projection is done such that the distance between $\boldsymbol{s}_i$ and $\boldsymbol{s}_{\text{av}}$ on the hypersphere is equal to the distance between $\acute{\boldsymbol{s}}_i$ and $\boldsymbol{s}_{\text{av}}$ on the tangent plane. Then, the point $\boldsymbol{s}_{\text{av}}$ is the average of points $\boldsymbol{s}_i$ in the spherical coordinate system, if $\boldsymbol{s}_{\text{av}}$ is the average of the points $\acute{\boldsymbol{s}}_i$ in the Cartesian coordinate system.

Filtering on a hypersphere cannot be done in a single step but requires iterations [5]. In [5], some iterative methods are proposed based on the similarity mentioned above for filtering in spherical and Cartesian coordinate systems. Here, we employ the algorithm called *Algorithm A1* in [5]. For a hypersphere with radius 1, the algorithm is summarized as follows [5]:

1. Select an initial point $\boldsymbol{s}_{\text{av}}$. A good initial guess is $\frac{\sum_i f_{n-i}\boldsymbol{s}_i}{\|\sum_i f_{n-i}\boldsymbol{s}_i\|}$.
2. Project all the points on hypersphere to the tangent plane at the point $\boldsymbol{s}_{\text{av}}$. We call them $\acute{\boldsymbol{s}}_i$.
3. Calculate the weighted average $\acute{\boldsymbol{s}}_{\text{av}} = \sum_i f_{n-i}\acute{\boldsymbol{s}}_i$.
4. Unless $\acute{\boldsymbol{s}}_{\text{av}}$ is close enough to $\boldsymbol{s}_{\text{av}}$, project $\acute{\boldsymbol{s}}_{\text{av}}$ back to the surface of hypersphere, call it $\boldsymbol{s}_{\text{av}}$, and go back to step 2.

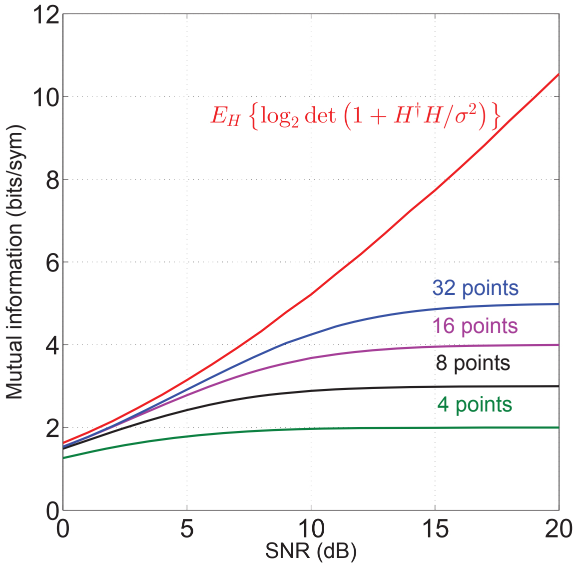

Fig. 2. The mutual information of discrete PMH with 2 antennas vs. SNR in i.i.d. Gaussian channel.

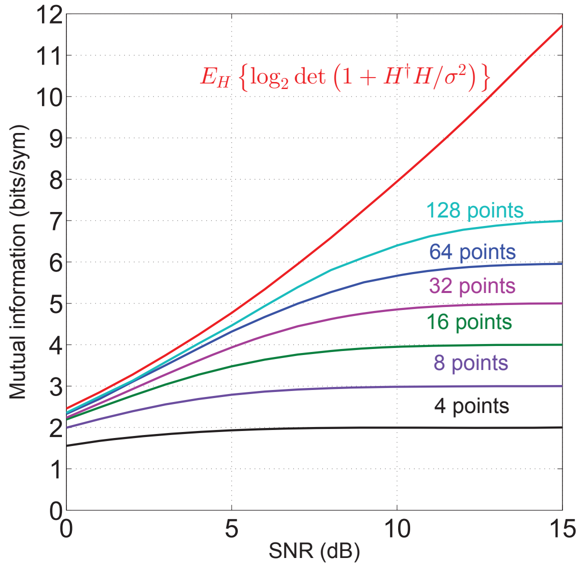

Fig. 3. The mutual information of discrete PMH with 3 antennas vs. SNR in i.i.d. Gaussian channel.

In this paper, we use the filter coefficients $\frac{g_k^2}{\sum_i g_i^2}$ for spherical filtering, where g_i are the RRC filter coefficients. We call this filter RRC^2. Our investigations show that filtering in the spherical domain results in insufficient sidelobe suppression; therefore, we propose to use a low-pass filter in the spherical domain followed by a low-pass filter in the Cartesian domain. The good point is that the second filter hardly increases the PASPR since it only affects sidelobes with very low energy.

5. NUMERICAL RESULTS

5.1. Numerical results on mutual information of discrete constellation PMH in Gaussian i.i.d. channels

To show the performance of discrete constellation PMH, we estimate the mutual information of it using the toolbox proposed in [8]. Furthermore, we use the spherical codes as the constellation points as explained in [3].

The channel coefficients are assumed to be i.i.d. Gaussian. For each realization of the channel, we first estimate the mutual information using 10^5 symbols, and then, averaging is done over 10^4 channel realizations. The number of the antennas at receiver and transmitter are assumed to be equal. The results for 2 and 3 antennas are shown in Fig. 2 and Fig. 3, respectively. The mutual information for Gaussian input is also plotted for sake of comparison. The figures show that PMH with fixed constellation has an acceptable mutual information compared to the mutual information of Gaussian input.

5.2. Numerical results on spectral shaping

Here, the numerical results of the spherical filtering are presented. An oversampling factor of 32 was used to simulate continuous time signals. We consider a standard RRC filter with length 1024 and roll-off factor 0.3 .

Fig. 4 shows the result for 4 antennas. The figure shows that the RRC^2 filter in the spherical domain results in only 17dB sidelobe suppression. However, by accepting a small PASPR, we can use a post filtering by adding an RRC filter in the Cartesian coordinate system, i.e., per antenna filtering. In fact, the spherical filter shapes the spectrum approximately, and the second filter kills the side lobes. The main point is that the first filter keeps PASPR at 0dB and the second filter operates on side lobes and changes PASPR only slightly. Note that for the spherical RRC filter, reducing the roll-off factor does not improve the spectrum and a roll-off around 0.3 has the best performance.

Due to lack of space, further investigations are referred to the extended version of this paper.

Next, the resulted PASPR due to using the second filter is plotted in Fig. 5 versus the number of antennas. It is observed that the spherical filtering followed by a Cartesian post filtering reduces PASPR by about $0.5\mathrm{dB} - 0.8\mathrm{dB}$ compared to the Cartesian RRC filtering. Note that in this paper we use spherical codes as constellation points which have phase shifts of up to π. In the longer version of this paper, we propose some other mapping with better PASPR properties.

Note that the best Inter Symbol Interference (ISI) free low pass filter for CPMH is unknown and designing appropriate filters can still be done in the future.

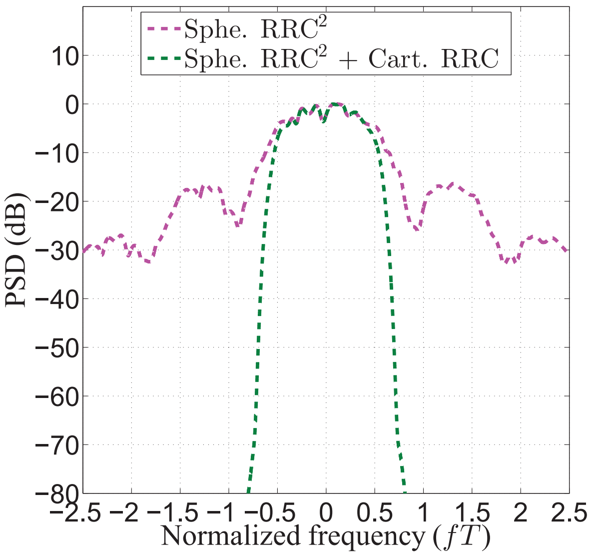

Fig. 4. PSD versus normalized frequency for different filters for 4 antennas.

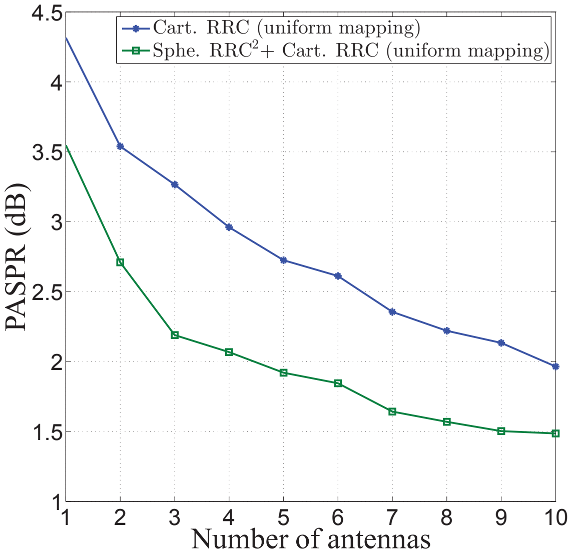

Fig. 5. PASPR versus the number of antennas for spherical RRC^2 filtering followed by Cartesian post filtering and a Cartesian RRC filtering.

6. CONCLUSION

In this paper, some new results on the capacity of PMH in Gaussian channels were presented. It was proven that in the uplink of massive MIMO with one user, the capacity achieving input is uniformly distributed on a hypersphere when channel coefficients are i.i.d. Gaussian. Numerical results on mutual information of discrete constellation PMH in an i.i.d. Gaussian channel were also presented.

Furthermore, a new spectral shaping method for CPMH was proposed using spherical filtering. It was shown that using two filters in spherical and Cartesian domains results in excellent spectral properties while the PASPR of the systems is hardly affected.

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