BAYESIAN MULTI-TARGET TRACKING WITH SUPERPOSITIONAL MEASUREMENTS USING LABELED RANDOM FINITE SETS

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ABSTRACT

In this paper we present a general solution for multi-target tracking problems with *superpositional measurements*. In a superpositional sensor model, the measurement collected by the sensor at each time step is a superposition of measurements generated by each of the targets present in the surveillance area. We use the Bayes multi-target filter with Labeled Random Finite Set (RFS) in order to jointly estimate the number of targets and their trajectories. We propose an implementation of this filter using Sequential Monte Carlo (SMC) methods with an efficient multi-target sampling strategy based on the Approximate Superpositional Cardinalized Probability Hypothesis Density (CPHD) filter.

1. INTRODUCTION

Superpositional sensors are an important class of pre-detection sensor models which arise in a wide range of joint detection and estimation problems. In a superpositional sensor model, the measurement at each time step is a superposition of measurements generated by each of the targets present [1,2]. This is a different setting compared to the usual multi-target tracking problems using detections model [3–11].

In [1], Mahler derived a superpositional Cardinalized Probability Hypthesis Density (CPHD) filter, as an approximation to the Bayes multi-target filter for superpositional sensor. However, this approximate filter is numerically intractable due to the combinatorial nature of the solution. The first tractable superpositional approximate CPHD (SA-CPHD) filter was proposed in [2], and successfully demonstrated on an RF tomography application. The technique was also extended to multi-Bernoulli and a combination of multi-Bernouli and CPHD [12]. These filters, however, are not multi-target trackers because they rest on the premise that targets are indistinguishable. Moreover, they require at least two levels of approximations: analytic approximations of the Bayes multi-target filter and particles approximation of the obtained recursion.

Inspired by [1,2], this paper proposes a multi-target tracker for superpositional sensors which estimates target tracks and requires only one level of approximation. Our formulation is based on the class of labelled Random Finite Sets (RFSs) [13–15], which enables the estimation of target tracks as well as direct particle approximation of the (labeled) Bayes multi-target filter. To mitigate the depletion problem arising from sampling in high dimensional space we propose an efficient multi-target sampling strategy using the superpositional CPHD filter [1,2]. While both the CPHD and labeled RFS solutions require particle approximation, the latter has the advantage that it does not require particle clustering for the multi-target state estimation. Simulation results are not reported here due to space limitation. However, preliminary results in a challenging closely-spaced multi-target scenario using radar power measurements with low signal-to-noise (SNR) ratio [16–19] verify the applicability of the proposed approach and can be found in [20].

The paper is organised as follows: in Section 2 we recall multi-target Bayes filter, some definitions for Labeled RFSs, and superpositional CPHD filter. In Section 3 we discuss the rationale behind our approach, provide details on how to use the approximate CPHD filter on multi-target particles at time $k-1$, and present the multi-target particle tracker with Vo-Vo proposal distribution [1]. Conclusions and future directions are discussed in Section 4.

2. BACKGROUND

Suppose that at time k, there are N_k objects with their states denoted by $x_{k,1}, \ldots, x_{k,N_k}$, each taking values in a state space $\mathcal{X}$. An RFS is a random variable $X_k = \{x_{k,1}, \ldots, x_{k,N_k}\}$ that takes values in $\mathcal{F}(\mathcal{X})$, the space of all finite subsets of $\mathcal{X}$. Mahler's Finite Set Statistics (FISST) provides powerful mathematical tools for dealing with RFSs [4] based on a notion of integration/density that is consistent with point process theory [7]. In this work we are interested in the multi-object filtering density, which can be propagated by the *multi-object Bayes filter* as detailed in [4].

[1]The Vo-Vo density was originally called the Generalized Labeled Multi-Bernoulli density. However, we follow Mahler's last book [5] and call this the Vo-Vo density.

2.1. Labeled RFS

The labeled RFS model incorporates a unique label in the object's state vector to identify its trajectory [4]. The (single-target) state space $\mathcal{X}$ is a Cartesian product $\mathbb{X}\times\mathbb{L}$, where $\mathbb{X}$ is the feature/kinematic space and $\mathbb{L}$ is the (discrete) label space. A finite subset $\mathbf{X}$ of $\mathbb{X}\times\mathbb{L}$ has distinct labels if and only if $\mathbf{X}$ and its labels $\{\ell : (x,\ell) \in \mathbf{X}\}$ have the same cardinality [13]. A recently developed class of labeled RFS, known as the Vo-Vo distributions [5, 13, 14], is a conjugate prior that is also closed under the Chapman-Kolmogorov equation under the standard multi-target model. Let $\mathcal{L} : \mathbb{X}\times\mathbb{L} \to \mathbb{L}$ be the projection $\mathcal{L}((x,\ell)) = \ell$, and $\Delta(\mathbf{X}) \triangleq \delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)$ denote the *distinct label indicator*. A Vo-Vo multi-target density takes the form [13, 14]:

$$\boldsymbol{\pi}(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{c\in\Xi} w^{(c)}(\mathcal{L}(\mathbf{X})) \left[p^{(c)}\right]^{\mathbf{X}}, \tag{1}$$

where Ξ is a discrete index set. The density in (1) is a mixture of multi-object exponentials. In [13, 14] an analytic solution to the labeled version of the multi-target Bayes filter, known as the Vo-Vo filter [5], was derived using labeled RFS.

2.2. Superpositional Approximate CPHD

In a superpositional sensor model, the measurement z is a non-linear function of the sum of the contributions of individual targets and noise [1]. The SA-CPHD filter presented in [2] is an approximation to the multi-target Bayes filter for a multi-target likelihood function of the form

$$g_k(z|X) = \mathcal{N}_R\left(z - \sum_{x\in X} h(x)\right) \tag{2}$$

where $\mathcal{N}_R(\cdot)$ is a zero-mean Gaussian distribution with covariance R. Similar to the standard CPHD filter [4], the SA-CPHD filter [1, 2] is an analytic approximation of the Bayes multi-target filter [20, eqs.(2)-(3)] based on independently and identically distributed (iid) cluster RFS.

We are interested in using the approximate CPHD for superpositional measurements of the following form:

$$\tilde{z}_k = \left|\sum_{\mathbf{x}\in\mathbf{X}_k} \tilde{h}(\mathbf{x})\right|^2 + n_k \tag{3}$$

where $\mathbf{X}_k$ is the labeled multi-target state at time k, $n_k \sim \mathcal{N}(0,\sigma_n^2)$ is zero-mean white Gaussian noise, and $\tilde{h}(\mathbf{x})$ is a possibly nonlinear function of the single state vector $\mathbf{x} = (x,\ell)$. The model in eq. (3) can be used to approximate the radar power measurement model commonly used in Track-Before-Detect (TBD) problems [16–19] assuming a Gaussian. Obviously the model in eq. (3) is a strong approximation of the TBD model. However, it allows the use of a superpositional CPHD update step which can be used to evaluate measurement updated PHD $v(x)$ and cardinality distribution $\rho(n)$ for the targets set. In turns, the information in the updated $v(x)$ and $\rho(n)$, along with the targets labels from the previous step and birth model [13, 20], can be used to construct an approximate posterior density using the Vo-Vo distribution in (1). Finally, the obtained approximate posterior can be used as a proposal distribution for a multi-object particle filter. Following [1, 2], standard CPHD formulas are used for the prediction step, while the SA-CPHD update step is given by:

$$\rho_k(n) = \rho_{k|k-1}(n) \frac{\mathcal{N}_{\Sigma_r+\Sigma^n_{k|k-1}}\left(\tilde{z}_k - n\hat{\mu}_{k|k-1}\right)}{\mathcal{N}_{\Sigma_r+\Sigma_{k|k-1}}\left(\tilde{z}_k - N_{k|k-1}\hat{\mu}_{k|k-1}\right)} \tag{4}$$

$$v_k(x) = v_{k|k-1}(x) \frac{\mathcal{N}_{\Sigma_r+\Sigma^o_{k|k-1}}\left(\tilde{z}_k - \tilde{h}(x) - \mu^o_{k|k-1}\right)}{\mathcal{N}_{\Sigma_r+\Sigma_{k|k-1}}\left(\tilde{z}_k - N_{k|k-1}\hat{\mu}_{k|k-1}\right)} \tag{5}$$

Details on eqs. (4)-(5) can be found in [2]. In the sequel we describe how the updated PHD and cardinality distribution from the SA-CPHD can be used to design an efficient proposal distribution for multi-target tracking using particles.

3. BAYESIAN MULTI-TARGET TRACKING FOR SUPER-POSITIONAL SENSOR

In general, the propagation of the multi-target posterior involves the evaluation of multiple set integrals and hence the computational requirement is much more intensive than single-target filtering. Particle filtering techniques permits recursive propagation of the set of weighted particles that approximate the posterior. Following [7], suppose that at time $k-1$, a set of weighted particles $\{w^{(i)}_{k-1}, \mathbf{X}^{(i)}_{k-1}\}_{i=1}^{N_p}$ representing the multi-target posterior $\boldsymbol{\pi}_{k-1|k-1}$ is available, i.e.

$$\boldsymbol{\pi}_{k-1}(\mathbf{X}) \approx \sum_{i=1}^{N_p} w^{(i)}_{k-1}\delta(\mathbf{X};\mathbf{X}^{(i)}_{k-1}) \tag{6}$$

Note that $\delta(\cdot;\mathbf{X}^{(i)}_{k-1})$ denotes the Dirac-delta centered at $\mathbf{X}^{(i)}_{k-1}$. The particle filter approximates the multi-target posterior $\boldsymbol{\pi}_k$ by a new set of weighted particles $\{w^{(i)}_k, \mathbf{X}^{(i)}_k\}_{i=1}^{N_p}$ as follows

Multi-target Particle Filter

For time $k \geq 1$

- For $i = 1,\ldots,N_p$ sample $\tilde{\mathbf{X}}^{(i)}_k \sim \mathbf{q}(\cdot|\mathbf{X}^{(i)}_{k-1}, z_k)$ and set

$$\tilde{w}^{(i)}_k = \frac{g_k(z_k|\tilde{\mathbf{X}}^{(i)}_k)\mathbf{f}_{k|k-1}(\tilde{\mathbf{X}}^{(i)}_k|\mathbf{X}^{(i)}_{k-1})}{\mathbf{q}_k(\tilde{\mathbf{X}}^{(i)}_k|\mathbf{X}^{(i)}_{k-1}, z_k)} w^{(i)}_{k-1} \tag{7}$$

- Normalize the weights: $\tilde{w}^{(i)}_k = \frac{\tilde{w}^{(i)}_k}{\sum_{i=1}^{N_p}\tilde{w}^{(i)}_k}$

Resampling Step

- Resample $\{\tilde{w}^{(i)}_k, \tilde{\mathbf{X}}^{(i)}_k\}_{i=1}^{N_p}$ to get $\{w^{(i)}_k, \mathbf{X}^{(i)}_k\}_{i=1}^{N_p}$

The importance sampling density $\mathbf{q}_k(\cdot|\mathbf{X}_{k-1}, z_k)$ is a multi-target density and $\tilde{\mathbf{X}}_k$ is a sample from an RFS. The main practical problem with the multi-target particle filter is the need to perform importance sampling in very high dimensional spaces if many targets are present. A naive choice of importance density such as the transition density will typically lead to an algorithm whose efficiency decreases exponentially with the number of targets for a fixed number of particles [7]. In the following we design an efficient multi-target proposal distribution $\mathbf{q}_k(\cdot|\mathbf{X}_{k-1}, z_k)$ using the SA-CPHD [1, 2] and the Vo-Vo distribution [13, 14].

3.1. Superpositional CPHD proposal

In this section we discuss how to use the SA-CPHD filter to construct a relatively inexpensive and accurate proposal $\mathbf{q}(\cdot|\mathbf{X}_{k-1}, z_k)$ using the Vo-Vo density. The basic idea is to obtain the updated PHD $v_k(x)$ and cardinality distribution $\rho_k(n)$ at time k from the SA-CPHD filter and multi-target particles at time $k-1$. We then construct a proposal distribution $\mathbf{q}(\cdot|\mathbf{X}_{k-1}, z_k)$ that exploits the approximate posterior information contained in both the cardinality distribution $\rho_k(\cdot)$ and the state samples from $v_k(\cdot)$.

Assume a particles representation $\left\{\mathbf{X}_{k-1}^{(i)}, w_{k-1}^{(i)}\right\}_{i=1}^{N_p}$ of the posterior distribution $\boldsymbol{\pi}_{k-1}(\mathbf{X})$ is available at time $k-1$. Then, the cardinality distribution $\rho_{k-1}(\cdot)$ and the PHD $v_{k-1}(\cdot)$ at time $k-1$ are given by [7]:

$$\begin{aligned} \rho_{k-1}(n) &\propto \sum_{i:\left|\mathbf{X}_{k-1}^{(i)}\right|=n} w_{k-1}^{(i)} \\ v_{k-1}(x) &= \sum_{i=1}^{N_p} \sum_{\ell\in\mathcal{L}\left(\mathbf{X}_{k-1}^{(i)}\right)} w_{k-1}^{(i)}\, \delta\left(x; x_{k-1,\ell}^{(i)}\right) \end{aligned}$$

where $x_{k-1,\ell}^{(i)}$ denotes the kinematic part of each $\left(x_{k-1}^{(i)}, \ell_{k-1}^{(i)}\right) \in \mathbf{X}_{k-1}^{(i)}$. The SA-CPHD is then used to obtain the updated cardinality distribution $\rho_k(\cdot)$ and and PHD $v_k(\cdot)$ using the measurement z_k collected at time k. Unlike in standard unlabeled CPHD filtering, there is a natural labeling/clustering of particles due to the existing labels at time $k-1$ and the chosen iid cluster process with implicit cluster labels for the birth model. In fact we have:

$$v_k(x) = \sum_{i=1}^{N_p} \sum_{\ell\in\mathcal{L}\left(\mathbf{X}_k^{(i)}\right)} w_k^{(i)}\, \delta\left(x; x_{k,\ell}^{(i)}\right)$$

Then we can rewrite the PHD according to target labels as

$$\begin{aligned} v_k(x) &= \sum_{\ell\in\mathbb{L}_{0:k}} v_{k,\ell}(x) \\ v_{k,\ell}(x) &= \sum_{i=1}^{N_p} \sum_{\ell'\in\mathcal{L}\left(\mathbf{X}_k^{(i)}\right)} \delta_\ell(\ell')\, w_k^{(i)}\, \delta\left(x; x_{k,\ell}^{(i)}\right) \end{aligned}$$

where $v_{k,\ell}(x)$ is the contribution to the PHD of track ℓ. Note that the above is not the PHD of a labeled RFS but the PHD mass from a specific label representing a survival or birth target. This means that at time k we can extract $|\mathbb{L}_{0:k}|$ clusters of particles from the posterior PHD. Here, $|\mathbb{L}_{0:k}|$ is the set of labels at time k, which is built recursively as the union of surviving and newborn labels [13]. Furthermore, a continuous approximation to each cluster can be obtained by evaluating sample mean and covariance for a Gaussian approximation,

$$v_{k,\ell}(x) = p_k^+(\ell)\, \mathcal{N}\left(x; \mu_{k,\ell}, Q_{k,\ell}\right) \tag{8}$$

where $p_k^+(\ell)$ is the PHD mass of the ℓ^{th} cluster. The obtained posterior cardinality and posterior target clusters can be used to construct a proposal distribution $\mathbf{q}(\cdot|\mathbf{X}_{k-1}, z_k)$ using the Vo-Vo density in (1).

3.2. Vo-Vo Proposal Distribution

We seek a proposal distribution that matches the CPHD cardinality exactly while exploiting the weights of individual labeled target clusters as computed from the approximate posterior PHD. A single component Vo-Vo density can be used,

$$\mathbf{q}_k(\mathbf{X}_k|\mathbf{X}_{k-1}, z_k) = \Delta(\mathbf{X}_k)\omega(\mathcal{L}(\mathbf{X}_k))\, [p(\cdot)]^{\mathbf{X}_k} \tag{9}$$

We now specify the component weight $\omega(\mathcal{L}(\mathbf{X}_k))$ and the multi-object exponential $[p(\cdot)]^{\mathbf{X}_k}$. The single-target densities $p(\cdot)$ are obtained directly from the Gaussian clusters,

$$p_k(x, \ell) = \mathcal{N}(x; \mu_{k,\ell}, Q_{k,\ell}),\ \ell \in \mathbb{L}_{0:k} \tag{10}$$

The weight $\omega(\mathcal{L}(\mathbf{X}_k))$ is then chosen to preserve the CPHD cardinality distribution, and for a given cardinality, to sample labels proportionally to the product of the posterior PHD masses of any possible label combinations. Specifically, from the posterior PHD mass of each cluster $p_k^+(\ell)$ we construct approximate "existence" probabilities as:

$$r_k^+(j) = \frac{p_k^+(j)}{\sum_{\ell=1}^{|\tilde{\mathbb{L}}_{0:k}|} p_k^+(\ell)},\ \ j = 1, \ldots, |\tilde{\mathbb{L}}_{0:k}| \tag{11}$$

$$\tilde{\mathbb{L}}_{0:k} = \bigcup_{i=1}^{N_p} \mathcal{L}(\mathbf{X}_{k-1}^{(i)}) \bigcup \mathbb{L}_k \tag{12}$$

where $\tilde{\mathbb{L}}_{0:k}$ is the set of labels from resampled particles at time $k-1$ and newborn targets at time k. The weight $\omega(\mathcal{L}(\mathbf{X}_k))$ in (9) is then defined as:

$$\omega(\mathcal{L}(\mathbf{X}_k)) = \rho_k(|\mathcal{L}(\mathbf{X}_k)|)\frac{[r_k^+(\cdot)]^{\mathcal{L}(\mathbf{X}_k)}}{e_{|\mathcal{L}(\mathbf{X}_k)|}(R_k)} \quad (13)$$

where $R_k = \{r_k^+(j)\}_{j\in\tilde{\mathbb{L}}_{0:k}}$ denotes the set of "existence" probabilities for all current tracks and $e_n(\cdot)$ is the elementary symmetric function of order n. The construction of the proposal in (9) leads to a simple and efficient strategy for sampling. Specifically, to sample from (9) we: (1) sample the cardinality $\left|\mathbf{X}_k^{(i)}\right|$ of the newly proposed particle according to the distribution $\rho_k(n)$; (2) sample $\left|\mathbf{X}_k^{(i)}\right|$ labels $\mathcal{L}(\mathbf{X}_k^{(i)})$ from $\tilde{\mathbb{L}}_{0:k}$ using the distribution defined by $\frac{[r_k^+(\cdot)]^{\mathcal{L}(\mathbf{X}_k)}}{e_{|\mathcal{L}(\mathbf{X}_k)|}(R_k)}$; (3) for each $\ell \in \mathcal{L}(\mathbf{X}_k^{(i)})$ we sample the kinematic part $x_{k,\ell}^{(i)}$ from $p_k(\cdot,\ell) = \mathcal{N}(\cdot;\mu_{k,\ell},Q_{k,\ell})$.

Multi-Target Particle Filter with Vo-Vo Proposal Distribution

Initialize particles $\mathbf{X}_0^{(i)} \sim p_0(\cdot)$
For $k = 1,\ldots,K$

- For $i = 1,\ldots,N_p$
  - Sample the cardinality for the new particle $\left|\mathbf{X}_k^{(i)}\right| \sim \rho_k(n)$
  - Sample the set of labels $\mathcal{L}\left(\mathbf{X}_k^{(i)}\right)$ uniformly from $\tilde{\mathbb{L}}_{0:k}$
  - For each $\ell \in \mathcal{L}\left(\mathbf{X}_k^{(i)}\right)$ generate $x_{k,\ell}^{(i)} \sim \mathcal{N}\left(\cdot;\mu_{k,\ell},Q_{k,\ell}\right)$
  - For $j = 1,\ldots,N_p$ evaluate the transition kernel
  $$\mathbf{f}_{k|k-1}\left(\mathbf{X}_k^{(i)}|\mathbf{X}_{k-1}^{(j)}\right) = \prod_{\ell\in\mathcal{L}_D^{(i)}}(1-p_S(\ell))\prod_{\ell\in\mathcal{L}_{NB}^{(i)}}(1-p_B(\ell))\times \prod_{\ell\in\mathcal{L}_S^{(i)}} p_S(\ell)\, f_{k|k-1}(x_{k,\ell}^{(i)}|x_{k-1,\ell}^{(j)}) \prod_{\ell\in\mathcal{L}_B^{(i)}} p_B(\ell)\, p_B(x_{k,\ell}^{(i)})$$
  - Evaluate the proposal distribution
  $$\mathbf{q}_{k|k-1}\left(\mathbf{X}_k^{(i)}|\mathbf{X}_{k-1},z_k\right) = \omega\left(\mathcal{L}(\mathbf{X}_k^{(i)})\right)\prod_{\ell\in I}\mathcal{N}\left(x_k^{(i)};\mu_{k,\ell},Q_{k,\ell}\right)$$
  - Evaluate the multi-object likelihood $g_k\left(z_k|\mathbf{X}_k^{(i)}\right)$
  - Update the particle weight $w_k^{(i)}$ using
  $$w_k^{(i)} = \frac{g_k(z_k|\mathbf{X}_k^{(i)})\sum_{j=1}^{N_p}\mathbf{f}_{k|k-1}(\mathbf{X}_k^{(i)}|\mathbf{X}_{k-1}^{(j)})w_{k-1}^{(j)}}{\mathbf{q}_k(\mathbf{X}_k^{(i)}|\mathbf{X}_{k-1},z_k)}$$
- Normalize the weights and resample as usual

Notice that in the pseudo-code we require the evaluation of the multi-target transition sum kernel $\mathbf{f}_{k|k-1}(\mathbf{X}_k^{(i)}|\mathbf{X}_{k-1})$ with respect to the previous set of particles. This is due to the fact that for each new particle $\mathbf{X}_k^{(i)}$ the cardinality $\left|\mathbf{X}_k^{(i)}\right|$ and set of labels $\mathcal{L}\left(\mathbf{X}_k^{(i)}\right)$ are not sampled directly from the previous particle $\mathbf{X}_{k-1}^{(i)}$. This obviously increases the computational load but generally leads to improved performance.

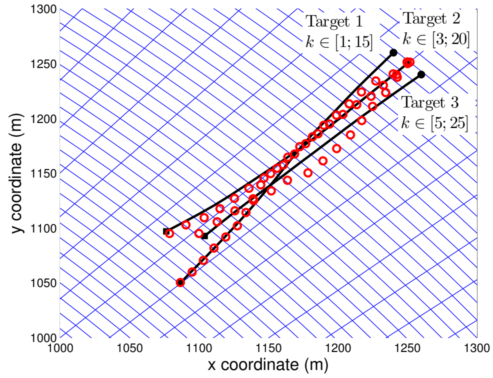


Fig. 1. Simulated scenario and estimated trajectories. The radar is positioned at the Cartesian origin and there are 3 closely spaced targets moving towards the origin.

Furthermore, efficient approximation techniques can be used to mitigate the computational load due to the sum kernel problem [18]. We refer the reader to [20] for additional details.

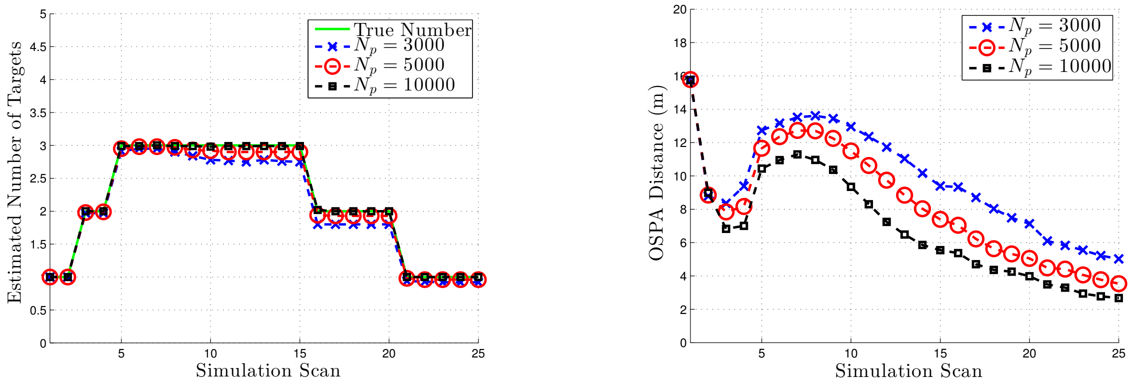
Simulation results for a multi-target TBD problem with closely spaced targets are reported in Figs. 1-2. Due to space limitation, we only report the simulation results for SNR $= 10dB$. Additional details including the single-target dynamics and the TBD measurement model can be found in [20]. Notice that a comparison between the proposed approach and the SA-CPHD filter would be unfair. In fact, the SA-CPHD filter was developed for a superpositional measurement model with additive Gaussian noise, while the Radar TBD is a superpositional model with Rayleigh noise. Thus, while the SA-CPHD is a good approximation for designing a proposal distribution for this case, it cannot handle this type of measurement model on its on. From Figs. 2(a)-2(b) we can verify the applicability of the proposed approach and performance improvement for increasing number of particles.

4. CONCLUSIONS AND FUTURE RESEARCH

In this paper we discussed a general solution for multi-target tracking with *superpositional measurements*. The proposed approach aims at evaluating the multi-target Bayes filter using SMC methods. The critical enabling step was the definition of an efficient proposal distribution based on the Approximate CPHD filter for *superpositional measurements*. Preliminary simulation results verify the applicability of the proposed approach. Future research will focus on validating the approach for challenging multi-target TBD problems with low SNR.

REFERENCES

[1] R. Mahler and A. El-Fallah, "An approximate CPHD



(a) Monte Carlo results: No. of Targets for SNR $= 10dB$

(b) Monte Carlo results: OSPA Distance for SNR $= 10dB$

Fig. 2. Monte Carlo results for SNR $= 10dB$

filter for superpositional sensors," in *Proc. SPIE*, vol. 8392, 2012, pp. 83 920K–83 920K–11.

[2] S. Nannuru, M. Coates, and R. Mahler, "Computationally-Tractable Approximate PHD and CPHD Filters for Superpositional Sensors," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 7, no. 3, pp. 410–420, June 2013.

[3] Y. Bar-Shalom and T. Fortmann, *Tracking and Data Association*. Academic Press, 1988.

[4] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Artech House, 2007.

[5] ——, *Advances in Statistical Multisource-Multitarget Information Fusion*. Artech House, 2014.

[6] B.-N. Vo and W.-K. Ma, "The Gaussian mixture probability hypothesis density filter," *IEEE Trans. Signal Processing*, vol. 54, no. 11, pp. 4091–4104, 2006.

[7] B.-N. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo methods for Multi-target filtering with Random Finite Sets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 4, pp. 1224–1245, 2005.

[8] B.-N. Vo, B.-N. Vo, and A. Cantoni, "Analytic implementations of the Cardinalized Probability Hypothesis Density filter," *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3553–3567, 2007.

[9] B.-T. Vo, B.-N. Vo, and A. Cantoni, "The Cardinality Balanced Multi-target Multi-Bernoulli filter and its implementations," *IEEE Trans. Sig. Proc.*, vol. 57(2): 409-423, 2009.

[10] G. Battistelli, L. Chisci, F. Papi, A. Benavoli, A. Farina, and A. Graziano, "Optimal flow models for multiscan data association," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 47, no. 4, pp. 2405–2422, 2011.

[11] G. Battistelli, L. Chisci, S. Morrocchi, F. Papi, A. Farina, and A. Graziano, "Robust multisensor multitarget tracker with application to passive multistatic radar tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 4, pp. 3450–3472, 2012.

[12] S. Nannuru and M. Coates, "Hybrid multi-Bernoulli CPHD filter for superpositional sensors," in *SPIE Defense + Security*, I. S. for Optics and Photonics, Eds., June 2014, pp. 90 910D–90 910D.

[13] B.-T. Vo and B.-N. Vo, "Labeled Random Finite Sets and Multi-Object Conjugate Priors," *IEEE Trans. Sig. Proc.*, vol. 61(13), pp. 3460–3475, 2013.

[14] B.-N. Vo, B.-T. Vo, and D. Phung, "Labeled Random Finite Sets and the Bayes Multi-Target Tracking Filter," *IEEE Trans. Sig. Proc.*, vol. 62(24), pp. 6554–6567.

[15] F. Papi, B.-N. Vo, B.-T. Vo, C. Fantacci, and M. Beard, "Generalized Labeled Multi-Bernoulli Approximation of Multi-Object Densities," *arXiv preprint*, 2014, arXiv:1412.5294.

[16] Y. Boers and J. N. Driessen, "Multitarget Particle Filter Track-Before-Detect application," *IEE Proc. on Radar, Sonar and Navigation*, vol. 151, pp. 351–357, 2004.

[17] M. Bocquel, F. Papi, M. Podt, and Y. Driessen, "Multitarget Tracking With Multiscan Knowledge Exploitation Using Sequential MCMC Sampling," *IEEE J. Selected Topics in Sig. Proc.*, vol. 7(3), pp. 532–542, 2013.

[18] F. Papi, M. Bocquel, M. Podt, and Y. Boers, "Fixed-Lag Smoothing for Bayes Optimal Knowledge Exploitation in Target Tracking," *Signal Processing, IEEE Transactions on*, vol. 62, no. 12, pp. 3143–3152, June 2014.

[19] F. Papi, V. Kyovtorov, R. Giuliani, F. Oliveri, and D. Tarchi, "Bernoulli Filter for Track-Before-Detect using MIMO Radar," *IEEE Sig. Proc. Letters*, 2014.

[20] F. Papi and D.-Y. Kim, "A Particle Multi-Target Tracker for Superpositional Measurements using Labeled Random Finite Sets," *arXiv preprint arXiv:1501.02248*.