

COEXISTENCE OF G.FAST AND VDSL IN FTTDP AND FTTC DEPLOYMENTS

Rainer Strobel*[†], Wolfgang Utschick*

*Fachgebiet Methoden der Signalverarbeitung
Technische Universität München, 80290 München, Germany
{rainer.strobel,utschick}@tum.de

[†]Lantiq Deutschland GmbH
85579 Neubiberg, Germany
rainer.strobel@lantiq.com

ABSTRACT

Hybrid copper/fiber networks bridge the gap between the fiber link and the customer by using copper wires over the last meters. This solution combines energy efficiency and low cost of the copper network with higher fiber data rates. ITU recently finished the G.fast standard for high speed data transmission on copper wires for this application.

Coexistence with legacy VDSL2 systems is an important topic for the introduction of the new technology, as the systems share a significant part of the frequency spectrum.

This paper investigates the performance of G.fast coexisting with VDSL2. Methods for decentralized spectrum optimization and protection of legacy services are presented.

Index Terms— spectrum optimization, alien crosstalk, iterative water-filling, FTTdp, coexistence

1. INTRODUCTION

Coexistence of G.fast [1] with legacy ADSL and VDSL2 [2] is a key to success of the new technology. The VDSL2 fiber to the curb-architecture (FTTC) is extended [3] by the fiber to the distribution point (FTTdp) architecture using G.fast for the copper link between the distribution point (DP) and the customer premises equipment (CPE). Some of the subscribers are still served with the legacy service, while others have been upgraded to G.fast. But they share the same cable binder, and therefore, there is crosstalk between the services (see Fig. 1).

Another coexistence scenario is shown in Fig. 2. The FTTC locations can be upgraded to serve G.fast. Higher data rates are available for subscribers located close to the cabinet, while subscribers with longer lines or with legacy equipment are served with the legacy service.

Both G.fast deployment strategies result in mutual couplings between G.fast and VDSL2 services. This paper investigates the performance impact of such couplings and shows methods for crosstalk management, which are primarily implemented on the G.fast side and do not require coordination between G.fast and VDSL2.

This work has been founded by the research project “FlexDP - Flexible Breitband Distribution Points”, funded by the Bayerische Forschungsstiftung

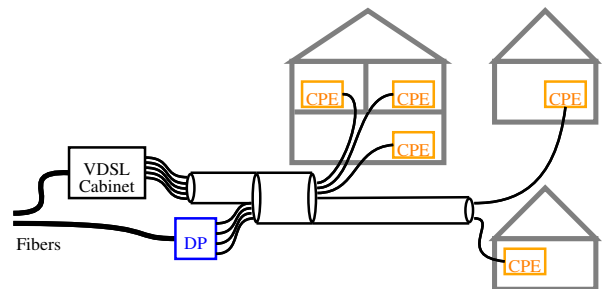


Fig. 1. FTTdp coexisting with FTTC in one cable bundle

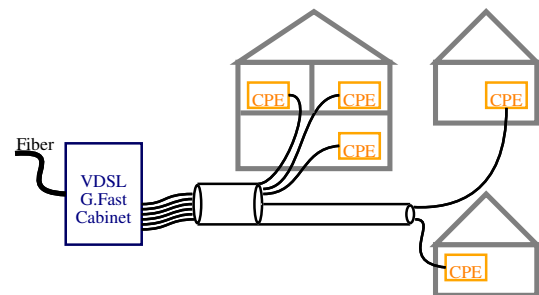


Fig. 2. VDSL2/G.fast multi-mode cabinet

2. SYSTEM MODEL

2.1. Transmission Technology Basics

G.fast as well as VDSL2 are based on DMT modulation [2] [1]. We assume multi-carrier transmission with K carriers on L lines in upstream (us) and downstream (ds) direction. The transmit power $p_{ds\ l}^{(k)}$ for downstream and $p_{us\ l}^{(k)}$ can be configured for each line $l = 1, \dots, L$ and carrier $k = 1, \dots, K$ to achieve the desired power spectral densities (PSD) $\psi_{ds\ l}(f)$ and $\psi_{us\ l}(f)$ according to

$$p_{(ds/us)\ l}^{(k)} = \int_{f_k - \Delta f_l / 2}^{f_k + \Delta f_l / 2} \psi_{(ds/us)\ l}(f) df \quad (1)$$

with a subcarrier spacing Δf_l and a carrier frequency $f_k = k\Delta f_l$. The line indices are grouped into lines using VDSL2

$l \in \mathbb{I}_V$ and G.fast $l \in \mathbb{I}_F$. VDSL2 and G.fast use different subcarrier spacings, $\Delta f_l = 4.3125$ kHz for $l \in \mathbb{I}_V$ and $\Delta f_l = 51.75$ kHz for $l \in \mathbb{I}_F$. While A/VDSL use frequency division duplexing (FDD), G.fast uses time division duplexing (TDD).

The spectral characteristics of both services are very different. G.fast uses a low per-line sum transmit power of $p_{\text{sum } l} = 4$ dBm for $l \in \mathbb{I}_F$, which is spread over a wide frequency band from 2.2 MHz to 106 MHz, while VDSL2 uses more than 10 times higher transmit power of $p_{\text{sum } l} = 14.5$ dBm for $l \in \mathbb{I}_V$ which is concentrated on a smaller frequency band from 25 kHz to 17.6 MHz. VDSL2 runs a DMT symbol rate of $1/t_{\text{sym } l} = 4$ kHz for $l \in \mathbb{I}_V$ while G.fast uses a much higher symbol rate of $1/t_{\text{sym } l} = 48$ kHz for $l \in \mathbb{I}_F$.

For each line l , subcarrier k and direction (us, ds), a certain signal-to-interference-noise ratio $SINR_{(\text{ds/us})l}^{(k)}$ is present.

The number of bits $b_{(\text{ds/us})l}^{(k)}$ per symbol is obtained by

$$b_{(\text{ds/us})l}^{(k)} = \min \left(\left\lfloor \log_2 \left(1 + \frac{SINR_{(\text{ds/us})l}^{(k)}}{\Gamma} \right) \right\rfloor, b_{\text{max } l} \right) \quad (2)$$

with Γ that accounts for the SNR gap [4] and a maximum supported bit loading $b_{\text{max } l} = 12$ for $l \in \mathbb{I}_F$ and $b_{\text{max } l} = 15$ for $l \in \mathbb{I}_V$. The data rate of line l is given by

$$R_{(\text{ds/us})l} = \frac{\eta_{(\text{ds/us})l}}{t_{\text{sym } l}} \sum_{k=1}^K b_{(\text{ds/us})l}^{(k)}. \quad (3)$$

Transmission overhead is incorporated by an efficiency factor η where the values $\eta_{\text{ds } l} = 0.675$ and $\eta_{\text{us } l} = 0.2625$ for $l \in \mathbb{I}_F$ are used for G.fast while for VDSL2, $\eta_{\text{ds } l} = \eta_{\text{us } l} = 0.925$ for $l \in \mathbb{I}_V$ is assumed. Additional cyclic extension overhead is considered within t_{sym} .

2.2. Mutual Couplings between G.fast and VDSL2

Due to the different duplexing schemes and the overlapped frequency spectrum between 2.2 and 17.6 MHz, there is crosstalk between G.fast and VDSL2, called alien crosstalk. As shown in Fig. 3, there are four different coupling paths.

Each of the four receiving points (VDSL2 Cabinet, VDSL2 CPE, G.fast DP, G.fast CPE) experiences noise from three sources. For the affected (victim) line v , it is caused by NEXT (near end crosstalk) $\psi_{\text{NEXT } vd}(f)$ and far-end crosstalk (FEXT) $\psi_{\text{FEXT } vd}(f)$ from the interfering (disturber) line d and receiver noise ψ_n .

Self-FEXT, the far-end crosstalk between lines of the same service is reduced by crosstalk cancellation. Therefore, G.fast victim lines $v \in \mathbb{I}_F$ are disturbed by VDSL2 lines $d \in \mathbb{I}_V$ and the vice versa. We distinguish between downstream $H_{\text{FEXT ds } vd}^{(k)}$ and upstream $H_{\text{FEXT us } vd}^{(k)}$ FEXT and DP-side $H_{\text{NEXT dp } vd}^{(k)}$ and CPE-side $H_{\text{NEXT cpe } vd}^{(k)}$ NEXT couplings for subcarrier k .

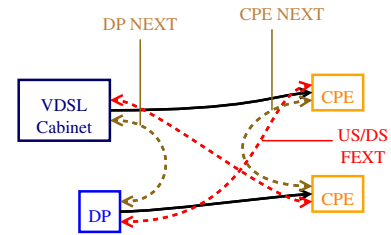


Fig. 3. Crosstalk couplings between G.fast and VDSL2

The interference plus noise PSD $\psi_{\text{in } v}(f)$ is given by

$$\psi_{(\text{ds/us})v}^{\text{in}}(f) = \sum_{d \in \mathbb{I}_{\text{dist } v}} \left(\psi_{(\text{ds/us})vd}^{\text{FEXT}}(f) + \psi_{(\text{dp/cpe})vd}^{\text{NEXT}}(f) \right) + \psi_n \quad (4)$$

where $\mathbb{I}_{\text{dist } v}$ are the indices of lines interfering v . The background noise is assumed to be additive white Gaussian (AWGN) zero-mean noise with a flat noise PSD ψ_n . The NEXT and FEXT PSDs used in Eq. (4) are derived by multiplication of the disturber transmit PSD $\psi_d(f)$ with the squared crosstalk transfer function, e. g., $H_{\text{NEXT dp } vd}(f)$ to be $\psi_{\text{NEXT dp } vd}(f) = \psi_{\text{ds } d}(f) |H_{\text{NEXT dp } vd}(f)|^2$. The crosstalk from disturber d to victim v is modeled according to [5].

The interference plus noise PSD can be converted into a noise power per tone $p_{\text{in } v}^{(k)} = \int_{f_k - \Delta f/2}^{f_k + \Delta f/2} \psi_{\text{in } v}(f) df$, which gives the $SINR$ to be

$$SINR_{(\text{ds/us})v}^{(k)} = \frac{|H_{(\text{ds/us})v}^{(k)}|^2 p_{(\text{ds/us})v}^{(k)}}{p_{\text{in } (\text{ds/us})v}^{(k)}} \quad (5)$$

with a direct channel gain $H_{\text{ds } v}^{(k)}$ in downstream and $H_{\text{us } v}^{(k)}$ in upstream direction for line v .

2.3. Spectral Masks

The used transmit spectrum $\psi(f)$ is bounded by limit PSDs or masks $\psi_{\text{limit}}(f)$, which are defined in the standards for G.fast [6] and for VDSL2 [2]. G.fast uses only one limit PSD due to TDD while DSL uses different limit PSDs and different in-band frequencies $f \in \mathcal{F}_{\text{Vds}}$ and $f \in \mathcal{F}_{\text{Vus}}$ for downstream and upstream direction. The in-band PSD holds for frequencies where data is transmitted. It is controlled in frequency domain with per-tone power values $p_l^{(k)}$. To protect the legacy service, power back-off masks $\psi_{\text{pbo ds}}(f)$ for downstream and $\psi_{\text{pbo us}}(f)$ for upstream may be used, as described later.

The out-of-band transmit spectrum cannot be reduced to zero. It is reduced with time domain transmit filters according to the standard requirements. Fig. 4 shows the G.fast limit PSD for the 106 MHz profile together with an example of a measured transmit PSD with the corresponding in-band and out-of-band characteristics. The actual out-of band

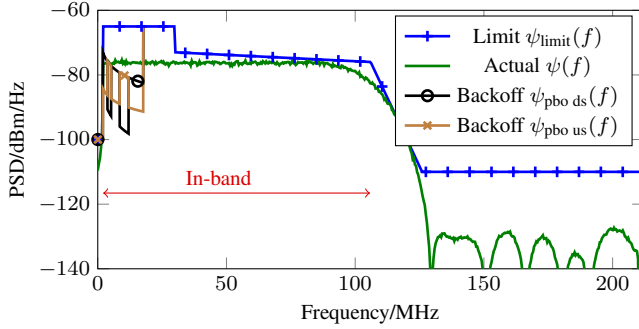


Fig. 4. In-band and out-of-band spectrum for 106 MHz G.fast

PSD depends on the filters used in a specific implementation. $\psi(f)$ must also satisfy the per-line sum-power $p_{\text{sum } l}$ constraint. Therefore, $\psi(f)$ in Fig. 4 is below the limit mask in most cases.

2.4. Long Range G.fast and 35 MHz VDSL2

Some operators prefer an intermediate upgrade step between vectored VDSL2 and G.fast FTTdp. There are two approaches to serve this demand. Besides the idea of an extended VDSL2 using 35 MHz bandwidth, which increases the overlapping frequency band between G.fast and VDSL2, we focus on long reach G.fast as an alternative solution. It allows higher transmit power for G.fast, e.g., 8 dBm or 14.5 dBm in combination with the higher VDSL2 limit mask for frequencies below 30 MHz and a longer cyclic extension to support long lines. This system is compatible to a future upgrade to G.fast FTTdp, but it increases the alien interference into legacy VDSL2 lines when no further actions are taken.

3. COEXISTENCE STRATEGIES

This analysis focuses on two coexistence strategies, crosstalk avoidance and optimized overlapped spectrum.

3.1. Crosstalk Avoidance Strategy

The straight-forward strategy to guarantee spectral compatibility between G.fast and VDSL2 is crosstalk avoidance. In this case, the VDSL2 frequency bands are completely excluded from the used G.fast frequency band and only the out-of-band spectrum overlaps. This approach results in a minimum disturbance of the legacy service, but it causes a substantial reduction of the G.fast data rates. It shall be noted that there is still some interference between both services due to their out-of-band transmit power.

3.2. Overlapped Spectrum Strategy

A better approach is to allow a spectral overlap between the services and optimize the transmit spectrum with spectral

constraints. This is a similar approach as for cognitive radio with temperature-interference constraints in the wireless context [7]. The general optimization problem with spectral mask and per-line sum-power constraints of each service reads as

$$\begin{aligned} & \max_{p_l^{(k)} \forall k=1, \dots, K; l=1, \dots, L} \sum_{l=1}^L R_{\text{ds } l} + R_{\text{us } l} & (6) \\ & \text{s.t. } 0 \leq p_{(\text{ds/us}) l}^{(k)} \leq p_{\text{mask } (\text{ds/us}) l}^{(k)} \forall k = 1, \dots, K; l = 1, \dots, L \\ & \text{s.t. } \sum_{k=1}^K p_{\text{ds } l}^{(k)} \leq p_{\text{sum } l} \forall l = 1, \dots, L \\ & \text{s.t. } \sum_{k=1}^K p_{\text{us } l}^{(k)} \leq p_{\text{sum } l} \forall l = 1, \dots, L \end{aligned}$$

with a sum-power limit $p_{\text{sum } l}$ and a spectral mask constraint $p_{\text{mask } l}^{(k)} = \int_{f_k - \Delta f/2}^{f_k + \Delta f/2} \psi_{\text{limit } l}(f) df$.

No coordination between legacy lines and G.fast lines is assumed. Each group of lines determines the optimized transmit spectrum independent of the others. The systems are only coupled by the crosstalk which each of the receivers observes.

Spectrum Optimization: The optimal transmit spectrum is obtained using an iterative water-filling approach [8]. G.fast as well as VDSL2 have spectral mask constraints and sum-power constraints. Water-filling with support of spectral mask constraints has been investigated in [9]. A further extension of the algorithm is required to include the bit loading upper bound b_{max} into the optimization. Otherwise, power is wasted on carriers with high SINR.

The maximum bit loading is transformed into an equivalent spectral mask constraint. It must be noted that, in opposite to the spectral mask constraint [9], the maximum bit loading constraint may affect convergence. The maximum bit loading is converted into a maximum required SINR $\text{SINR}_{\text{max } l} = 2^{b_{\text{max } l}} + 1$ and incorporated into the spectral mask constraint

$$\tilde{p}_{(\text{ds/us}) l}^{(k)} = \min \left(p_{(\text{ds/us}) l}^{(k)}, \text{SINR}_{\text{max } l} \cdot \Delta_{\text{SNR}}, p_{\text{mask } (\text{ds/us}) l}^{(k)} \right) \quad (7)$$

with an additional margin $\Delta_{\text{SNR}} \geq 1$ to relax the bit loading constraint and avoid rate reductions during iterative water-filling convergence.

For water-filling with spectral mask and sum-power constraints, the carriers $k = 1, \dots, K$ for each line l are grouped into three groups. Carriers $k \in \mathbb{I}_{l,0}$ with zero power, carriers $k \in \mathbb{I}_{l,\text{mask}}$ where the spectral mask constraint $p_l^{(k)} \leq \tilde{p}_{\text{mask } l}^{(k)}$ is active and the remaining carriers $\mathbb{I}_{l,\text{fill}}$. The power per carrier and line for each of these groups is given by

$$p_{(\text{ds/us}) l}^{(k)} = \begin{cases} 0 & \text{for } k \in \mathbb{I}_{l,0} (\text{ds/us}) \\ \tilde{p}_{\text{mask } (\text{ds/us}) l}^{(k)} & \text{for } k \in \mathbb{I}_{l,\text{mask}} (\text{ds/us}) \\ \mu_{(\text{ds/us}) l} - \frac{p_{\text{ni } (\text{ds/us}) l}^{(k)}}{|H_{(\text{ds/us}) l}^{(k)}|^2} & \text{otherwise} \end{cases} \quad (8)$$

and μ_l for line l and direction (us or ds) is given by the water-filling solution

$$\mu_l = \frac{1}{|\mathbb{I}_{l,\text{fill}}|} \left(p_{\text{sum } l} + \sum_{k \in \mathbb{I}_{l,\text{fill}}} \frac{p_{\text{in } l}^{(k)}}{|H_l^{(k)}|^2} - \sum_{k \in \mathbb{I}_{l,\text{mask}}} \tilde{p}_{\text{mask } l}^{(k)} \right). \quad (9)$$

Power Back-Off: Sometimes, sum-rate optimization is not sufficient, because it is required to protect the legacy service. This is done by limiting the crosstalk into VDSL2 to a maximum value p_{nmax} according to

$$p_{\text{ni}(\text{ds/us})v}^{(k)} \leq p_{\text{nmax}} \quad (10)$$

for all G.fast lines $d \in \mathbb{I}_F$ disturbing legacy line $v \in \mathbb{I}_V$. Solving Eq. (6) including the constraint (10) is not feasible in practice, as it requires knowledge of the crosstalk between G.fast and VDSL2 lines as well as a central coordination for both services.

With the help of crosstalk statistics, more precisely, with worst case values for aggregate DP-side NEXT $h_{\text{NEXTsum dp}}^{(k)}$, CPE-side NEXT $h_{\text{NEXTsum cpe}}^{(k)}$ and FEXT $h_{\text{FEXTsum}}^{(k)}$, a more strict limit PSD for the G.fast lines is defined, which guarantees that (10) is satisfied with a certain probability.

Approximations for worst case cabinet NEXT and FEXT have been proposed in [10]. The channel model [5], which is used in this paper, indicates that the worst case approximations presented there can be applied with small changes for this application. Therefore, the following approximations are used for the worst case couplings

$$|h_{\text{NEXTsum dp}}(f)|^2 = N_{\text{dist}}^{0.6} \left(\frac{f}{f_0} \right)^{1.5} 10^{-44/10} \quad (11)$$

$$|h_{\text{NEXTsum cpe}}(f)|^2 = N_{\text{dist}}^{0.6} \left(\frac{f}{f_0} \right)^{0.75} 10^{-44/10} \quad (12)$$

$$|h_{\text{FEXTsum}}(f)|^2 = N_{\text{dist}}^{0.6} \left(\frac{f}{f_0} \right) \frac{d_{\text{avg}}}{1\text{km}} 10^{-39/10} \quad (13)$$

with $f_0 = 1$ MHz and a number of N_{dist} disturbers and the average disturber line length d_{avg} . Fig. 5 shows the comparison between these approximations and the crosstalk model.

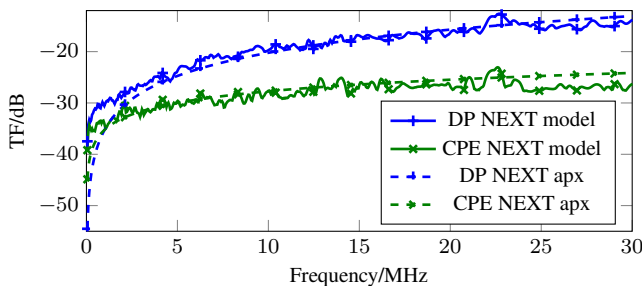


Fig. 5. Worst case NEXT and FEXT for 0.5mm PE line

The power limit $\hat{p}_{\text{mask}}^{(k)}$ including the power back-off for G.fast lines for Eq. (6) is

$$\hat{p}_{\text{mask}}^{(k)} = \begin{cases} \min \left(p_{\text{pbo}}^{(k)}, \tilde{p}_{\text{mask}}^{(k)} \right) & \forall k : f_k \in \mathcal{F}_{\text{Vus}} \cup \mathcal{F}_{\text{Vds}} \\ \tilde{p}_{\text{mask}}^{(k)} & \forall l \in \mathbb{I}_F \\ \tilde{p}_{\text{mask}}^{(k)} & \text{otherwise} \end{cases} \quad (14)$$

The upstream and downstream power constraint values $p_{\text{pbo ds}}^{(k)}$ and $p_{\text{pbo us}}^{(k)}$ are derived from upstream and downstream back-off PSD masks for G.fast $\psi_{\text{pbo ds}}(f)$ and $\psi_{\text{pbo us}}(f)$ using Eq. (1) and the masks are given by

$$\psi_{\text{pbo ds}}(f) = \begin{cases} \frac{\psi_{\text{nmax}}}{|h_{\text{NEXTsum dp}}(f)|^2 |h_{\text{dp cab}}(f)|^2} & \text{for } f \in \mathcal{F}_{\text{Vus}} \\ \frac{\psi_{\text{nmax}} |h_{\text{dp cab}}(f)|^2}{|h_{\text{FEXTsum}}(f)|^2} & \text{for } f \in \mathcal{F}_{\text{Vds}} \end{cases} \quad (15)$$

$$\psi_{\text{pbo us}}(f) = \begin{cases} \frac{\psi_{\text{nmax}}}{|h_{\text{FEXTsum}}(f)|^2 |h_{\text{dp cab}}(f)|^2} & \text{for } f \in \mathcal{F}_{\text{Vus}} \\ \frac{\psi_{\text{nmax}}}{|h_{\text{NEXTsum cpe}}(f)|^2} & \text{for } f \in \mathcal{F}_{\text{Vds}} \end{cases} \quad (16)$$

where $h_{\text{dp cab}}(f)$ is the transfer function of the cable between DP and street cabinet and $\psi_{\text{nmax}} = p_{\text{nmax}}/\Delta f$. The resulting back-off PSDs for a multi-mode cabinet are shown in Fig. 4.

4. SIMULATIONS

For both scenarios, Fig. 1 and 2, crosstalk avoidance is compared with overlapped spectrum with and without power back-off for G.fast. In Fig. 6, 7 and 8, the dashed lines are data rates for overlapped spectrum while the solid lines are for crosstalk avoidance. Each figure shows downstream (DS) and upstream (US) rates for DSL and G.fast.

Simulation conditions are the 0.5mm PE quad cable with up to 30 pairs according to [11], a colored background noise of -140 dBm/Hz below 30 MHz and -150 dBm/Hz above 30 MHz according to [12] and $\Delta SNR = 2$ dB. Power back-off settings for G.fast use $\psi_{\text{nmax}} = -120$ dBm/Hz.

4.1. FTTdp Scenario

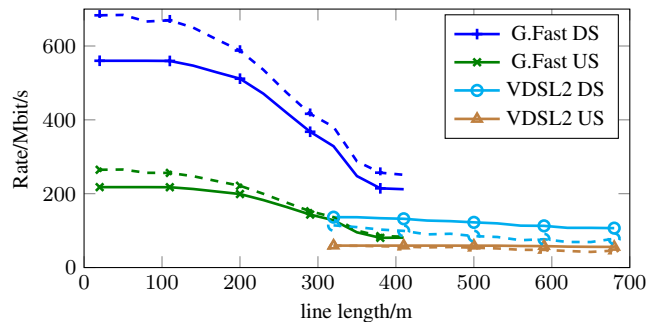


Fig. 6. Rate vs. reach curves for G.fast from the DP

For the FTTdp scenario, a cable length of 300 m between VDSL2 street cabinet and G.fast DP is assumed. The final

drop between DP and the CPEs is assumed to be random uniformly distributed between 10 and 400 m. The VDSL2 and G.fast lines are randomly placed within the binder. The scenarios of interest are crosstalk avoidance with G.fast starting at 23 MHz, and overlapped spectrum applying iterative water-filling. Fig. 6 shows the resulting data rates. The max. downstream gain for G.fast using the overlapped spectrum is about 120 Mbit/s with an mean gain of 75 Mbit/s compared to crosstalk avoidance.

However, there is a substantial rate loss for the legacy lines due to alien crosstalk, as can be seen in the lower right of Fig. 6. With power back-off, the rate vs. reach curves as shown in Fig. 7 are achieved. The downstream rate gain is still up to 80 Mbit/s and 50 Mbit/s on average, but with a small loss for legacy lines.

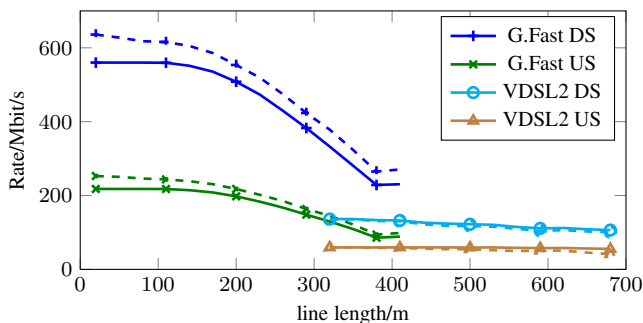


Fig. 7. G.fast from the DP with power back-off

4.2. Multi-Mode Cabinet Scenario

In the multi-mode cabinet scenario, G.fast and VDSL2 lines start at the same point which is the worst case in terms of alien NEXT. The line length from the cabinet to the CPEs is uniformly distributed between 10 m and 700 m. Long range G.fast as described in Sec. 2.4 is used for this scenario, because standard G.fast doesn't support lines longer than 400 m.

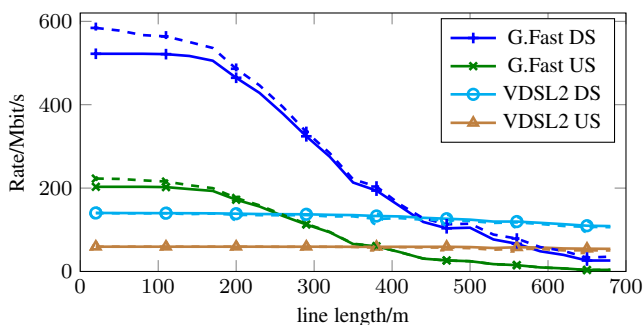


Fig. 8. G.fast and VDSL2 FTTC with power back-off

Power back-off avoids disturbance of the legacy lines and still gives some gain for G.fast lines, which is max. 50 Mbit/s in downstream, as Fig. 8 shows.

5. CONCLUSION

For FTTdp, the overlapped spectrum approach gives a significant gain, compared to crosstalk avoidance. The NEXT between VDSL2 and G.fast is attenuated through the line length between cabinet and DP. FEXT from G.fast is small, because the transmit power of G.fast is small.

For co-located scenarios, crosstalk avoidance causes performance losses for VDSL2, especially in upstream direction. This is maintained by the proposed power back-off, which allows to adjust the performance loss. Fig. 8 indicates that very long lines may benefit from switching to VDSL2 rather than using long reach G.fast with the selected power back-off.

Overlapped spectrum still outperforms crosstalk avoidance, and, in the worst case, converges to the crosstalk avoidance rates with conservative power back-off settings.

REFERENCES

- [1] ITU-T Rec. G.9701, "Fast Access to Subscriber Terminals - Physical layer specification," 2013.
- [2] ITU-T Rec. G.993.2, "Very high speed digital subscriber line transceivers 3 (VDSL2)," 2006.
- [3] F. Mazzenga, M. Petracca, F. Vatalaro, R. Giuliano, and G. Ciccarella, "Coexistence of fttc and ftdp network architectures in different vdsl2 scenarios," *Transactions on Emerging Telecommunications Technologies*, 2014.
- [4] J. Cioffi, "A multicarrier primer," *ANSI TIE1*, vol. 4, pp. 91–157, 1991.
- [5] R. Strobel, R. Stolle, and W. Utschick, "Wideband Modeling of Twisted-Pair Cables for MIMO Applications," in *IEEE Globecom 2013 - Symposium on Selected Areas in Communications (GCI3 SAC)*, 2013.
- [6] ITU-T Rec. G.9700, "Fast access to subscriber terminals (FAST) - Power spectral density specification," 2013.
- [7] J.-S. Pang, G. Scutari, D. P. Palomar, and F. Facchinei, "Design of cognitive radio systems under temperature-interference constraints: A variational inequality approach," *Signal Processing, IEEE Transactions on*, vol. 58, no. 6, pp. 3251–3271, 2010.
- [8] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, "Iterative water-filling for Gaussian vector multiple-access channels," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 145–152, 2004.
- [9] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels," *IEEE Transactions on Information Theory*, vol. 54, no. 7, pp. 2868–2878, 2008.
- [10] ETSI Standard, "Ts 101 270-1," *Transmission and Multiplexing (TM); Access transmission systems on metallic access cables; Very high speed Digital Subscriber Line (VDSL); Part 1: Functional requirements*, 2003.
- [11] Broadband Forum, "TR-285 Broadband Copper Cable Models," 2015.
- [12] L. Humphrey, "On VHF background noise assumptions for G:FAST," 2013.